RYDBERG ATOMS AS GRAVITATIONAL-WAVE ANTENNAS

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Abstract

I show that highly excited Rydberg atoms nearby astrophysical gravitational wave sources are expected to emit significant electromagnetic radiation in the radio through a process of gravitationally induced resonance fluorescence. Semiclassical arguments are discussed and a quantum-mechanical expression for the differential cross section is obtained. This process could provide a new observational tool for the remote detection and study of gravitational waves.

Introduction

Recent studies have rekindled interest in the effects of gravitation on atomic systems [1]. These studies have shown that such effects, typically considered negligible under most astrophysical circumstances, may play a role in perturbing atoms if other processes are sufficiently depressed. In particular, it has been shown that the gravitational tidal energy shifts of freely falling Rydberg atoms of principal quantum number $n \sim 900$ or larger at the surface of neutron stars could in principle be resolved by means of radiotelescopic observations [1] (Parker *et al.* first suggested this possibility [2]).

Rydberg atoms are very fragile and thus lend themselves to being used as probes of external fields. Rybderg atoms with n up to ~ 700 are currently studied in the laboratory; atoms with n up to ~ 350 have been detected in the interstellar medium [3].

However, such highly excited atoms are extremely sensitive to other perturbations due to external magnetic fields and to atom-ion and atom-electron interactions. Therefore, the constraints on the temperature, density, and magnetic field of environments where observations could be successful are very strict and difficult to satisfy in the magnetosphere of a neutron star.

From this point of view, observations of Rydberg systems in the interstellar medium are more likely candidates to measure the effects of gravitation on atoms. Of course in this case the difficulty is that the gravitational fields involved are hopelessly weak. Leen *et. al.* have analyzed the possibility of detecting gravitational waves by directly measuring the periodic energy shifts caused on Rydberg atoms at distances $r_s \sim 10^9$ cm from the source. These studies imply that such effects are beyond the reach of present technologies.

In this essay I discuss the possibility of using Rydberg atoms as resonant quantum mechanical antennas for the detection of gravitational waves. In this case, the action of gravitational fields is not treated as quasi-time-independent as in all previous studies on the subject. Instead, it is shown that the very high values of the quality factor Q cause the time-dependent dynamical response of atoms acted upon by periodic gravitational waves to be quite dramatic even at distances r_s from the source ~ 1 A.U.

Semiclassical Arguments

The foundations of the study of atoms in curved space-time were laid by Parker [2]. He expanded the Dirac equation to first order in the Riemann tensor and wrote a fully covariant perturbative Hamiltonian in Fermi normal coordinates. In the case of low electron speeds ($v/c \ll 1$), the most important correction corresponds to the tidal gravitational force stretching and squeezing the freely-falling atom.

In order to evaluate the importance of time-dependent gravitational perturbations on atoms, we shall consider the effects of a plane gravitational wave travelling toward the positive z axis on an electron harmonically bound to a freely falling massive particle in the presence of radiation dampening.

The electron-particle system is in the $(x,\, y)$ plane at the initial time. The equation of motion then reads:

$$\ddot{x}_i + \gamma \dot{x}_i + \omega_0^2 x_i = c^2 R_{0i0j}(t) X^j(t)\,, \tag{1}$$

where γ is the friction coefficient related to the lifetime of the state τ as $\gamma = 1/\tau$, ω_0^2 is the natural frequency, R_{0i0j} is the Riemann tensor, x_i is the displacement from equilibrium, and X^j is the total distance between the nucleus and the electron. The time dependence of the Riemann tensor is related to the gravitational wave emission mechanism. In the weak field approximation, the Riemann tensor is related to the metric perturbation as $R_{0i0j} = -(1/2c^2) h_{ij}^{TT}$, where the transverse-traceless gauge (TT) was chosen. Let us consider the case of a rapidly rotating compact object whose highly ellipsoidal figure of equilibrium can be schematized as being composed of two masses M at a distance l_o from each other rotating rigidly at an angular speed ω. The order of magnitude of the metric is in this case $h_{ij}^{TT} \sim (GM/c^4) l_0^2 \omega^2 \exp(2i\omega t)/r_s$ [6].

By substituting the above relationships into Eq. (1), we find

$$\ddot{x}_i + \gamma \dot{x}_i + \omega_0^2 x_i \sim \frac{GM}{c^4} l_0^2 \omega^4 \frac{e^{2i\omega t}}{r_s} X^j(t)\,. \tag{2}$$

As the right-hand-side term involves $X^j(t)$, which in turn is related to x_i, this differential equation resembles those of the Mathieu kind with an ordinary forcing term added. Consequently, one cannot rule out the existence of unbound solutions typical of parametric oscillations even in the presence of friction [7]. However, in order to just assess the role of the tidal gravitational force in determining the dynamics of this system, we shall assume that the displacement x_i is small compared to the total average distance X^j of the two particles. In this case Eq. (2) reduces to the typical forced harmonic oscillator equation and one can immediately write the steady-state solution as

$$x_i(t) \sim \frac{(GM/c^4) l_0^2 \, \omega^4}{[(\omega_0^2 - \omega_g^2)^2 + (\omega_g/\tau)^2]^{1/2}} \frac{e^{2i\omega t}}{r_s} < X^j > , \tag{3}$$

where $\omega_g = 2\omega$. For numerical estimates, let us consider a Rydberg atom at a distance of $r_s = 10^{12}$ cm from a $3M_\odot$ compact object rotating with a period $T = 1$ms ($\omega \approx 6.3 \times 10^3$ s^{-1}). We will assume that the object can be represented as two fragments at a distance $l_0 = 10$ km from each other.

In order for the gravitational waves to produce the largest possible effect, resonance must occur. We shall therefore concentrate on the particular $n \longrightarrow (n+1)$ transition which has a frequency as close as possible to twice that of the spinning star. The frequency of such transitions is $\omega_{n,n+1} \approx (Z^2 e^4 m_e/\hbar^3)(1/n^3)$, where Z is the atomic number, m_e is the electron mass and the principal quantum number n is assumed to be $\gg 1$. In our case, we find $n \approx 1.5 \times 10^4$ (see below for a comment on the real significance of such high value). The semiclassical Bohr radius of such an orbit is $a_n \approx 1$ cm and its lifetime is extremely long as it scales as n^3 [8]. Substituting these results into Eq. (3), we find a steady-state response of the order of

$$< x_i > \sim \frac{GM}{c^4} l_0^4 \, \omega^3 \tau \sim 10^{-4} \, \tau \, \text{cm} \tag{4}$$

which, even for $\tau \sim 1$ s, corresponds to the order of magnitude of the distance between two contiguous Bohr orbits in the range of n considered.

The conclusion that one must draw from these order of magnitude considerations is that the time-dependent tidal gravitational effect is so large that it should affect atoms even out of resonance. This is due to the extremely high value of the quality factor Q of an atom, which one can naively estimate as

$$Q = \omega_{n,n+1}\tau_n \sim \frac{Z^2 e^4 m_e}{\hbar^3}(10^{-8}\mathrm{s}\, n^3) \sim 10^8 \,. \qquad (5)$$

It is important to point out that the high principal quantum number obtained above was not due to the need to deal with extremely fragile or large Rydberg atoms, but just to the condition that the $n \longrightarrow (n+1)$ transition frequencies match twice those of the rotating star. As it is now clear that the effect of gravitational waves on Rydberg atoms can be quite large, this condition can be safely relaxed to include atoms with smaller, and more realistic values of n, for instance in the 10^{2-3} range.

Quantum Mechanical Cross Section

As astronomical observations are typically performed by detecting electromagnetic radiation, we are interested in determining what processes correspond to the destruction of one graviton and the creation of a photon. This corresponds to calculating the power emitted by the electron as it accelerates under the action of the gravitational wave in our semiclassical model above.

The non-relativistic interaction Hamiltonian for an atom coupled to a quantized EM-field in the presence of a weak tidal gravitational interaction can be written by generalizing Parker's results in [4] as:

$$H_{int} = -\frac{e}{m_e c}\mathbf{A}\cdot\mathbf{p} + \frac{1}{2}m_e c^2 R_{0i0j}(t)x^i x^j + (\mathrm{g}-\mathrm{EM})\,\mathrm{terms}\,, \tag{6}$$

where $\mathbf{A}$ is the quantized vector potential and the (g-EM terms) describe the distortion of electromagnetic waves due to the spacetime curvature and are therefore responsible for the coupling of the two fields.

If we concentrate on incoming quantized plane waves, we can write the Riemann tensor field operator as

$$R_{0i0j}(t) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k_g}}\sum_{\alpha_g} R^{\mathbf{k_g}}_{0i0j} d_{\mathbf{k_g},\alpha_g}(0)\epsilon^{(\alpha_g)}_{ij} e^{ik_g z} e^{-i\omega_g t}\,, \tag{7}$$

where $R^{\mathbf{k_g}}_{0i0j}$ is a normalization factor, $d_{\mathbf{k_g},\alpha_g}$ is a graviton annihilation operator, $\epsilon^{(\alpha_g)}_{ij}$ is the polarization tensor, $\mathbf{k_g}$ is the graviton momentum, and α_g is the appropriate polarization ($+$ or $\times$).

It is clear from the well known expression for the quantized vector potential and from the above equation, that four processes are possible which involve the destruction of one graviton: (1) We have first order quadruple transitions described by $< f|\frac{1}{2}m_e c^2 R_{0i0j}(t)x^i x^j|i >$, whereby the atom makes a transition from an initial state $|i >$ to a final state $|f >$. However, this process does not create a photon until the atom spontaneously decays away from $|f >$; (2) we then have a first order process corresponding to the destruction of a graviton and the simultaneous creation of a photon,

$< f|\mathrm{g} - \mathrm{EM\,terms}|i >$. In this essay this process will not be investigated any further as it does not play a key role close to resonance; (4) then we have a graph corresponding to the destruction of a graviton at time t_1, followed by the creation of a photon at t_2 and (5) another graph corresponding to the creation of a photon at time t_1, followed by the destruction of a graviton at t_2.

By using standard second order perturbation theory techniques, it is simple to calculate the differential cross section corresponding to processes (4) and (5), which both contribute to the same final state. One finds the following cross section for a photon of momentum $\mathbf{k}_\gamma$ and polarization α_γ emitted into the solid angle $d\Omega$:

$$\frac{d\sigma}{d\Omega} = C(\mathbf{k_g}, \mathbf{k}_\gamma) \left| \sum_R \frac{< f|\mathbf{p} \cdot \epsilon_\gamma^{\alpha_\gamma}|R >< R|\epsilon_{ij}^{\alpha_g} x^i x^j|i >}{E_R - E_i - \hbar\omega_g - i\Gamma_R/2} \right|^2 , \tag{8}$$

where the sum is over all intermediate states $|R >$, Γ_R is related to the radiative lifetime of $|R >$ as $\Gamma = \hbar/\tau_R$, and $C(\mathbf{k_g}, \mathbf{k}_\gamma)$ is a global normalization factor.

The above expression simplifies considerably if the gravitational wave frequency ω_g is close to $(E_R - E_i)/\hbar$ for some state $|R >$. In this case, analogous to resonance fluorescence, the cross section rises sharply and is bound to a finite value only by the limited lifetime of the intermediate state. In the simple case in which only one state $|R >$ resonates with $|i >$ for a given ω_g, Eq. (8) takes on a form very close to what one would obtain from the semiclassical model (proportional to $(\ddot{x})^2$ in Eq. (2)):

$$\frac{d\sigma}{d\Omega} = C(\mathbf{k}_g, \mathbf{k}_\gamma) \frac{| < f|\mathbf{p} \cdot \epsilon_\gamma^{\alpha_\gamma}|R >< R|\epsilon_{ij}^{\alpha_g} x^i x^j|i > |^2}{(E_R - E_i - \hbar\omega_g)^2 + \Gamma_R^2/4} . \tag{9}$$

Possible Observations

One can reasonably expect several effects of such gravitationally-induced resonance fluorescence on the overall radiation output of atoms in the interstellar medium. I am presently conducting a detailed numerical study of the cross section for Rydberg atoms nearby gravitational wave sources. This includes polarization of the wave and of the EM-radiation, line broadening due to altered lifetime, different geometrical arrangements of the star, atom, observer, etc. I am also studying the effects on statistical state populations which would again be affected by these induced transitions.

As the effects described in this essay appear to be rather common, one cannot not rule out the possibility that radio emission from atoms might turn out to be a useful tool for the remote study of gravitational waves in the kHz frequency range.

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