1972

OBSERVATIONS OF THE CURVATURE OF SPACE-TIME

ABSTRACT

We describe in this essay a practical method of observing the curvature of space-time due to gravity based on the scattering of light away from null geodesics.

The theory of the method is based on a perturbation technique developed by de Witt and de Witt. Calculations show that events in the near neighbourhood of dense massive objects give rise to multiple images. We show how to exploit these images for measuring masses.

The method is used to interpret the light curve of the Crab Pulsar. Its mass turns out to be about three solar masses.

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Gravitational mass is clearly an important parameter for assisting the understanding of the structure of astronomical objects.Our present kowledge of gravitational masses of stars is limited. It is obtained largely by three methods with different areas of application,viz:

(1) Direct determination based on red-shift measurements,

(2) Direct determination based on observation of gravitational interaction between near neighbouring objects,

(3) Indirect inference from the success of stellar models containing mass as a parameter.

Problems to which more than one of these methods are applicable are important because agreement between measurements has served to confirm the theories on which they are based. The agreement between (1) and (2) for Sirius was important for Relativity as was that between (2) and (3) for the companion of Sirius important for stellar structure.However such problems are rare.For a large class of objects one method only is available preventing confirmation and for an increasingly large class of objects none of these methods is applicable so that theorists have to construct models in ignorance of mass. Clearly then, our understanding of astronomy would be increased enormously if another method of mass determination were available. It is the purpose of this essay to point out that current gravitational theory shows that a fourth method is indeed possible and to describe the uses to which it has already been put.

The new method depends on the curvature of space-time due to gravity and is effective when emission of light takes placein regions where the gravitational potential is high. A dimensionless measure of gravitational potential is the red-shift z.A distance r from an object of mass m we have $z=\frac{Gm}{c^2 r}$.[1] We expect our method to work whenever z exceeds about 0.1 % at the emission point. Most models of pulsars satisfy this criterion (although z cannot be measured by red-shift because we do not know enough about pulsar spectra) and it is with pulsars that we have achieved most success with our method. Quasars also have high red-shift probable ascribable in part to gravitation. We have attempted to apply our method to quasars but have obtained so far only negative inequalities as results.

Red-shift can be understood as the passage of light along the null geodesics of curved space-time which are themselves curved in the sense that the distance between parallel geodesics is not constant.The hypothesis that light disturbances are restricted to null geodesics predicts that a pulse of radiation is observed at each point as a single pulse. It implies

Huygens's principle that the disturbance at each point of space-time can be inferred from the disturbance a time t earlier all over a sphere radius ct surrounding the point. It is responsible for our intuitive inference of events from what we see. It is true in flat space-time because a retarded Green's functionwwhich satisfies

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)G^{i}_{j'} = \delta^{(4)}(r,t,r',t')\,\delta^{i}_{j'} \qquad (1)$$

and so allows us to build up solutions of the flat space-time wave equation by linear combinations in the usual way, is

$$G\,(r,t,r,t) = \frac{1}{4\pi|r-r'|}\,\delta\left(|r-r'| - c(t-t')\right)\delta^{i}_{j'}$$

i.e. is different from zero only when (r,t) and (r',t') lie on each other's light-cones. This is a special property of the (3+1) dimensional flat space-time on whiich (1) is the wave equation. This property is not shared by a (2+1) dimensional flat space-time as the continuing outward ripples from a splash in still water may serve to demonstrate.Nor is it shared by the curved space-time of general relativity. An example whose analytical verification is simple is that the scalar function

$$\bar{G} = H^2\left(\frac{tt'}{|r-r'|}\,\delta(|r-r'|-t-t') + \theta(t-t'-|r-r'|)\right) \qquad (3)$$

satisfi~~fi~~es

$$\Box\bar{G} = g^{ij}\bar{G}_{;ij} = \sqrt{-g}\,g^{ij}\delta^{4}(r,t,r',t') \qquad (4)$$

where g^{ij} is the metric of de Sitter space, $g^{ij} = e^{-Ht}\,\mathrm{diag}(-1,-1,-1,+1)$

The second term representing a disturbance which lags behind the light-cone is called the "tail". It is zero only in a few special spaces [2] of which flat space-time is one.

De Witt and de Witt [3] discovered a perturbation technique for evaluating the tail for the vector wave equation in curved space-time whose departure from flatness is small. By applying their technique [4] we found how much light is left behind the light-cone when a pulse is emitted in a Schwarzschild metric, a distace r' from a mass m. The tail consists of:

(1) A continuous smeared out disturbance which dies off with distance according to an inverse square law

(2) A second pulse lagging a time $\frac{2r'}{c}$ behind the first with an amplitude $\frac{Gm}{c^2 r}$ times that of the first.

The continuous disturbance (1) cannot be observed as its associated energy is the square of its amplitude and thus has a fourth power fall-off with distance making it unobservable. This is in line with a general theorem due to Gunther [5] on the fall-off rate of continuous tails.

The second pulse (2) could be interpreted as a fraction $\frac{Gm}{c^2 r}$ bouncing off the central mass. It is in line with the work of Darwin [6] who showed how ghost images can occur from emmissions near to dense objects. Our perturbation technique and interpretation as a breakdown of Huygens's principle has allowed us to assess its magnitude and delay time. This interpretation is not free from controversy but the success of the prediction in one area has led us to seek the effect in others.

We sought the predicted effect at first in the light curves of quasars. If the effect is real it should give rise to a maximum in the autocorrelation of the light curve at a time $\frac{2r'}{c}$. We evaluated the autocorrelation of a series of 100 day means of the light curve of 3C273 by means of a computer programme. The maximum was tested for significance but it turned out not to be significant. The negative result shows that either the radius of 3C273 is less than 100 light days or that the parameter $\frac{Gm}{c^2r'}$ is less than the R.M.S. deviation of the sequence of 100 day means -- about 1%-- so that the autocorrelation peak is swamped by random deviation.

It was around this time that we published the paper reference (4). The paper showed how to apply the technique of mass measurement to pulsars by exploiting the very rapid intensity fluctuations manifested by their light-curves. A special feature of the Crab pulsar is the rotation of the angle of polarization of its light output simultaneous with very rapid fluctuations in its radio output. We suggested that the radio output should be examined for rotating polarization and predicted that the degree of polarization would drop a time $\frac{2r'}{c}$ after the formation of the peak of the main pulse as thepeak in the predicted tail pulse arrived. We showed how to calculate the mass of the Crab pulsar from such a drop. We do not know whether such an experiment has been carried out.

Since this publication Schönhardt(7) has communicated to us his measurements made at Jodrell Bank of the detailed structure of the Crab pulsar pulse stucture. They manifested exactly the features expected by the theory. A time 3mS after the formation of the main peak a second peak was formed of relative amplitude 1%. This occurs at each of the radio frequencies measured and the correspondence in shape between main and secondary peaks is quite marked. If we are observing the predicted tail image here our meassure of the distance from the centre of mass to the emmission point is about 1000km and of the mass of the Crab pulsar is about 3 solar masses. Both these numbers agree in orderof magnitude with current models.

This success leads us to suggest:

(1) The experiment proposed above on the polarization should be carried out not only for the Crab but also for other pulsars.

(2) That the autocorrelation of light curves of quasars should be examined in more detail.

(3) That our method should be added to the repertory of available methods of mass measurement.

Notes and references:

(1) Throughout this essay G stands for the constant of gravitation; c for the velocity of light; g_{ij} for the metric tensor; $\bar{G}$ and $\bar{G}_j^i$ for Green's functions; $\underline{r}',t'$ for the coordinates of an emission event and $\underline{r},t$ for the coordinates of its observation; H for Hubble's constant; δ for the Dirac delta function and θ for the Heaviside step function.

(2) McLenaghan R. Ph.D. Thesis Univ. Cambridge 1964

(3) De Witt B. and De Witt C. Physics $\underline{1}$ 1 145 1964

(4) Roe P.E. Nature 227 5254 154-156

(5) Günther Proc. Univ. Leipzig 1965

(6) Darwin Proc. Roy. Soc. 1940

(7) Schönhardt Private communication (of measurements only)

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