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George M. Rideout, President
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Dear Sir:

I wish to submit the enclosed paper, "A New Test of Weak Equivalence", to this year's essay competition. I understand the rules, and will abide by them. Should my paper be accepted for publication, I would appreciate the opportunity to redraw my figure, and add an acknowledgement. If you have any questions, please contact me.

Sincerely,

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A New Test of the Weak Equivalence Principle

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Summary

The Eötvös, Dicke, and Braginski experiments do not rule out the recent suggestion that the weak equivalence principle (WEP) might be violated at intermediate ranges (10^{-1} m $\leq r \leq 10^{4}$ m). I briefly discuss the problems inherent to Eötvös-type apparatus in searches for WEP-violating forces ("hyperforces") between laboratory masses, and suggest an alternative detector free of such problems. The proposed detector is driven by a "hyperforce" torque at the rotational frequency. If the detector is tuned to this frequency, the signal, enhanced by resonance, may be detected synchronously. I derive the response equations for the detector, and discuss how spurious responses due to gravity torques may be suppressed.

I. Introduction

The strong and weak equivalence principles (SEP and WEP) are linchpins of modern gravitational theory. The SEP almost uniquely yields Einstein's theory [1,2], while the WEP has been verified to high accuracy (for distant sources) by Eötvös-type experiments [2,3].

Recently, two arguments appeared for violation of the WEP at intermediate ranges (10^{-1} m $\lesssim r \lesssim 10^{4}$ m). Ref. 4 discussed (possibly spin-dependent) forces due to psuedo-goldstone boson exchange. Ref. 5 re-analyzed the original Eötvös experiment and found evidence for a difference in free-fall acceleration proportional to the difference in Baryon number* per unit mass.

To determine if such WEP-violating forces ("hyperforces") exist, experiments using laboratory-sized masses will likely be needed. In this essay, I describe such an experiment.

II) Eötvös experiments and their drawbacks

Eötvös, Pekár, and Fekete [6] compared inertial and gravitational mass using a torsion balance sensitive to the difference in the horizontal components of gravitational and centrifugal acceleration (due to the Earth's rotation) for two differing materials. The Eötvös experiment suffers from (a) an apparatus which is very sensitive to gravity gradients, (b) an uncontrollable torque (since the rotation and mass of the Earth are fixed), and (c) a data analysis which may be flawed [2].

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*or lepton number, or hypercharge; for every element except hydrogen, these ratio are nearly proportional.

Roll, Krotkov, and Dicke [2] and Braginski and Panov [3] solved (a) by using a mass distribution with vanishing quadrupole moment, and (b) by looking for a 24-hour periodicity due to the Sun. However, the required high sensitivity and large number of normal modes of their apparatus made it quite susceptible to seismic and thermal noise. Furthermore, the Sun is too distant to interact with the apparatus via intermediate range forces.

III) Resonant detectors

To avoid the deficiencies above, while retaining the positive features, consider another device -- namely, the rotating gravity gradiometer, or mass detector [7]. It consists of two perpendicular "dumbbells" coupled by a central torsion spring and magnetically suspended from a turntable. As the detector rotates about the spring axis, any external masses produce periodic tidal torques on the dumbbells at twice the rotational frequency. If the frequency of the tidal torque is tuned to the resonant frequency of the detector, an oscillation proportional to the product of the torque and the mechanical "Q" results; this may be detected synchronously using a strain-gauge and a lock-in amplifier. The detector described in [7] (Q $\approx$ 400) easily senses a 15-kg mass at 0.5 m.

A similar detector consisting of two bodies having "hyperdipole" moments may be used to search for "hyperforces". It has the following advantages over an Eötvös apparatus:

1. robust design
2. single normal mode
3. more massive test-bodies

4. resonance amplification of oscillation

5. synchronous detection

6. controllable rotation rate and source distribution

7. magnetic suspension seismically isolating the detector

8. negligible thermal effects

IV) Detector response equations

Lacking a specific "hyperforce" model, assume (in the static limit) a "hyperpotential" linearly coupled to a "hypercharge" density. The total (gravitation plus "hypercharge") interaction energy of the a^{th} test body is then

$$U^{(a)} = \int \{\Phi\rho^{(a)} + \Psi\eta^{(a)}\}\, dV \;. \qquad (1)$$

Here, Φ and Ψ are external gravitational and "hyper" potentials, and ρ and η, the mass and "hypercharge" densities. Also assume that Ψ satisfies a Yukawa (modified Helmholtz) equation --- $(\nabla^2 - \mu^2)\psi = 0$, with inverse range μ. (I shall not treat here gradient-coupled spin-dependent forces, but the extension should be straightforward). The general solutions for Φ and ψ are

$$\Phi(r,\theta,\phi) = \sum A_{\ell m} r^{\ell} Y^{*}_{\ell m}(\theta,\phi) \;. \qquad (2)$$

and, for $\mu r \ll 1$,

$$\Psi(r,\theta,\phi) = \sum B_{\ell m} r^{\ell} Y^{*}_{\ell m}(\theta,\phi) \;. \qquad (3)$$

(For $\mu r \gtrsim 1$, replace r^{ℓ} in (3) by a modified spherical Bessel function). Summations run over $0 \leq \ell < \infty$ and $-\ell \leq m \leq \ell$ unless otherwise shown. Under the Condon-Shortley phase convention, $A_{\ell,-m} = (-1)^m A^{*}_{\ell m}$ for real Φ, and similarly for $B_{\ell m}$. Inserting

(2) and (3) into (1), and replacing ϕ by $\phi' + \psi_a(t)$, where ϕ' is the body-frame azimuth and $\psi_a(t)$ the instantaneous rotation angle of (a), $U^{(a)}$ becomes

$$U^{(a)} = \sum \exp[-im\psi_a]\{A_{\ell m} J^{(a)}_{\ell m} + B_{\ell m} K^{(a)}_{\ell m}\} , \tag{4}$$

where $J^{(a)}_{\ell m}$ and $K^{(a)}_{\ell m}$ are the mass and "hypercharge" moments of (a) in the body frame.

The detector response follows from the lagrangian

$$\begin{aligned} L = {} & \frac{1}{2} I_1\dot{\psi}_1^2 + \frac{1}{2} I_2\dot{\psi}_2^2 - \frac{1}{2}\kappa(\psi_2 - \psi_1)^2 \\ & - \sum \exp[-im\psi_1]\{A_{\ell m} J^{(1)}_{\ell m} + B_{\ell m} K^{(2)}_{\ell m}\} \\ & - \sum \exp[-im\psi_2]\{A_{\ell m} J^{(2)}_{\ell m} + B_{\ell m} K^{(2)}_{\ell m}\} , \end{aligned} \tag{5}$$

where I_1 and I_2 are the moments of inertia of (1) and (2) about the z-axis, and κ is the torsion spring constant. The equations of motion are

$$\begin{aligned} & I_1\ddot{\psi}_1 - \nu(\dot{\psi}_2 - \dot{\psi}_1) - \kappa(\psi_1 - \psi_2) \\ & \qquad = \sum im \exp[-im\psi_1]\{A_{\ell m} J^{(1)}_{\ell m} + B_{\ell m} K^{(1)}_{\ell m}\} , \end{aligned} \tag{6}$$

$$\begin{aligned} & I_2\ddot{\psi}_2 + \nu(\dot{\psi}_2 - \dot{\psi}_1) + \kappa(\psi_2 - \psi_1) \\ & \qquad = \sum im \exp[-im\psi_2]\{A_{\ell m} J^{(2)}_{\ell m} + B_{\ell m} K^{(2)}_{\ell m}\} , \end{aligned} \tag{7}$$

where the damping term (with coefficient ν) accounts for bearing friction. Introducing the relative and "center-of- moment" angles δ and θ, and the abbreviations

$$\psi_1 = (\theta - \beta\delta), \qquad \psi_2 = (\theta + \beta\delta),$$

$$\alpha \equiv (I_1/I_o), \qquad \beta \equiv (I_2/I_o),$$

$$I_o \equiv (I_1 + I_2), \qquad I_{12} \equiv (I_1 I_2/I_o),$$

the response equations (to first order in δ) become

$$I_o\ddot{\theta} = \sum im\, e^{-im\theta}\left\{A_{\ell m}\left[J^{(2)}_{\ell m} + J^{(1)}_{\ell m}\right] + B_{\ell m}\left[K^{(2)}_{\ell m} + K^{(1)}_{\ell m}\right]\right\} \qquad (8)$$
$$- \delta\sum m^2 e^{-im\theta}\left\{A_{\ell m}\left[\alpha J^{(2)}_{\ell m} - \beta J^{(1)}_{\ell m}\right] + B_{\ell m}\left[\alpha K^{(2)}_{\ell m} - \beta K^{(1)}_{\ell m}\right]\right\},$$

$$I_{12}\left(\ddot{\delta} + \frac{\omega_o}{Q}\dot{\delta} + \omega_o^2\delta\right) \qquad (9)$$
$$= \sum ime^{-im\theta}\left\{A_{\ell m}\left[\alpha J^{(2)}_{\ell m} - \beta J^{(2)}_{\ell m}\right] + B_{\ell m}\left[\alpha K^{(2)}_{\ell m} - \beta K^{(1)}_{\ell m}\right]\right\}$$
$$- \delta\sum m^2 e^{-im\theta}\left\{A_{\ell m}\left[\alpha^2 J^{(2)}_{\ell m} + \beta^2 J^{(1)}_{\ell m}\right] + B_{\ell m}\left[\alpha^2 K^{(2)}_{\ell m} + \beta^2 K^{(1)}_{\ell m}\right]\right\}.$$

Here, $\omega_o = (\kappa/I_{12})^{1/2}$ is the natural frequency of the detector, and $Q = \omega_o I_{12}/\nu$ its mechanical quality factor. There are two types of forcing terms. Those without the factor δ, excite the detector <u>directly</u>; those with it, excite it <u>parametricly</u> [8].

V) <u>Analysis</u>

If the r.m.s. value of the right-hand side of (7) is sufficiently small that $\dot{\theta} \cong \{\text{const}\} = \omega_o$, the dominant Fourier component of the right-hand side of (8) will be at the resonant frequency of the detector. The Fourier series for δ converges <u>very</u> rapidly; thus, setting $\delta(t) = \Re e\{\delta\, e^{-i\omega_o t}\}$ in (9) and neglecting non-resonant terms,

$$\delta_o = -Q \left\{ \frac{I_{12}\omega_o^2 D - iQD^* P}{I_{12}^2\omega_o^4 - Q^2 |P|^2} \right\} , \qquad (10)$$

where

$$D \equiv \sum_{\ell=1}^{\infty} \left\{ A_{\ell 1}\left[\alpha J_{\ell 1}^{(2)} - \beta J_{\ell 1}^{(1)}\right] + B_{\ell 1}\left[\alpha K_{\ell 1}^{(2)} - \beta K_{\ell 1}^{(1)}\right]\right\} ,$$

$$P \equiv 4\sum_{\ell=2}^{\infty} \left\{ A_{\ell 2}\left[\alpha^2 J_{\ell 2}^{(2)} + \beta^2 J_{\ell 2}^{(1)}\right] + B_{\ell 2}\left[\alpha^2 K_{\ell 2}^{(2)} + \beta^2 K_{\ell 2}^{(1)}\right]\right\} .$$

To suppress a spurious response, we seek to minimize gravity terms and maximize "hyperforce" terms through symmetry of the source and test-bodies. This is necessary for the first few ℓ only.

First, choose test-bodies (1) and (2) to be identical (so (so $\alpha = \beta = 1/2$), and oriented such that (1) is rotated by 180° from (2) when $\delta = 0$. Then $J_{\ell m} \equiv J_{\ell m}^{(2)} = (-1)^m J_{\ell m}^{(1)}$, and

$$[\alpha J_{\ell m}^{(2)} - \beta J_{\ell m}^{(1)}] = \begin{cases} 0 & , \ m \ \underline{\text{even}} \\ J_{\ell m} & , \ m \ \underline{\text{odd}} \end{cases}$$

$$[\alpha^2 J_{\ell m}^{(2)} + \beta^2 J_{\ell m}^{(1)}] = \begin{cases} \tfrac{1}{2} J_{\ell m} & , \ m \ \underline{\text{even}} \\ 0 & , \ m \ \underline{\text{odd}} \end{cases}$$

and likewise for $K_{\ell m}$. Since (primes denote body frame)

$$J_{11} = -\sqrt{(3/8\pi)}\, M_{total}(X'_{cm} - iY'_{cm}) ,$$

$$J_{12} = \frac{1}{3}\sqrt{(15/8\pi)}\, (I_{x'z'} - iI_{y'z'}) ,$$

$$J_{22} = \frac{1}{12}\sqrt{(15/2\pi)}\, (I_{x'x'} - I_{y'y'} - 2iI_{x'y'}) ,$$

if the rotation axis passes through the center of mass, the body and principal axis are aligned, and $I_{x'x'} = I_{y'y'}$, the lowest non-vanishing J-term is $\ell = 3$. Dicke's arrangement [2] very nearly satisfies this; furthermore, its' J_{31} and J_{32} are also quite small. (The two largest terms are J_{20} and J_{33}, which don't contribute to δ_o).

To suppress unwanted $A_{\ell m}$, place 2N long cylinders of equal mass at the vertices of an equilateral polygon, normal to and symmetric about the midplane. Then the $A_{\ell m}$ vanish for $m \neq \pm 2jN$ (j an integer), and $\ell = 2jN$ dominate. If $N \geq 2$, the $A_{\ell m}$ do not contribute to δ_o at all!

To maximize $B_{\ell m}$, make the first N cylinders of substance I and the last N of substance II. Then $B_{\ell m} = 0$ for $m = \pm 2k$ (k an integer) and $\ell = 2k$ dominates; only $\ell = 1$, $m = \pm 1$ contributes to δ_o. To check any positive response, place substance I at even vertices and substance II at odd vertices. Then $B_{\ell m} = 0$ for $m = \pm(2k+1)$, and δ_o should vanish. Figure 1 shows a conceptual diagram of the apparatus.

VI. Conclusion

I have shown how the lowest order response of a resonant detector to external gravitational and "hypercharge" fields may be expressed in terms of its multipole moments, and discussed how spurious gravitational responses may be suppressed. Such a detector is inherently immune to the major sources of error in Eötvös experiments, and may provide a viable alternative to them for investigating the existence of equivalence-principle violating forces between laboratory-scale masses.

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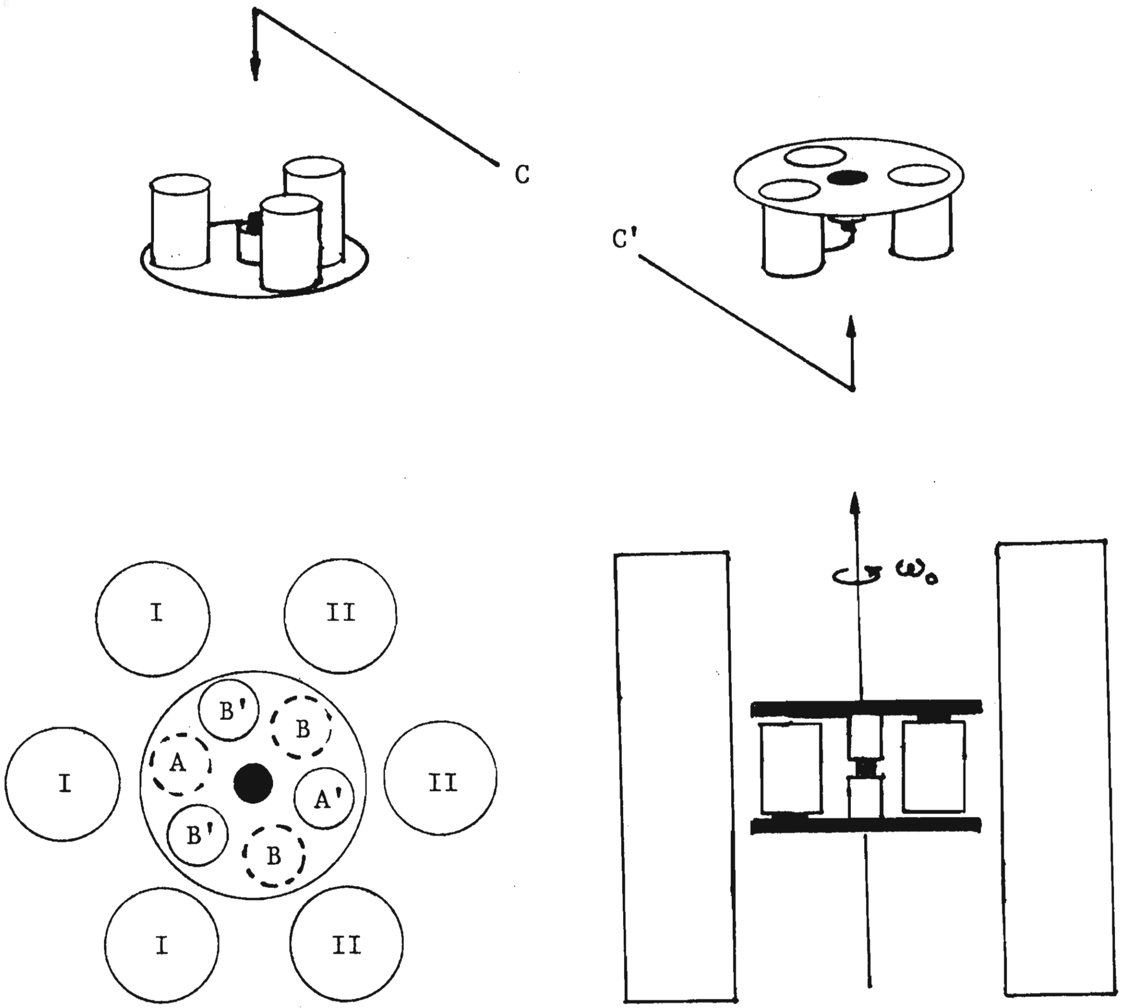

A - substance A
B - substance B
C - hub (spring, strain gauge)

I - substance I
II - substance II

Unprimed (primed) denotes test-body 1 (2).

Fig. 1 - Conceptual diagram of detector.