Black Hole Hawking Radiation may never be observed!

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SUMMARY

Thermal Hawking emission from black holes is a remarkable consequence of unity of quantum physics and gravitation. Black holes of a few solar masses are the only ones which can form in the present universe. However, having temperatures million times smaller than the ambient cosmic background radiation they cannot evaporate. Primordial black holes of $M \sim 10^{14} gm$, would evaporate over a Hubble age and considerable ongoing effort is on to detect such explosions. I point out, however, that at the early universe epochs when such black holes form, the ambient radiation temperature considerably execeeds their corresponding Hawking temperature. This results in rapid continual accretion (absorption) of ambient radiation by these holes. Consequently by the end of the radiation era their **masses grow much greater** so that their lifetimes (scaling as M^3) would now be enormously greater than the Hubble age implying **undetectably small emission.**

Hawking showed in 1974 that [1,2] black holes can evaporate by the emission of low temperature thermal radiation, now named Hawking radiation. A black hole with a mass M, emits thermal radiation with a temperature given by:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \simeq 10^{-7}(\frac{M}{M_\odot})k \quad (1)$$

Such a steady thermal emission would imply an energy emission rate of : (mass loss rate of) :

$$\frac{dM}{dt} \simeq \frac{\hbar c^6}{G^2 M^2} \simeq 10^{-17}(\frac{M}{M_\odot})^{-2} \frac{ergs}{sec} \quad (2)$$

This gives a lifetime for the evaporation of a black hole of mass M:

$$t_H \simeq \frac{G^2 M^3}{\hbar c^4} \simeq 10^{71}(\frac{M}{M_\odot})^3 secs \quad (3)$$

Eqs (2) and (3) are supplemented by a factor $f(s) \sim 1$, depending on the various species and spins(s) of particles emitted [3].

From eqs. (1) - (3), we can see that the emission from solar mass black holes is utterly negligible. However, in our present universe, the only black holes which can currently form would be of a few solar masses. The conclusive work of Chandrasekhar on stellar degeneracy and white dwarfs [4] shows that black holes of less than a few solar masses cannot be formed in the present astronomical universe. Thus primordial low mass black holes of the mass range ($10^{14} - 10^{15} gm$) which according to eq.(3) would evaporate over a Hubble age ($\sim 10^{10} years$) as envisaged by Hawking could form only in the early universe [5]. However there is one more aspect to all this. This is often ignored when talking of Hawking radiation. The point is that the ambient cosmic microwave background which is at a present tempeature of $T_{cb} \simeq 2.73k$, would effectively suppress emission from the solar mass range black holes which are at temperatures millions of times smaller (as seen from eq.(1)). The radiation falling onto these black holes would be considerably greater than their thermal emission. Indeed in a recent paper [6], non-equilibrium thermodynamics developed for glassy systems has been applied to black holes by bringing in the cosmic background radiation temperature (temperature T_{cb}) as a heat bath. The drawback in the present equilibrium formulation of black hole thermodynamics is that it assumes the same temperature for black hole and heat bath. Thus it is important to consider not only the quantum evaporation of the black hole but also its absorption of the cosmic background radiation, so that for the present $T_{cb} = 2.73k$, only black hole masses smaller than $2.2 \times 10^{-8} M_\odot$ evaporate. In order for solar mass black holes which are the only ones that can astrophysically form at present to start evaporating, the universe must expand to several million times its present size (so that T_{cb} drops by the same amount !). Thus an observer cannot hope to detect Hawking radiation from solar mass black holes till T_{cb} drops to $< 10^{-7}k$, which would take $\sim 10^{18}yrs$. However during this time these black holes are also absorbing the background radiation and with the change in their mass their temperature drops continually in proportion (see eq.(1)). We shall quantify this later. The constraints for an evaporating black hole to accrete (absorb) either ambient radiation or matter are treated in [7]. The above arguments also hold for the formation of primordial black holes in the early universe. These have a mass $M \simeq 10^{14}gm$, which by eq.(1) would correspond to a temperature of $T_H \simeq 10^{12}k$ and evaporation time $\sim 10^{10}yr$ (the Hubble age). Still lower mass black holes would have already evaporated (recall lifetime scales as M^3). It is probably a coincidence that such black holes (of mass $M \sim 10^{14}gm$ and $T \sim 10^{12}$ k) with equal quantum and classical entropies evaporate over the Hubble age [8]. For such cosmological constraints on black hole temperatures see [9]. Thus the suggestion to look for signatures of black hole explosions (of $M \sim 10^{14}g\ holes$) annihilating into a burst of high energy photons and particles [10]. Constraints on the number of such black holes have been put from the background gamma ray flux [11, 12] and even short duration gamma ray bursts have been associated with such mini black holes [13]. Constraints on still lower mass primordial black holes (i.e. those evaporating during big bang nucleosynthesis) have been put based on light element abundances (i.e. D, He-4 etc.) [for an update see [14]].

In the early universe, primordial black holes are formed when the metric fluctuations exceed unity [8, 9]. This for instance could happen in the early universe if the external radiation pressure forced material inside the Schwarzschild radius provided it began with a density sufficiently in excess of the ambient average density ρ [15]. However, the radiation temperature T_R changes rapidly with time (t) in the early universe and is known to have a time-temperature relation of the form:

$$T_R \simeq \frac{10^{10}k}{t^{1/2}(s)} \tag{4}$$

Briefly this comes about by considering the evolution of the scale factor with the time-dependent density ρ as:

$$\frac{\dot{R}}{R} = \left(\frac{8\pi G\rho}{3}\right)^{1/2} \tag{5}$$

Substituting for $\rho = aT_R^4/c^2$ (a the Stefan Boltztnan constant) and using the relation (RT = constant) $T/T = -R/R$, for expanding blackbody radiation, we have:

$$\frac{\dot{T}}{T} = -(\frac{8\pi Ga}{3c^2})^{1/2} T^2 \qquad (6)$$

which upon integration gives:

$$T_R = (\frac{3c^2}{32r\pi Ga})^{1/4} \cdot \frac{1}{t^{1/2}(s)} \qquad (7)$$

(We have eliminited the integration constant, by putting $T \to \infty$ *as* $t \to o$.).

The quantity within parenthesis in eq.(7) made up of constants, c, G and 'a' yields a value $\approx 10^{10}$. We note that $a = \frac{\pi^2 k_B^4}{45\hbar^3 c^3} = 7.2 \times 10^{-14} erg cm^{-3} deg^{-4}$, K_B the Boltzmann constant.

Eq.(7) can thus be written: (this gives the ambient radiation temperature T_R at any epoch t (in secs) in the earl;y radiation dominated universe.)

$$T_R = \frac{10^{10} K}{t^{1/2}(s)}$$

so that a T_R of 10^{12} k occurs at $t \approx 10^{-4}s$. The energy density of ambient radiation in the radiation dominated early universe as a function of time is given by:

$$\rho_R = \frac{3}{32\pi G t^2}$$

Thus the mass of radiation energy in causal contact after time t is:

$$M_H = \frac{4\pi}{3} c^3 t^3 \rho_R$$

and substitutions for ρ_R, this implies that at an epoch less than t, only black holes of mass: given by:

$$M_H = \frac{c^3 t}{8G} \qquad (8)$$

form Eq(8) implies that for a radiation temperture of $10^{12}k$ occurring at $t \simeq 10^{-6}s$ (from eq. (4)). Only black holes with mass $M_H \simeq 10^{32}$gm. can form, which however have a Hawking tempeature (for this mass) of only $T_H \sim 10^{-6}k$. Indeed the $M_H \simeq 10^{14}g.$, primordial black holes which are supposed to be now exploding (lifetimes $\sim$ Hubble age) with $T_H \approx 10^{12}k$, actually form at $t \approx 10^{-24}s$ (eq.(8)), when the ambient radiation temperature $T_R \approx 10^{22}k$. ($T_R >> T_H$). Combing the relations for M_H and T_R, above, it turns out that for black holes formed in the early universe, the hole Hawking temperature and the background radiation temperature are comparably the same only for $T_R = T_H \simeq 10^{20}\frac{\pi K_B}{\hbar} \simeq 10^{32}k$, or in other words only for Planck mass black holes with $M \simeq 10^{-5}$ gm produced at $t \simeq 10^{-43}s$. This result is obtained from substituting eq.(8) for the black hole mass M_H (forming at epoch t) in the Hawking formula (eq.(1)), which gives (for the primordial black holes formed at t)

$$T_H = \frac{\hbar}{\pi k_B t} \tag{9}$$

Now equating T_R and T_H above we have (from eqs.(9) and (4)).

$$T_R \;=\; T_H \;\simeq\; 10^{20}\;\frac{\pi k_B}{\hbar} \;\simeq\; 10^{32}k \tag{10}$$

(The constant 10^{20} has dimensions of (temperature)^2 $\times time and k_B/\hbar$ had dims. of (temp $\times time)^{-1}$ so their product gives the temperature).

The above results (see also [7]) imply that since primordial black holes formed in early universe have a much smaller temperature than the ambient radiation, would tend to absorb (accrete) the radiation energy at a rate given by (the usual formula):

$$\dot{M} \;=\frac{dM}{dt}\;=\;\frac{16\pi G^2 M^2}{c^5}\;aT_R^4 \tag{11}$$

As T_R is a function of time (t) given by eq.(4) we have: (using (4) in (11)):

$$\frac{dM}{dt} \;=\; \frac{16\pi G^2}{c^5}\;M^2\;.\;a\;\frac{(10^{40})}{t^2} \tag{12}$$

Integration of (12), gives for the mass of the primordial hole after a time t:

$$M \;\simeq\; \frac{c^5}{16\pi G^2 a.10^{40}}\;.\;t \tag{13}$$

Eq.(13) shows that a primordial black hole, formed at much earlier epochs would by the time $t \approx 10^{-4}s$ (when $T_R \approx 10^{12}k$), have already a mass, $M > .10^{32}gm$ So no such holes survive the hot evolving ambient universe. Even if (13) is an over estimate by orders of magnitude it follows in any case that the resulting growing mass >> primordial black hole mass, so that virtually no Hawking emission occurs. A hole of mass M formed when the ambient radiation temp is T_R, would lose mass by Hawking radiation (prop. to $\sim \hbar c^6/G^2M^2$) equal to the mass of radiation accreted only if M satisfies,

$$M^4 < \frac{\hbar c^9}{16\pi G^4 a T_R^4} \tag{14}$$

(obtained by comparing eq. (11) with eq.(2)

So for $T_R \approx 10^{22}k$, this implies $M < 10^4 gm$; but as seen above at this value of T_R, only holes of $M \simeq 10^{14}$ form, whose Hawking radiation is too weak to avoid the influx of the $T_R >> T_H$ ambient radiation.

As eqs. (13) and (1), show, T_H falls as t^{-1} whereas T_R falls only as $t^{-1/2}$, so that $T_R >> T_H$ always, so that the holes keep growing in mass rather than radiate.

The **net result** is that the **currently popular** supposition that primordial black holes formed in the early universe with masses $\sim 10^{14} gm$ are in their **exploding stage now** due to Hawking radiation has **ignored that** fact that the ambient radiation temperatures (at their formation) are always much larger than the hole's Hawking temperature. This implies that their **masses grow** rapidly with time in the hot evolving universe. Thus there would be no such primordial holes exploding in the present universe, implying that Hawking radiation from black holes may never be seen . No black holes formed at any epoch in our universe would ever begin to evaporate. .

References

1. Hawking, S.W. (1974) Nature **248**, 30.
2. Hawking, S.W. (1975) Commun. Math. Phys. **43**, 199.
3. MacGibbon, J.H. (1991) Phys. Rev. **D44**, 376.
4. Chandrasekhar, S. (1984) (Nobel Lecture), Revs. Mod. Phys. **56**, 137.
5. Carr, B.J. (1975) Ap. J. **201**, 1.
6. Nieuwenhuizen, Th. M. (1998) Phys. Rev. Lett. **81**, 2201.
7. Sivaram, C. (2000): Phys. Rev. Lett. **85**, (in press).
8. For eg. Barrow, J. and Tipler F.: The Anthropic Principle (1991) P.353
9. Sivaram, C. (1983) Amer. J. Phys. **51**, 277.
10. Rees, M. (1977) Nature, **266**, 333.
11. Page, D.N. and Hawking, S. Ap. J. **206**, 1.
12. Carr, B.J. Ap. J. **206**, 8.
13. Cline, D. (1993) Proc. of 23rd ICRC (Calgary) Ed. D.Leahy. Sivaram, C. (1994) NATO ASI Series, M.Shapiro ed Kluwer, PP. 177-97.
14. Kohri, K. and Yokoyama, J. (1999) Phys. Rev. D 61 023501.
15. Sivaram, c. (1994) Physics Essays **7**, 103, Carr B.J. (1976) Ap. J. **206), 1.**
16. **Sivaram, C. (1999) Ap. J. 520, 454; Phys. Rev. Lett. (to appear) 86, (2000).**