The thermodynamics of gravitational radiation

Lee Smolin

The Institute for Advanced Study
Princeton, New Jersey 08540

ABSTRACT

I conjecture, and show for a large class of cases, that given a spacelike hypersurface on which is an arbitrary distribution of linearized gravitons and matter, satisfying the positive energy condition, the probability that within any finite time each of the gravitons has scattered from, or been absorbed by the matter is strictly less than one (except for a set of initial configurations of measure zero.) Consequences of this result are: 1) the impossibility of any system containing gravitational radiation reaching thermal equilibrium in a finite time, 2) the absence of an ultraviolet catastrophe for gravitational radiation, 3) the impossibility of measuring accurately the quantum state of the linearized gravitational field and 4) the impossibility of constructing a gravitational wave laser.

In this essay I propose, and prove for a large class of cases a new conjecture concerning the properties of gravitational radiation which, if generally true, has far reaching consequences for the problems of the relationship between gravitation, quantum mechanics and thermodynamics. The statement of this conjecture is as follows:

C) Consider any region, R, of a spacelike Cauchy surface of a spacetime, on which is found some distribution of linearized gravitational radiation, which is described quantum mechanically as consisting of some distribution of free graviton states, and some distribution of matter, satisfying the local positive energy condition. Let $<N>$ be the expected number of gravitons in this initial configuration and let $N_c(t)$ be the

number of events in which one of the gravitons is scattered from or absorbed by the matter within the time t. Then,

$$\frac{N_c(t)}{<N>} < 1$$

for all time t to the future of the initial configuration, (except for a special set of configurations of measure zero in the space of initial configurations for which there are precise correlations between the gravitons and the matter such as would arise if the initial configuration is the time reverse of one which evolved from a distribution with no gravitational radiation present.)

In particular, except for situations in which the gravitational radiation is strictly monocromatic the probability that all of the gravitons will have scattered or been absorbed by the matter will be much less than one.

Before discussing the motivation and support for this conjecture I would like to indicate what some of its consequences are. (More details are given in the indicated references.) I begin with the consequences for thermodynamics.

1) Given the positive energy conditions, and any initial distribution of matter and gravitational radiation, a state of thermal equilibrium involving both the matter and the gravitational radiation will not be reached in any finite time (unless the initial distribution is to begin with an equilibrium distribution.)[1,2]

2) Consequently, the notion of thermodynamic equilibrium for any system is only an approximate one which is applicable to the extent that coupling to the radiative modes of the gravitational field may be ignored. While this is a very good approximation in most circumstances this could be a problem for discussions of thermal equilibrium between black holes and radiation or in the early universe.

3) As is true of processes in any system which is away from a state of thermal equilibrium, processes in which gravitational radiation is emited or scattered by matter must be considered to be, from the point of view of statistical thermodynamics, *irreversible* because processes involving gravitational radiation are more likely to bring the system closer to a state of equilibrium than away from it. However, processes involving

gravitational radiation differ from other irreversible processes in that as the state of equilibrium cannot be reached in a finite time from any configuration; all interactions involving gravitational radiation are irreversible in the sense of statistical thermodynamics. (The only exception being in those cases in which the system is artificially started off in an equilibrium distribution.) In particular it is possible to assign an entropy to any distribution of gravitational radiation which is a measure of the unlikeness of the occurence of interactions in which the energy carried by the gravitational radiation is entirely transferred to matter degrees of freedom.[1]

In addition, there are several important consequences for the quantum theory of gravity.

4) Because a state of thermal equilibrium involving gravitational radiation cannot be reached in a finite time from a generic initial configuration there is no ultraviolet catastrophe for gravitational radiation[2]. That is to say, we cannot argue, as did Planck and Einstein for the case of the electromagnetic field, from the existence of an equilibrium state involving radiation and matter to the necessity that the energy of the field is carried and transferred to matter in quanta satisfying $E = \hbar \omega$ [3]. This circumstance is particularly important as, given the unlikeleyhood of any experimental observation of gravitons in the forseeable future, such an argument, if possible, would have been the only possible way of arguing that not to quantize the gravitational field is in direct conflict with observation.

5) According to the information-theoretic definition of entropy an increase in the entropy of a system involves an equal loss of information which is accessible to macroscopic observers concerning the microstate of that system. Thus, the reader may wonder whether the irreversibility associated with the production of gravitational radiation in 3) above is also associated with a loss of information.

There is, indeed a loss of information associated with the emission of gravitational radiation because, if the conjecture is true, it is impossible to construct aconfiguration of matter in any region of spacetime that could be used as a detector to determine the precise quantum state of linearized gravitational radiation in that region[1]. The reason is that, no matter how the detector is configured, by the conjecture some

- 4 -

proportion of the gravitons carried by the wave will pass through it without interacting with it. Thus different pure quantum states with the same average spectral distribution will not be distinguishable by any configuration of detectors. Indeed, it is clear that the impossibility of constructing detectors to determine the quantum state of gravitational radiation in any given region corresponds to the impossibility of arranging matter in that region so that the probability that all of the free gravitons interact at least once with the matter is close to one, and that, therefor both measures of irreversibility, the thermodynamic based on the impossibility of recovering energy carried by gravitational radiation and the information-theoretic, based on the impossibility of measuring the precise quantum state of the radiation coincide.

6) In particular, experiments in which coherent states of gravitational radiation are produced, or in which such states could be distinguished by quantum correlation effects from chaotic states with the same average spectral distributions are impossible.

I turn now to a discussion of the conjecture itself. I will here sketch a proof of the conjecture for a large class of situations. The details are contained in papers that have appeared[1,2] or will shortly[4].

We begin by showing that it is imposible to construct, out of uniform material that satisfies the positive energy condition, a good conductor or absorber for gravitational radiation. Consider a uniform slab of material of density ρ on which gravitational radiation of wavelength λ is incident and let the depth of the material in the direction of incidence be L. Assume that $L > \lambda$ and $L > R_{Schw.} = (\frac{4\pi}{3}G\rho)^{-\frac{1}{2}}$ If $L(\lambda)_{abs.}$ is the absorption length or skin depth of the radiation in the slab then

$$L(\lambda)_{abs.} > L \qquad (1)$$

Thus, not all of the gravitational radiation can be absorbed or reflected by the slab. The proof of equation (1) is by a series of calculations in which the various mechanics for absorption of gravitational radiation by matter are considered. These include classical absorption by a uniform material[1,2,5,6], conversion to electromagnetic radiation in a constant electric or magnetic field[1] or in the field or a charged black hole[4], and quantum absorption by the analogue of the photoelectric effect or by the

conversion of gravitons into phonons[1]. Furthermore in each case one finds either a strong wavelength dependence, such as in the case of classical absorption

$$\frac{L}{L(\lambda)_{abs.}} = (\frac{\lambda}{L})^2(\frac{R_{Schw.}}{L})(\frac{v^3}{c^3})$$ (2)

or else that there is a small numerical constant multiplying $L(\lambda)_{abs.}$ in the inequality. For example, in the case of absorption by conversion in a constant electromagnetic field (which is the most favorable case) one finds that the maximum energy loss by the gravitational wave is approximately .012 of the energy carried by the wave.

Thus, except for the case of monocromatic radiation, it is not possible for more than a small fraction of the gravitons to interact during their passage through the slab.

We now show, for regions of static spacetimes, without apparent horizons, which contain some large number of particles which can serve as absorbers or scatterers of gravitational radiation that the conjecture is true. The particles may be individual atoms or uniform blocks of material, what we need to assume about each of them is that they each have a crosssection σ_i to scatter or absorb gravitational radiation. For example, for atoms which will absorb gravitons by ionization,

$$\sigma_i = \alpha^2(a_0)(\frac{Gm_e}{c^2})\frac{\lambda_{grav.}}{\lambda_{electron}} + O(\frac{a_0}{\lambda_{grav.}})$$ (3)

It is sufficient to show that not all gravitons emitted in spherical modes from any point p of R will be absorbed during their passage through R. (By a spherical mode we mean one which is spherical in a small neighborhood of p.) We will label the wavefronts of such a spherical mode by $r = (A/4\pi)^{\frac{1}{2}}$, where A is the area of the wavefront. Then the total probability that a graviton emitted in a spherical mode from p will be absorbed is,

$$P = \sum_i \frac{\sigma_i}{4\pi r_i^2}$$ (4)

Now let us consider a wavefront labeled by r' which completely encloses R, and let us call the region it encloses R'. Now, R' may contain some

particles not in R so $P'>P$. We now replace the distribution of particles in R' with a distribution which is spherically symmetric around p by moving the particles just as long as they are moved to positions of equal or smaller r. Then the probability, P_s that the graviton is absorbed by this new distribution is $P_s>P$. Further, by moving the particles to smaller r we can give them a uniform, spherically symmetric distribution of radius r_0. Now there are two cases, depending on whether by making the distribution uniform at a given time we must move the particles inside the static limit or not. If we can make the distribution uniform without violating the static limit then we may conclude that the probability, $P_{uniform}$ again satisfies $P_{uniform}>P$. Now for this distribution $P_{uniform}<1$ if $\int_0^{r_0} \frac{d(properlength)}{L(\lambda)_{abs.}}$ is bounded by a constant of order unity. But this may be easily shown using previously computed expressions for $L(\lambda)_{abs.}$ [1,7] and the positive energy conditions.

If we cannot make the distribution uniform without violating the static limit we pass to the second case, which is that of initial distributions inside the static limit. In these cases we can use an argument similar to that used for the first case to show that the probability that a specifically chosen distribution of gravitons interact is not decreased by replacing the initial distribution of matter by a homogeneous distribution inside the static limit with some density ρ'. The region R may then be described by a region of a Freedman-Robertson-Walker cosmology. However, in a FRW cosmology without cosmological constant it is easy to show that $\tau_g(t)$, the average mean free time between graviton interactions at proper time t since the initial singularity satisfies,

$$\tau_g(t) \simeq \frac{1}{\sqrt{G p(t)}} > \frac{1}{\sqrt{G \rho(t)}} > t \qquad (5)$$

Here, the first equality follows from calculations of the propagation of linearized gravitational radiation in FRW cosmologies,[7] the first inequality is the positive energy condition and the second is a property of FRW cosmologies. Thus, in this case also most of the gravitons in the region R never interact.

This establishes the conjecture for a large class of initial configurations. More generaly, one might hope to show that any initial

- 7 -

distribution of matter will be in one of two classes, such that either it can be replaced by a static uniform distribution or a uniform distribution which is like a region of a FRW cosmology without decreasing the probability for a random distribution of gravitons which may be present in the initial distribution to interact. Doing so would constitute a general proof of the conjecture.

Acknowledgements

REFERENCES

1. L. Smolin, On the intrinsic entropy of the gravitational field, IAS preprint, 1982.

2. A. Rechel-Cohn and L. Smolin, Is there an ultraviolet catastrophe for gravitational radiation? Santa Barbara preprint NSF-ITP-81-87.

3. M. Planck, *The Theory of Heat Radiation* (New York, Dover Publications, 1959); A. Einstein, Ann. d. Phys. 17 (1905) 132.

4. L. Smolin, to appear

5. F. J. Dyson, Astrophysical J. 156 (1969) 529.

6. F. P. Esposito, Astrophysical J. 165 (1971) 165.

7. L. P. Grishchuk and A. G. Bolnarev, in *General Relativity and Gravitation: One Hundred Years After the Birth of Einstein* vol. 2, ed. A. Held (New York and London: Plenum Press, 1980) 393-434, and references contained within.