March 13, 1973

Dr. Andrzej Staruszkiewicz
UNIWERSYTET JAGIELLONSKI
INSTYTUT FIZYKI
30-059 Krakow 16, ul. Reymonta 4
POLAND

Dear Dr. Staruszkiewicz:

This will acknowledge and thank you for your entry in our essay competition for 1973. The winners will be announced May 15. You willbbe notified of the results.

Sincerely,

Secretary

BMPelchat/bmp

Dr Andrzej Staruszkiewicz

UNIWERSYTET JAGIELLOŃSKI
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30-059 KRAKÓW 16, ul. Reymonta 4

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George M. Rideout, President
Gravity Research Foundation
New Boston, N. H. 03070

Mr President,

I would like to compete for 1973 Awards for Essays on Gravitation; I submit the enclosed essay "On Retardation Effects in the Planetary System", with two copies. The essay has not been published and will not be submitted for publication before May 15, 1973.

Sincerely yours

A. Staruszkiewicz

Dr Andrzej Staruszkiewicz

1973

On Retardation Effects in the Planetary System

By Andrzej Staruszkiewicz
Institute of Physics, Jagellonian University, Cracow, Poland

Summary

The author has discovered that a system of two bodies interacting by means of retarded forces has an infinite number of nonclassical normal modes of motion. In this essay the physical nature of those modes is described; they turn out to correspond to the oscillations of an external shock between two bodies. Some applications in the theory of planetary motions are suggested.

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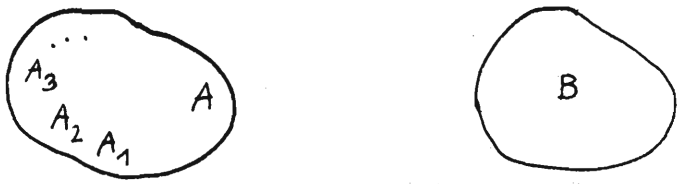
The phenomenon of retardation

It is now virtually certain that gravitation propagates with a finite velocity. Some consequences of this can be seen from the following.

Consider two bodies, A and B, and suppose that a part of A, say A_1, is slightly disturbed. The disturbance will be felt at A_2 after



some finite time; even longer time will elapse before the disturbance reaches B. Clearly, the retarded influence of A_1 on A_2, A_3 etc. is the same phenomenon as the influence of A on B. However, because of immense distances between heavenly bodies it is reasonable to divide the problem of retarded interactions on two parts: /I/ the influence of retardation within a body on the motion of this body; /II/ the influence of retardation in the space between two bodies on the motion of those two bodies. Since the classical work of Lorentz problem /I/,i.e. the problem of radiation reaction,has occupied attention of numerous investigators; it is therefore surprising that the closely related problem /II/ has received little or no attention at all.

Why Laplace was wrong?

Daniel Bernoulli was the first who tried to estimate the velocity of gravitation but the first estimate based on quantitative arguments was given by Laplace[1]. Laplace finds that the velocity

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[1] Oeuvres, vol. IV, p. 355, Paris 1845.

of gravitation cannot be smaller than 10^7c, where c is the velocity of light. A similar estimate was made by Lehmann-Filhes at the end of XIX century: suppose that the gravitational force f propagates with the velocity V; then, if $\underline{x}$ denotes position of a planet with respect to the Sun, the force acting on the planet at the moment t is $f\left(\underline{x}\left(t-\frac{|\underline{x}|}{V}\right)\right)$ rather than $f\left(\underline{x}(t)\right)$. Assuming V to be very large we put

$$f\left(\underline{x}\left(t-\frac{|\underline{x}|}{V}\right)\right)=f\left(\underline{x}-\frac{|\underline{x}|}{V}\dot{\underline{x}}\right)=f(\underline{x})-\frac{|\underline{x}|}{V}\dot{\underline{x}}\cdot \operatorname{grad} f(\underline{x})\,;$$

the additional term cannot fail to produce a secular acceleration of the planet. Comparing this acceleration with the astronomical data one finds $V > 10^6c$.

The source of error is clearly seen: the only effect taken into account is retardation of the static force, while we know from electrodynamics that static force may be accompanied by a force of magnetic type. Calculations of Laplace or Lehmann-Filhes - correctly reinterpreted - show simply that the "magnetic" component of the gravitational force cancels out exactly the first order correction to the static force.

The Einstein-Infeld-Hoffmann approximation

To calculate velocity-dependent corrections we need a field theory of gravitation. Many theories have been proposed but Einstein's theory is certainly the most satisfactory one. It follows from Einstein's theory that gravitation propagates with the velocity of light. Unfortunately, to calculate the retardation effects in Einstein's theory is an impossible task: the two-body problem in this theory is not just difficult; it is not posed, as yet, as an analytically meaningful problem. We have to use some approximation; the only approximation used up to now in the theory of planetary motions is the Einstein-Infeld-Hoffmann method. The EIH method is "admirably adapted to solving all problems related to the motion of slowly moving gravitating bodies"[2]. Nevertheless, some doubts about the method have been recently expressed. Driver[3] says that "the mathematical pitfalls in this "method" are abundant". Quotation marks in this statement are too aggressive perhaps, but doubts are certainly well founded; they centre on the treatment of retardation.

The equations of motion in the EIH method are ordinary, second order differential equations. On the other hand, forces in the planetary system are retarded /or advanced but not instantaneous/; in the derivation of the EIH equations one introduces instantaneous forces in place of the retarded ones expanding all relevant functions in powers of retardation. Such a procedure is objectionable, as can be seen from the example of an oscillator with a retarded force; the relevant equation is

/1/ $$\ddot{x}(t) = -\omega^2 x(t-\tau), \quad 0 < \tau = const.$$

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[2] A. Trautman, Acta Phys. Polon. 33 /1968/ 165.

[3] Phys. Rev. 178 /1969/ 2051.

In the EIH method we assume τ to be small, put $x(t-\tau) = x(t) - \tau\dot{x}(t) + \frac{\tau^2}{2}\ddot{x}(t)$ and obtain the approximate equation $\left[1+\frac{1}{2}(\omega\tau)^2\right]\ddot{x}(t) = -\omega^2 x(t) + \frac{1}{2}\omega^2\tau\dot{x}(t)$, $\omega\tau \ll 1$. One might say that retardation causes two effects: the appearance of a damping force and a small increase of inertia.

Now, the original equation can be solved exactly. Suppose that for $0 \leq t \leq \tau$, $x(t) = \varphi(t)$, where $\varphi(t)$ is a given function; then for $\tau \leq t \leq 2\tau$ the right hand side of /1/ is known and the equation may be solved with the initial data $x(\tau) = \varphi(\tau)$, $\dot{x}(\tau) = \dot{\varphi}(\tau)$. Obviously, the procedure may be repeated for $2\tau \leq t \leq 3\tau$ etc. The initial value problem $x(t) = \varphi(t)$ for $0 \leq t \leq \tau$ is a well posed one[4]. We see that the class of solutions of the well posed initial value problem contains an arbitrary function; moreover, the solutions are as a rule non-analytic and cannot be continuously extended for $t < 0$. It is difficult to see a connection between this class and the EIH solutions which form a two-parameter set of analytic functions. For the simple, purely retarded[5] eq. /1/ the situation is not as bad as it looks. One can show that the EIH solution approximates the exact solution for $t \gg \tau$. Should this be the case for the equations of motion of heavenly bodies, the following picture

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[4] L. E. Elsgolc, Introduction into the Theory of Differential-Difference Equations, Moscow 1964 /in Russian/.

[5] An equation with retarded arguments is said to be of retarded type, if it does not contain the leading derivative taken at the retarded argument, see Elsgolc, op. cit., for details.

would emerge: the EIH equations describe correctly the motion, when all disturbances created in the formative period of the solar system have settled down. This point of view cannot be entirely wrong, but serious difficulties remain: the equations of motion of heavenly bodies are not of retarded type, even if purely retarded forces are assumed; for such equations the existence of a well behaved asymptotic solution in general should not be expected.

The case of nearly circular orbits

The author[6] developed a method which allows to calculate exactly some retardation effects. Actually, the author considered the electromagnetic two-body problem, but the method can be applied to the gravitational case. The essential idea of the method is very simple.

Suppose that two bodies move along a circular orbit; in this case the retardation is constant. Suppose now, that the motion is infinitesimally perturbed. The retardation will be also perturbed but the perturbation is of the second order. Consequently, even for the perturbed motion we have unperturbed i.e. constant retardations - a circumstance which makes the problem analytically tractable.

The equations of the perturbed motion were calculated by the author for two charges of equal mass and by Andersen and von Baeyer[7] for arbitrary masses; Pyragas and Zhdanov[8] calculated

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[6]Acta Phys. Polon. 33 /1968/ 1007.

[7]Phys. Rev. 5D /1972/ 802.

[8]Acta Phys. Polon. B3 /1972/ 585, 599.

equations of motion in the gravitational case but again for equal masses. In all cases half-retarded, half-advanced forces have been assumed but the method can be applied for retarded forces also. In the electromagnetic case the interaction is described by the exact Green function of the Maxwell equations. In the gravitational case the exact treatment of Einstein's equations is impossible; Pyragas and Zhdanov use the approximate Green function; the approximation involved is certainly adequate in the conditions prevailing in the solar system.

The equations are too complex to be discussed here, but the nature of the dynamical problem may be fairly well explained on an example of the half-retarded, half-advanced harmonic oscillator with the equation of motion

$$\ddot{x}(t) + \frac{1}{2}\omega^2\left[x(t-\tau) + x(t+\tau)\right] = 0.$$

We assume retardation to be small: $\omega\tau << 1$. The function $\exp(i\lambda t)$ is a solution provided λ is a root of the characterictic equation $\lambda^2 = \omega^2 \cos\lambda\tau$. For $\tau = 0$ there are two roots $\lambda = \pm\omega$ corresponding to two degrees of freedom of the harmonic oscillator, but for $\tau \neq 0$ there is an infinite number of roots. Two roots are real and for $\tau \to 0$ go over into $+\omega$ and $-\omega$ respectively; all other roots are complex and tend to infinity for $\tau \to 0$. We see again, that even arbitrarily small retardation changes qualitatively the space of solutions: there are infinitely many normal modes associated with the retarded oscillator and only two of them have the classical counterpart!

A similar situation exists in the two-body problem. Two bodies moving on a plane have eight degrees of freedom; the characteristic equation of the two-body problem has eight "classical" solutions and infinitely many "nonclassical" ones. The nonclassical modes of motion are unstable with the half-of-life period much shorter then the revolution period. We do not observe such violent instabilities in the solar system and must conclude that the nonclassical modes are not "switched on". This does not mean, however, that they do not make themselves felt.

Suppose that the two-body system receives an external impulse. The response of the system is described by its Green function; some peculiarities of this function may be again explained on the example of the oscillator. The condition that the unstable modes should be "switched off" determines the Green function which was calculated by the auther[9]. The function is numerically very close to the classical one but analytically it is very different: for $t = \pm n\tau$, $n = 1,2,\ldots$, it has only $2n + 1$ continuous derivatives while the next derivative is discontinuous. Such a behaviour of the response function may seem strange; actually it can be easily understood. Consider again the two--body system and suppose that an external shock is communicated to the body A at $t = 0$; at $t = \tau$ the shock will be felt at B and at $t = 2\tau$ again at A. In this way an external shock, once communicated to the system, never dies out. The nonclassical normal modes of the two-body system are connected with this "ticking" of the initial shock between two bodies.

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[9] Annalen der Physik /Leipzig/ 23 /1969/ 66.

Applications

Two bodies on a nearly circular orbit form the simplest system in which retardation plays a role. Nevertheless many questions remain unsolved. In particular, a well posed initial or boundary value problem is not known. It is reasonable to ask, what can be expected from those difficult investigations?

One result is certain: a deeper understanding and possibly a justification of the EIH method. But there may be other, more interesting applications. There are peculiarities in the planetary system which cannot be explained by the classical mechanics; a notable example is the Titius-Bode law of planetary distances. In a recent review[10] it is concluded that regularities described by the law are of dynamical origin, but could not have been formed during the planet period of the solar system. We think that this conclusion can be challenged because - as far as we know - no one has considered retardation as a source of regularities. It is possible that the "ticking" of an external shock, which is the most characteristic feature of retarded forces, is particularly disruptive for some orbits. Unfortunately, for the time being too little is known to make some quantitative statements.

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[10] M. M. Nieto, Astron. and Astrophys. 8 /1970/ 105.

Andrzej Staruszkiewicz : a biographical sketch

Born January 15, 1940 in Rymanów, Poland[+].

Graduated from the Jagellonian University in 1961.

Ph. D. - Jagellonian University 1965.

1966-67 - research associate at Syracuse University, Syracuse, NY.

From 1967 up to now - adjunkt /lecturer/ of mathematical physics at the Jagellonian University.

Seventeen papers devoted mainly to the many-body problem in the special and general theory of relativity.

Married, one daughter.

[+]It is remarkable that in this very little place another physicist was born: Isaac Rabi.