THE SHEARING OF THE UNIVERSE

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Summary

Density fluctuations in the early Universe generate random shearing motions via their non-linear gravitational iteractions. The associated primeval adiabatic and shear-generated temperature fluctuations in the microwave background radiation are estimated, and recent observational constraints are found to require that the deceleration parameter q_o must exceed 0.1, _independently_ of whether rescattering by an ionized intergalactic plasma is assumed to weaken the primeval temperature fluctuations. This is because the shear motions generate fluctuations of comparable magnitude. The residual shear flow at the present epoch cannot be reconciled with the galaxy distribution in the local supercluster unless q_o is less than 0.5.

Large scale structure is observed in the Universe over scales of up to 20-30 Mpc, corresponding to superclusters of galaxies. The presence of such structure provides a gravitational interaction that necessarily generates shearing motions,which can in principle be observed. The magnitude of the resultant effect depends strongly on the cosmological model, and can consequently be used to provide an important cosmological test. In this Essay, we discuss the origin and evolution of the shear, and we then describe observable consequences of cosmological significance.

Primordial shear can arise by at least two mechanisms. It is generally accepted that galaxy clusters arise from small fluctuations in the distribution of galaxies. These fluctuations grow with time relative to the expansion of the Universe, until eventually bound self-gravitating systems are formed. It will be convenient to assume that the galaxies themselves are also produced by a similar mechanism, involving small primordial fluctuations in a gaseous expanding substratum; however this is not essential to our argument.

One finds that the density enhancements of the galaxy distribution relative to the mean density of the Universe decrease as a function of scale, and approach unity on the largest scales surveyed. The density enhancements grow with time by virtue of the well-known gravitational instability of the expanding Universe, and we can accordingly trace back the observed large-scale structure to early epochs when only small amplitude density fluctuations were present. These fluctuations are inherently asymmetrical, as would be expected if, for example, they originated from a white noise spectrum. Consequently, any given fluctuation will exert a tidal torque on its nearest neighboring fluctuations. These non-linear gravitational interactions result in shearing motions.

Another source of shear arises in the gravitational instability itself. The local enhancement in the gravitational potential tends to decelerate a fluctuation relative to the cosmic expansion. This dragging effect results in shearing motions that can be comparable to those generated by the tidal torques.

In order to estimate the magnitude of the shear, we make use of a simple theorem that applies to a statistically uniform and isotropic turbulent flow. The theorem states that the mean square of the shear scalar σ is equal to the sum of the mean square of the vorticity scalar and one half the mean square divergence.[1]

Since vorticity is conserved in the expansion of the ideal fluid presumed to characterize the Universe at early epochs, we may neglect any initial vorticity, and can now express the shearing rate σ in terms of the divergence of the velocity field. Furthermore, the law of conservation of mass enables us to relate the divergence of the velocity field to the rate of change of the density perturbation. Since the dominant mode of the density contrast δ associated with small inhomogeneities in a spatially flat expanding universe increases with time as $t^{2/3}$, we obtain the estimate that the shear increases according to the relation $\langle \sigma^2 \rangle = \frac{2}{9} \langle \delta^2 \rangle t^{-2}$, where t is the cosmological epoch at which the shear σ and relative density fluctuation δ are evaluated. A detailed analysis of the Newtonian hydrodynamical equations can be shown to yield a similar expression for the shear, provided that one adopts a suitable approximation for the higher order correlations of the various perturbed quantities.

We shall make two specific applications of this fundamental result. Consider first the question of the small-scale angular anisotropy of the microwave background radiation. Calculations have previously been made of the residual temperature fluctuations associated with primeval adiabatic density inhomogeneities at the decoupling epoch[2]. However although the predicted temperature fluctuations according to many of these discussions can be comparable to recent observational upper limits,[3] one has hitherto been left with the unsatisfactory situation that any primordial temperature fluctuations could, at least in principle, be smoothed out by rescattering at a subsequent epoch. Such a rescattering could be associated with a secondary heating and reionization of the intergalactic plasma.

In fact, any secondary ionization of intergalactic gas and rescattering of the background radiation must inevitably produce sizeable temperature fluctuations owing to random shear motions of the scattering plasma. As we have already demonstrated, the shear is generated by non-linear gravitational interactions of density fluctuations whose presence and, more specifically, power spectrum can be inferred from the observed distribution of galaxies and galaxy clusters.

The predicted temperature fluctuations are strongly enhanced for cosmological models with a lower value of the deceleration parameter q_o, principally because larger density fluctuations are required in such models at any given redshift in order to account for the structure observed at the present epoch. Hence our main utilization of shear and its observable effects will be to derive a restricted range of possible values for q_o.

In the absence of any secondary rescattering of the primeval temperature fluctuations, the fairly conservative assumption that the most massive bound systems to have formed by the present epoch contain some 10^{15} solar masses allows us to predict the magnitude of the associated residual temperature fluctuation produced at the recombination era to be $\tau \simeq 10^{-4} q_o^{-1}$. This estimate of the relative temperature fluctuation is appropriate for an angular scale of approximately $3 q_o^{-1/2}$ arc min; over smaller scales the fluctuations are rapidly attenuated because of scattering associated with the residual ionization after decoupling, and over larger scales τ decreases because of the reasonable requirement that the power in the primordial spectrum of density irregularities should not increase towards larger wavelengths.

Recent observational upper limits[3] on τ are 7×10^{-4} over an angular scale of 2.3 arc min, and 4×10^{-5} over 25 arc min. Using a simple method for extrapolating the observational rssults to intermediate angular scales,[4] we infer that $q_o \gtrsim 0.25$.

Allowance for possible rescattering may nullify this result, however, and in order to obtain an unambiguous estimate of q_o, it is necessary to solve the transfer equation for shear-generated temperature fluctuations in a scattering medium. A straightforward analysis leads to a value for τ that is a factor of 10 lower than the previous estimate, if we assume that the secondary heating and rescattering occurs no earlier than a redshift of 100. Since this result can be shown to be applicable over angular scales of between 1 and 10 arc min, the observational data allow us to conclude that q_o must exceed 0.1.

Consider next direct observations of local shear motions by deviations from the linear Hubble relation between velocity and distance. The expected amount of residual shear amounts to $\sim q_o^2 H_o$ over scales of 10–30 Mpc; this result follows by virtue of the facts that the shear evolves as $\sigma \propto (1+z)^2$, and that the shear should be a maximum over scales that have most recently achieved density contrast unity. In fact, the deviations from the uniform Hubble flow is the local supercluster (out to the Virgo cluster), where we observe a density contrast of approximately 2.5 relative to averaging over larger regions,[5] amount to about 35 percent according to de Vaucouleurs and Peters,[6] or somewhat less (about 20 percent) according to the data of Humason, Mayall and Sandage.[7] Consequently we infer that $q_o \lesssim 0.5$.

We have attempted to make this estimate more precise by means of the following argument. There exists an exact solution which describes the growth and eventual recollapse of a density fluctuation in an expanding universe of arbitrary spatial curvature.[8] The amplitude of the density contrast can easily be related to the associated deviation in velocity from the Hubble expansion at an arbitrary stage of the evolution of the inhomogeneity.

We find in particular that a $q_o = 0.5$ universe is marginally consistent with the observational constraints previously given. The effect of reducing q_o is to enhance the density contrast while the shear is not significantly increased at a given stage of evolution of a fluctuation. We can infer, for example, that the best fit to the data on the local shear ($\sigma = 0.2\ H_o$, $\delta = 1.5$) is obtained if $q_o = 0.3$.

To summarize the preceding discussion, we have argued that convertional assumptions about the origin of galaxy clusters lead inevitably to the generation of shearing motions. Two important consequences follow. If the Universe becomes opaque subsequent to the recombination epoch, shearing motions on the surface of last scattering produce sizeable temperature fluctuations over scales of a few arc minutes in the microwave background radiation. Together with the inferred spectrum of primeval adiabatic temperature fluctuations, this enables us to use recent observational upper limits to estimate that $q_o \gtrsim 0.1$.

A second means of observing the shear is by comparing the mean density of galaxies in a large volume of space with deviations from the Hubble flow over that region. This test can be performed for the local supercluster, and we conclude that a cosmological model with $q_o \lesssim 0.5$ is necessary in order to reconcile the observations of moderate amounts of shear with the local excess of galaxies.

Thus the shearing of the Universe restricts q_o to lie between 0.1 and 0.5. The upper limit is marginally consistent with the possibility of a closed universe, whereas the lower limit implies an average mass-to-luminosity ratio for the matter in the Universe that exceeds that of ordinary spiral or elliptical galaxies by an order-of-magnitude.

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BIOGRAPHY

Joseph I. Silk

Joseph Silk was born in London, England, on December 3, 1942. He received a B.A. from Clare College, Cambridge University, in 1963 and the Ph.D. degree from Harvard University in 1968. His doctoral thesis was a study of galaxy formation. After spending 1968-1969 as a visiting fellow at the Institute for Theoretical Astronomy, Cambridge, England, and the following academic year as a research associate of the Princeton University Observatory, Dr. Silk joined the Astronomy Department at the University of California, Berkeley, in the fall of 1970 as an Assistant Professor. He is presently an Associate Professor of Astronomy and an Alfred P. Sloan Research Fellow. Dr. Silk's research has dealt with cosmological problems involving the growth of inhomogeneities in the early universe. He has also done extensive work in the areas of high energy astrophysics and interstellar matter.

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