Gravitons and Photons:

The Methodological Unification of Source Theory

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It was Einstein's great ambition, in his later years, to construct a unified theory of gravitation and electromagnetism. In this he failed. The reason for that is quite clear once the problem is considered to be a physical one, and not an exercise in pure mathematics. There is no known physical phenomenon in which the gravitational action of the electromagnetic field differs in any significant way from that of any other energy-momentum bearing substance; all that counts is the distribution of these mechanical quantities. Accordingly, if it is not to run afoul of the simplest experimental tests, any formal union of the gravitational and electromagnetic fields must be physically empty. It is therefore of some interest that a unification, in quite a different sense, has been achieved in recent years through the application of source theory, a systematic phenomenological approach to all particle physics. In this short essay, we shall try to give an introductory account of that development.

The problem that Einstein set himself was posed in classical field language. Source theory uses quantum mechanical particle language. It was inspired initially by the necessity for providing a non-speculative attitude and formalism with which to describe the families of particles that have been artificially created in high energy physics experiments. But it was quickly recognized that this physically motivated technique also produced great simplifications in the theories that are built around two particular particles - the photon and the graviton - the theories of electromagnetism and gravitation. Indeed, the special values of mass and spin (helicity) possessed by these two particles, combined with the general laws of quantum mechanics and special relativity, are so restrictive that the essential frameworks of these two fundamental theories evolve inexorably with such parallel lines of reasoning that one can truly speak of a methodological unification.

Source theory finds an empirically secure basis for its reasoning in the idealization and abstraction of the physical manipulations used in the work of the experimentalist. The central concept of the source emerges in an idealization of the realistic collisions that are used to create and to detect the various types of particles. The associated source functions serve both as a descriptive device for characterizing the particles that enter into and emerge from the reactions of interest, and as initial models for the evolution of the detailed dynamics that is specific to each kind of particle. Since particles are considered to be created in order to study them, and annihilated through their eventual detection, the description of all processes begins and ends with a vacuum state (symbol: zero); this is represented by the quantum probability amplitude (wave function)

$$\langle 0_+ | 0_- \rangle^S = \exp[iW(S)] \quad .$$

Here the minus and plus tags on the vacuum symbol are causal labels, referring respectively to any time before and after the space-time region where the sources S are manipulated. The exponential form on the right is suggested by the existence of physically independent experimental arrangements, for which the associated probability amplitudes simply multiply and the corresponding W expressions therefore add. The historically important connection between a wave function and a mechanical action correctly suggests the general identification of W as this fundamental mechanical property of the system (our units are such that $\hbar = c = 1$).

In the simplest situation, a particle is exchanged between a pair of sources. We use the (physically empty) example of a massless, spinless particle (scalar source, symbol: K) to illustrate it:

$$W(K) = \frac{1}{2} \int (dx)(dx')\ K(x)\ D_+(x-x')\ K(x') \quad ,$$

where the propagation of the particle between space-time points x and x' (note the necessary symmetry: $D_+(x-x') = D_+(x'-x)$) is described by

$$x^0 > x^{0\prime} : \qquad D_+(x-x') = i \int d\omega_p \, e^{ip(x-x')} ,$$

$$d\omega_p = \frac{(d\underset{\sim}{p})}{(2\pi)^3} \frac{1}{2p^0} , \qquad p^0 = |\underset{\sim}{p}| ,$$

which is just an invariant summation of the plane waves representing the possible quantum states of the particle. We direct attention to the following immediately derived result,

$$|\langle 0_+|0_-\rangle^K|^2 = \exp[-\int d\omega_p \, K(p)^* K(p)] \le 1 ,$$

where K(p) is the Fourier transform of K(x), and note its consistency with the physical demand that a probability not exceed unity. And, we point to an arrangement where the source remains practically time-independent for a long time interval T. Then W basically reduces to W = -ET, where

$$E = -\frac{1}{2} \int (d\underset{\sim}{x})(d\underset{\sim}{x}') \, K(\underset{\sim}{x}) \frac{1}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} K(\underset{\sim}{x}')$$

incorporates the elementary time integral

$$\int_{-\infty}^{\infty} d(x^0 - x^{0\prime}) \, D_+(x-x') = \frac{1}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} .$$

The physical identification of E is evident in the form of the probability amplitude: exp[-iET]; it is an interaction energy. Two component sources, a and b, of dimensions that are small in comparison with their separation distance R, have the interaction energy

$$E_{ab} = -\frac{k_a k_b}{4\pi R} , \qquad k = \int (d\underset{\sim}{x}) \, K(\underset{\sim}{x}) ;$$

like sources attract with an inverse square law of force. What we see here is an early, and physically incorrect, theory of gravitation (Nordström).

We contact the real world, specifically that of photons and electromagnetism, by replacing the scalar source with a vector source, $J^{\mu}(x)$. If we do no more than that, as expressed by

$$W(J) = \frac{1}{2}\int (dx)(dx')\ J^{\mu}(x)\ D_{+}(x-x')\ J_{\mu}(x') \quad ,$$

with arbitrary $J^{\mu}(x)$, we encounter an unacceptable quantum mechanical difficulty:

$$|\langle 0_{+}|0_{-}\rangle^{J}|^{2} = \exp[-\int d\omega_{p}\ J^{\mu}(p)^{*}\ J_{\mu}(p)]$$

can exceed unity, since $J^{\mu}(p)^{*}\ J_{\mu}(p) = |\underset{\sim}{J}(p)|^{2} - |J^{0}(p)|^{2}$ is not necessarily positive. We cannot set J^{0} equal to zero - that is not a covariant statement. There must be a scalar restriction on $J^{\mu}(x)$, and there is only one possibility:

$$\partial_{\mu}J^{\mu}(x) = 0 \quad , \qquad p_{\mu}J^{\mu}(p) = 0 \quad .$$

To appreciate its effect, let the spatial momentum of a particular particle point along the third axis, so that $p_{3} = p^{0}$. Then we have $J_{3}(p) = J^{0}(p)$, and

$$J^{\mu}(p)^{*}\ J_{\mu}(p) = |J_{1}(p)|^{2} + |J_{2}(p)|^{2} = \left|\frac{J_{1} + iJ_{2}}{2^{1/2}}\right|^{2} + \left|\frac{J_{1} - iJ_{2}}{2^{1/2}}\right|^{2} \geq 0 \quad .$$

At the same time that we satisfy quantum requirements, the independent source components reduce to only two, corresponding to transverse linear or circular polarizations. We are indeed describing the photon, a unit helicity particle.

The restriction on photon sources is a differential conservation law. We know the name of that conserved physical property - electric charge. The interaction energy of two quasi-static charge distributions is

$$E_{ab} = - \int (d\underset{\sim}{x})(d\underset{\sim}{x}')\, J_a^{\ \mu}(\underset{\sim}{x}) \frac{1}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} J_{b\mu}(\underset{\sim}{x}')$$

$$= \int (d\underset{\sim}{x})(d\underset{\sim}{x}') \frac{J_a^{\ o}(\underset{\sim}{x})\, J_b^{\ o}(\underset{\sim}{x}') - \underset{\sim}{J}_a(\underset{\sim}{x})\cdot \underset{\sim}{J}_b(\underset{\sim}{x}')}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} \qquad ;$$

these are the Coulomb-Ampèrian laws, which include the statement that like charges repel with an inverse square law of force.

The next step in the progression that began with scalar and vector sources is a symmetrical tensor source,

$$T^{\mu\nu}(x) = T^{\nu\mu}(x) \qquad .$$

The same reasoning that produced the scalar restriction on the vector source will demand an analogous vector restriction on this tensor source,

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad .$$

There is also, now, the independent possibility of constructing a scalar,

$$T(x) = T^\nu_{\ \nu}(x)$$

which would represent a spinless particle. That is explicitly removed by writing

$$W(T) = \frac{1}{2} \int (dx)(dx')[T^{\mu\nu}(x)\, D_+(x-x')\, T_{\mu\nu}(x') - \frac{1}{2} T(x)\, D_+(x-x')\, T(x')] \quad .$$

The basis for the particular factor, 1/2, in the second term becomes transparent on evaluating $|\langle 0_+|0_-\rangle^T|^2$, and recognizing that the source restriction $p_\mu T^{\mu\nu}(p) = 0$, applied as before in the coordinate system where $p_3 = p^o$, immediately produces the reduction

$$T^{\mu\nu}(p)^* \, T_{\mu\nu}(p) - \frac{1}{2} T(p)^* \, T(p) = \sum_{a,b=1,2} |T_{ab}(p)|^2 - \frac{1}{2} |\sum_{a=1,2} T_{aa}(p)|^2 \geq 0 \quad .$$

This combination is just such as to remove the trace of the two-dimensional symmetrical tensor T_{ab}, thereby leaving only two independent components. The rearrangement into

$$\left| \frac{1}{2} (T_{11} - T_{22} + 2i\, T_{12}) \right|^2 + \left| \frac{1}{2} (T_{11} - T_{22} - 2i\, T_{12}) \right|^2$$

then makes explicit that we are describing a helicity two particle - the graviton.

The restriction on graviton sources is also a differential conservation law, and we know the name of this conserved physical property - it is the mechanical attribute of energy-momentum. Here, however, there is a conflict of units, and we relate the graviton source tensor to the mechanical stress tensor by

$$T^{\mu\nu}_{grav.} = \kappa^{1/2}\, T^{\mu\nu}_{mech.} \quad ,$$

where κ is necessarily positive and has dimensions of $(\text{length})^2$ in our atomic units. The interaction energy of two quasi-static mechanical distributions is, therefore,

$$E_{ab} = -\kappa \int (d\underset{\sim}{x})(d\underset{\sim}{x}') \left[T^{\mu\nu}_a(\underset{\sim}{x}) \frac{1}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} T_{b\mu\nu}(\underset{\sim}{x}') - \frac{1}{2} T_a(\underset{\sim}{x}) \frac{1}{4\pi|\underset{\sim}{x}-\underset{\sim}{x}'|} T_b(\underset{\sim}{x}') \right] \quad .$$

For two pure mass distributions (where only $T^{oo} \neq 0$), that are of dimensions negligible in comparison with their distance, this becomes

$$E_{ab} = -\frac{\kappa}{8\pi} \frac{M_a M_b}{R} \quad ;$$

here is the Newtonian law of attraction ($\kappa/8\pi = G$), with the algebraic sense of that inverse square law uniquely determined, whereas one must appeal to experiment in the traditional Einsteinian approach.

The complete interaction energy expression clearly contains more than Newtonian gravity and, indeed, it leads directly to all the familiar experimentally testable statements of general relativity. Since perihelion precession is the least elementary of these, we focus on it. The interaction energy between an arbitrary mechanical system and a mass M localized at the origin (the Sun) is given by

$$E = - GM \int \frac{(d\underset{\sim}{x})}{|\underset{\sim}{x}|} [T^{oo}(\underset{\sim}{x}) + \sum_{k=1}^{3} T_{kk}(\underset{\sim}{x})] \quad .$$

The neglect of T_{kk} and the replacement of T^{oo} with a point distribution of total mass m gives the Newtonian interaction energy $V = -GMm/R$. For a slowly moving planet, $|\underset{\sim}{v}| << 1$, the corrections to V include (a) the contribution of $\sum T_{kk} \cong |\underset{\sim}{v}|^2 \, T^{oo} = (2T/m)T^{oo}$, where T is the Newtonian kinetic energy - correction factor: $1 + (2T/m)$; (b) the contribution in T^{oo} of the kinetic energy T, supplementing the rest energy m - correction factor: $1 + (T/m)$; (c) the contribution in T^{oo} of the potential energy between planet and Sun - a one line calculation based on the Newtonian spatial distribution of this energy gives the additional energy: $V^2/2m$. To these effects is added the first relativistic correction to the kinetic energy, $-T^2/2m$, leading to the total perturbation

$$\frac{3T}{m} V + \frac{V^2}{2m} - \frac{T^2}{2m} \rightarrow - \frac{3V^2}{m} \quad .$$

The last step here uses the non-relativistic energy relation, $T + V = E$, and the fact that a perturbation proportional to V does not produce precession. Now we have only to recall that the kinetic energy effect by itself ($-T^2/2m \rightarrow -V^2/2m$) implies a precession rate that is exactly 1/6 of the Einsteinian prediction to see that we have neatly reproduced this characteristic consequence of the theory of general relativity.

An impression may have been conveyed that fields are exorcised in source theory. Not at all. The field concept appears naturally on examining the response of W to infinitesimal source variations. The characteristic gauge ambiguities of electromagnetic and gravitational fields follow from the respective source restrictions. And the field equations themselves are derived, straightforwardly for the Maxwell field, somewhat more elaborately for the Einstein field since there one must develop a formalism to convey the fact that the sources of the gravitational field include the gravitational field itself, as demanded by the conservation laws. The possibility of a geometrical interpretation of the gravitational field, at the classical level, emerges at this stage; it is not an input. We conclude that the source theory approach demystifies the gravitational field. Certainly it has very special properties. But they emerge from the particular mass and helicity values of a particle, the graviton, not from speculations about world geometry.

References

1. J. Schwinger, Particles, Sources, and Fields, Vol. I (Addison-Wesley, Reading, Mass., 1970). See particularly Sections 2-4 and 3-17.
2. J. Schwinger, Amer. Jour. Physics 42, 507 (1974), Precession Tests of General Relativity - Source Theory Derivations.

Summary

The phenomenological, non-speculative attitude of source theory is used in parallel developments of electromagnetism and gravitation, based on the analogous properties of the massless particles, photon and graviton, thus providing a methodological unification of these two areas of physics. The power and economy of the approach is illustrated by an application to perihelion precession.

Biographical Sketch

Julian Schwinger is now Professor of Physics at University of California, Los Angeles, after spending more than twenty-five years at Harvard University. He has published extensively in many areas of theoretical physics. Honors include the Presidential Medal of Science and the Nobel Prize.