1972

THE CASE FOR A CHAOTIC COSMOGONY

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Summary

It is argued that galaxy formation is an inevitable consequence of sufficiently chaotic initial conditions within the constraints implied by Einstein's field equations. Anisotropic rotation-free universes are shown to generate vorticity when, at early epochs, neutrino viscosity dissipates the anisotropic expansion. The vorticity becomes turbulent within the particle horizon, and the nonlinear density fluctuations thereby induced lead to the eventual formation of galaxies. Present limits on global vorticity restrict the primordial turbulence to velocities of a few percent of the speed of light. This nevertheless enables the present theory to account for many observed properties of galaxies.

Galaxies are the basic units of cosmology. Our knowledge of the expansion and isotropy of the Universe is attained by observations of remote galaxies. It is pertinent therefore to ask how essential galaxies are to the Universe. Are galaxies an inevitable consequence of gravitational theory applied to the large scale structure of the Universe, or are they merely local perturbations, superficialities in an otherwise homogeneous universe?

This latter viewpoint is adopted by advocates of the linearized theory of gravitational instability in an expanding universe.[1] As is well known, from the work of Lifshitz[2] and others, statistical initial fluctuations fail to give a satisfactory interpretation of the origin of galaxies. The outstanding uncertainty lies in choosing the initial conditions: both the amplitude of the initial fluctuations and the initial epoch at which they are presumed present must be specified. This necessarily requires a delicate adjustment of parameters: if one starts the perturbations prematurely, one forms galaxies too soon after decoupling. Alternatively, if one delays the perturbations one may not form any galaxies at all by the present epoch.

One suspects that the extreme sensitivity to specification of initial conditions may make this theory untenable. On the other hand, this result has been advocated recently as a reason for rejecting Einstein's field equations.[3] Clearly, before resorting to such drastic measures, one should ascertain whether there exists any means of manufacturing galaxies in the context of Einstein's theory that is relatively insensitive to the choice of initial conditions. Such a scheme can be developed, and a brief exposition is presented below.[4]

In such an approach, it is necessary to consider first the question of the uniqueness of the cosmological model adopted. The Universe presently appears isotropic, but at early epochs a variety of possible anisotropies may have existed. Misner has argued forcefully[5] that many anisotropic, rotation-free universes which expand from temperatures above 10^{10} °K become highly isotropic by the time the radiation temperature has cooled to 3 °K. The smoothing is due to the effect of neutrino viscosity at temperatures just above 10^{10} °K, when the neutrino collision rate is comparable to the expansion rate of the Universe.

At first sight, this result, obtained for spatially homogeneous models, may appear to have little in common with the problem of the origin of structure in the Universe. However there is a direct connection, as will presently be demonstrated. The argument consists of three stages.

The essence of the first stage lies in the following theorem[6]: that <u>viscous dissipation of shear anisotropy generates vorticity, even in an initially rotation-free universe</u>. This result has been derived for anisotropic cosmological models, which do not deviate grossly from Friedmann models at late epochs. Vorticity has thus been generated in a class of anisotropic cosmological models which allow a relatively wide range of initial conditions.

More general models would of course contain rotation ab initio. In fact, there is adequate reason to believe that rotation may be an essential feature of the early universe, as a consequence of the various singularity theorems.[7]

Consider therefore the fate of vorticity in an expanding universe. On sufficiently small scales, photon viscosity is effective at damping out any motions. The scale over which the damping occurs is just the random-walk distance of a photon (as it is scattered by free electrons) over a cosmological expansion time. The rotation rate ω is always small compared to the expansion rate, or at epoch t , $\omega t \ll 1$.

It is useful to study the evolution of a fixed comoving volume corresponding to a comoving length scale L . Since particles are neither created or destroyed, this corresponds to a specified total mass of matter. As the universe expands, the scale L is first encompassed by the particle horizon at an epoch L/c. The differential expansion velocity across L at subsequent epochs t is L/t , and the rotational velocity associated with the scale L is $\omega L \ll c$.

Rotation decays freely with the expansion, and angular momentum is conserved. It follows (cf. equation (1)) that the rotational velocity over any specified comoving length-scale remains constant until the epoch (denoted by redshift z_*) at which the mass-density of radiation first drops below the matter-density.[8]

At subsequent epochs, the rotational velocity decays inversely as the expansion scale factor. (This difference is due to the additional inertia of the radiation gas, which is always well coupled to the matter prior to the decoupling epoch at $z \approx 1000$).

On the other hand, during radiation-dominated epochs ($z > z_*$), the differential expansion velocity across a comoving scale L decreases as $1 + z$ (since $1 + z \propto t^{-\frac{1}{2}}$) . Consequently, as the redshift decreases, the comoving scale L will begin to undergo more and more rotation periods during the course of an expansion time. Moreover, since $L \ll c$, and the sound velocity prior to decoupling is close to c (the sound velocity in a radiation gas is $c/\sqrt{3}$, and the bulk of the pressure is contributed by the radiation) the fluid motion is highly incompressible.

There is one further clue that is needed to complete the second stage in the development of the main argument. One can define a Reynolds number associated with a fluid velocity v over a comoving scale L by $\mathcal{R} = 5vL\,(c\lambda)^{-1}$, where $\lambda \equiv (n_e\sigma)^{-1}$ is the mean free fath of a photon. It is readily shown that over mass-scales of interest, $\mathcal{R}$ is extremely large at early epochs. $\mathcal{R}$ decreases with time (as $(1 + z)^2$), but even at z_*, one finds that $\mathcal{R} \approx 4000\,(v/c)\,\Omega^{8/3}$ for a mass-scale corresponding to $10^{11}\,M_\odot$.

The magnitude of the Reynolds number attained at $z > z_*$ indicates that the fluid motion should develop into turbulence on scales $L \ll ct$. The maximum scale that participates in the turbulence is determined by requiring this scale L_{max} to interact with other turbulent eddies over a time comparable to the cosmological expansion time. If the velocity associated with this scale is v_{max}, it is apparent that one can estimate v_{max} by equating it to the differential expansion velocity over scale L_{max}, whence $v_{max} = L_{max}\,t^{-1}$.

The scales that participate in the turbulence extend down to those scales at which photon viscosity becomes an important source of dissipation of the vortical motions. It would accordingly seem to be an excellent assumption that over intermediate scales, homogeneous and isotropic turbulence develops, characterized by an energy flow from larger eddies to smaller eddies. The energy flow proceeds at a constant rate independent of eddy scale, and is determined only by the parameters of the largest scale. A hierarchy of eddies coexists at any given point in the flow, and the lifetime of an eddy is on the order of its scale divided by the eddy velocity.

Provided that $\mathcal{R} \gg 1$, the turbulent flow is continuously supplied by the decay and dissipation of the global rotation. As any fixed comoving scale is overtaken by the particle horizon, it is eventually drawn into the turbulent flow, and provides its energy to, and merges with, the residual turbulence.

A crucial moment in this scenario occurs at z_*. The rotational velocity is constant prior to this epoch, but subsequently decreases. One can show that the comoving mass-scale M_{max} (corresponding to L_{max}) increases with time as larger and larger mass-cales participate in the turbulence, until it reaches its maximum value at z_*. At this point $M_{max}(z_*)$ amounts to some $7 \times 10^{16} (v_{max}/c)^3 \Omega M_\odot$.

What is the eventual fate of turbulence that avoids dissipation? To answer this, note that viscous dissipation is effective through the decoupling epoch, when mass-scales of $10^{11} - 10^{12} M_\odot$ are damped.[9] Now the turbulent scale $M_{max}(z_*)$ is frozen out of the turbulent flow after z_*. Only considerably smaller scales, less than M_{max} evaluated at decoupling, have a sufficiently short turn-over time to be able to subsequently maintain the turbulence. However the corresponding mass-scale amounts to less than 1 percent of $M_{max}(z_*)$, if $\Omega = 1$, and would have been completely damped out by viscosity prior to decoupling unless $v_{max} > 0.1c$. Such a high value of v_{max} can be excluded since it would produce excessive anisotropy in the microwave background radiation, which sets the limit $v_{max} < 0.03c$.[10] Hence we conclude that turbulence is unlikely to persist after decoupling. This conclusion holds even for low density $(0.01 \leq \Omega \leq 1)$ cosmological models.[11]

At this point, it is appropriate to introduce the final stage in the main argument. Recall that the discussion has been for a spatially homogeneous universe. In fact, the onset of isotropic homogeneous turbulence must necessarily induce second-order density fluctuations. A recent study[12] of inhomogeneities generated by turbulence in an expanding universe found a non-linear growing mode of second order in v_{max} . On galactic mass-scales, the amplitude attained by this mode can be of order 0.1 to 1 percent at decoupling, provided that $v_{max} \sim 0.01c$ at z_* . In other words, one now has sizable density fluctuations, and can apply the standard theory of density perturbations of an expanding universe.[1] It follows that the density fluctuations should enter the non-linear growth regime, and presumably form galaxies, by $z = 1 - 10$.

It remains to indicate how certain of the observed properties of galaxies can be fitted into this scheme. The origin of the rotation of galaxies is a natural consequence[13], as are the positive energies observed in many groups of galaxies.[14] Small seed magnetic fields are generated by primordial vorticity[15], and these may be the precursors of galactic fields, when suitably amplified. Primeval turbulence in the fireball can even affect the cosmic helium abundance, and velocities $v_{max} \sim 10^{-2}c$ would produce fluctuations of a few percent in the primordial helium abundance.[16]

In summary, it has been shown that a wide range of possible anisotropic universes should generate a residual vorticity at $T \leqslant 10^{10}\ {}^{o}K$. As this vorticity enters the particle horizon, it forms a turbulent flow, which generates the density fluctuations that eventually give rise to galaxies. The present discussion is aimed at at serving as a pointer to the direction of future research into galaxy formation and anisotropic cosmologies in the context of Einstein's field equations. We have argued the case, and it cannot be over-emphasized, that galaxies are an inevitable consequence of sufficiently chaotic (but spatially homogeneous) cosmologies.

REFERENCES

1. A recent expression of this point of view is given by P. J. E. Peebles, Comments on Astrophys. and Space Phys. (in press, 1972).
2. E. Lifshitz, J. Phys. USSR, 10, 116 (1946).
3. W. C. Saslaw, Astrophys. J., 173, 1 (1972); F. Hoyle and J. V. Narlikar, Mon. Not. Roy. Astron. Soc., 155, 323 (1972).
4. The present discussion is a development of ideas on primordial turbulence eloquently advocated by C. F. Von Weizsacker (Astrophys. J., 114, 165 (1951)) and G. Gamow (Proc. Nat. Acad. Sci., 40, 480 (1954)) and more recently by L. M. Ozernoi and his colleagues (e.g., see L. M. Ozernoi and G. V. Chibisov, Astron Zh., 47, 769 (1970)).
5. C. W. Misner, Astrophys. J., 151, 431 (1968).
6. Proof of this theorem is based on the propagation equation for vorticity, which may be expressed as (e.g., see G. F. R. Ellis, in General Relativity and Cosmology, ed. R. K. Sachs, Academic Press, N. Y. (1971))

$$\frac{d}{dt}(a^2\omega^i) + a^2\omega^i \frac{\dot{p}}{\rho+p} = a^2\omega^i \sigma^i_j + \lambda\frac{a^2}{2}\,\epsilon^{iabc}\, u_a\left(\frac{\sigma^k_{b;k}}{\rho+p}\right)_{;c} - 2\lambda a^2\sigma^2 \frac{\omega^i}{\rho+p} . \qquad (1)$$

ω^i is the vorticity vector, σ^i_j is the shear tensor, λ is the neutrino collision rate (assumed constant), and $a(t)$ the expansion scale factor. At $t < t_o$, where $\lambda t_o = 1$, the vorticity is assumed to be zero, and the shear satisfies

$$\frac{d}{dt}(a^3 \sigma_{ab}) = -\lambda a^3 \sigma_{ab} .$$

In the limit of small anisotropy,

$$\sigma(\dot{a}/a) \ll 1 \quad \text{and} \quad \sigma \ll \lambda ,$$

only the second term on the right-hand side of (1) is significant, and one obtains a solution (for radiation-dominated expansion) for ω^i

which exhibits an exponentially decaying term, together with a term proportional to spatial gradients of the shear evaluated at t_o , and a time-dependence proportional to a^{-1} .

7. Several theorems on the existence of singularities conclude that a singularity is inevitable unless global causality is violated (cf. S. W. Hawking and G. F. R. Ellis, Astrophys. J., 152, 25 (1968)). (There is only one theorem that I am aware of due to Hawking that substitutes another condition for the causality constraint: it is not apparent how restrictive the new condition is with respect to causality).

Now K. Gödel (Proc. Int. Cong. Math., Camb., Mass., 1, 175 (1952)) has shown that the global causality condition (defined to be the nonexistence of closed time-like lines) is equivalent to the condition that a spatially homogeneous universe be non-rotating. Hence avoidance of a singularity may be feasible in a rotating universe. Since causality is locally inviolate, violation of global causality should not necessarily be an unacceptable physical concept, as others have maintained. Indeed, the Kerr exterior solution to a rotating mass possesses closed time-like lines.

8. z_* is approximately equal to the ratio of matter density to radiation mass density, evaluated at the present epoch, and is given by $z_* = 1.1 \times 10^4 \Omega$, where Ω is the ratio of the present mean mass density to the critical density required for closure of a Friedmann model with $q_o = \frac{1}{2}$.

9. J. Silk, Astrophys. J., 151, 459.

10. The upper limit on vorticity at the present epoch is 10^{-14} rad yr^{-1} (S. W. Hawking, Mon. Not. Roy. Astron. Soc., 142, 129 (1969); J. Silk, Mon. Not. Roy. Astron. Soc., 147, 13 (1970)). One can easily

show that, independently of cosmological model, this implies the quoted upper limit on v_{max} .

11. Ozernoi and co-workers have reached the opposite conclusion, apparently because they did not fully appreciate the importance of viscous damping of the turbulence prior to decoupling.

12. J. Silk and S. Ames, submitted to Astrophys. J. (1972).

13. To see this, note that the angular momentum of the turbulent velocity field is retained by the turbulence-generated density fluctuations. One finds that the angular momentum per unit mass associated with M_{max} is equal to

$$v_{max} L_{max} \approx 6 \times 10^{28} \Omega^{-4/3} (v_{max}/0.01c)(M_{max}/10^{11} M_{\odot})^{1/3} \text{ cm}^2 \text{ sec}^{-1}.$$

Provided that $\Omega \leqslant 0.1$, one can account for the angular momentum per unit mass of our own galaxy (amounting to some 10^{30} cm^2 sec^{-1}).

14. Since the density fluctuations generated by turbulence are found to scale inversely with size (reference 11), it follows that the larger mass-scales would not have entered the non-linear regime by the present epoch. Consequently the largest system (corresponding to groups of galaxies) should still be unbound. Note that if the present density of the universe were sufficiently low ($\Omega < 0.01$), observed peculiar motions could be attributable to a primordial origin with $v_{max} \sim 0.03c$ and $z_* \sim 100$.

15. E. R. Harrison, Mon. Not. Roy. Astron. Soc., 147, 279 (1970).

16. J. Silk and S. L. Shapiro, Astrophys. J., 166, 249 (1971).

Biographical Notes

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Joseph Silk is an astrophysicist who specializes in the areas of cosmology and high energy astrophysics. He is the author of numerous published papers in these fields. Recently, he has presented reviews of the diffuse x-and gamma radiation at the Fifth Texas Symposium on Relativistic Astrophysics, Austin, Texas, December 1970, and on primordial turbulence and intergalactic matter at the Cargèse Summer Institute on Cosmology and the Early Universe, July 1971. He is presently Assistant Professor in the Department of Astronomy, University of California, Berkeley.