Gravity and Grand Unified Theories

Leonard Parker and David J. Toms

Summary Paragraph:

We have carried out a renormalization group analysis of the gravitationally significant coupling constants in SU(5) grand unified theories in curved spacetime. We find that the effective values of many of these coupling constants at high curvature are determined by quantum effects depending on the numbers and types of elementary particle fields present. The resulting high curvature behavior of the theory evidently has interesting properties under conformal transformations of the spacetime metric.

Gravity and Grand Unified Theories

Leonard Parker and David J. Toms
Department of Physics
University of Wisconsin-Milwaukee
Milwaukee, Wisconsin 53201

It is currently believed that quantum effects play a dominant role in the evolution of the very early universe. The large curvature of spacetime at early times gives rise to unexpected phenomena, such as the creation of particles by the expanding universe, which have strong dynamical consequences. When realistic elementary particle interactions of the type occurring in grand unified theories (GUT's) are taken into account, further important processes are introduced, such as baryon number non-conservation and phase transitions between states of vastly different vacuum energies. These latter processes do not necessarily depend in an essential way on the curvature of spacetime.

We would like to discuss here some new consequences of grand unified theories that do depend in an essential way on curvature.[1] We have considered several GUT's, including the Georgi-Glashow SU(5) theory[2] and a related theory[3] in which the scalar self-interactions are asymptotically free. We find that at high curvature, such as existed in the early universe when particle energies were at the GUT scale of 10^{15} GeV, the effective values of the coupling constants which govern the strength of the interaction between curvature and matter are largely determined by the elementary particle content of the GUT under consideration. The coupling constants which behave in this way include the cosmological constant Λ, the coupling constants ξ_ϕ and ξ_H linking the scalar Higgs bosons to the curvature, and the coupling constants α_i which are coefficients of terms in the action that depend quadratically on the curvature tensor. The Newtonian constant G is exceptional, in that its value is evidently not significantly affected by curvature until the Planck scale (energies of 10^{19} GeV) is approached.

As early as 1962, Utiyama and DeWitt[4] showed that quantum field effects in curved spacetime will modify the Einstein field equations, introducing terms quadratic in the curvature. These quadratic terms are necessary for the renormalization of similar terms that appear in the part of the Lagrangian containing the quantum matter fields. Here, as in quantum electrodynamics, one obtains finite well defined results through a redefinition of the coupling constants of the theory. These renormalized values are the effective coupling constants that one measures.

The renormalization procedure is easily expressed in terms of the effective action,

$$\Gamma = I + \Gamma^{(1)} \ , \tag{1}$$

which consists of a part having the usual form,

$$I = \int dv_x \ L \ , \tag{2}$$

where L is the sum of the gravitational and matter field Lagrangians, and a part $\Gamma^{(1)}$ containing the quantum corrections. These quantum corrections include infinite terms, which can be made well defined or regular by working in a dimension d other than 4, in which case the infinite terms are replaced by terms having poles at d = 4. The terms in $\Gamma^{(1)}$ which have poles are of the same form as certain terms appearing in the action I of Eq. (2). Thus, by assuming that the original or "bare" coupling constants in I include parts (called counterterms) having similar poles, it is possible to cancel the pole terms in $\Gamma^{(1)}$, so that the effective action Γ is well defined at dimension d = 4.

In the process of regularization, it is necessary to introduce a parameter μ of dimension mass, in order to keep the action dimensionless (in units with $\hbar = c = 1$) when d is not equal to 4. The effective action and bare coupling constants are independent of the value of μ. However, the renormalized effective coupling constants do depend on μ. Typically, a bare coupling constant q_B is related to the corresponding effective coupling constant $q(\mu)$ through an equation of the form

$$q_B = \mu^{d-4}(q+\delta q) \ . \tag{3}$$

The quantity δq is the counterterm having a pole at dimension 4, which is required to cancel a corresponding pole in $\Gamma^{(1)}$. Differentiation with respect to μ of expressions such as Eq. (3) yield the renormalization group equations, a set of differential equations determining the dependence on μ of the effective coupling constants.

The renormalization group scaling in μ can be related to a scaling of the metric and of the curvature invariants of the spacetime. Because the effective action Γ is dimensionless, it must be possible to write it as a functional of dimensionless arguments,

$$\Gamma = F[\mu^{-\delta_i} q(\mu),\ \mu^2 g_{\mu\nu}] \ . \tag{4}$$

Here $q_i(\mu)$ is a quantity, such as a coupling constant, of mass dimension δ_i, and we have for convenience taken the coordinates as dimensionless. Because Γ does not depend on μ, one can then show[5,6,1] that the scaling of μ by a dimensionless parameter s, $\mu \to \mu s$, gives the values of the effective coupling constants appropriate to the spacetime obtained by scaling the metric as $g_{\mu\nu} \to s^{-2} g_{\mu\nu}$. Under this latter scaling, curvature invariants such as $R^{\mu\nu}R_{\mu\nu}$ scale as s^4, so that the large s limit corresponds to the high curvature limit. Thus, the renormalization group equations give information about the values of the effective coupling constants at high curvature.

The above techniques can be applied to various proposed unified elementary particle theories in order to find the form of the effective coupling constants which govern the interactions of matter with spacetime. One first calculates the relevant counterterms and then the renormalization group equations. These differential equations are integrated with appropriate boundary conditions to finally obtain the form of the effective coupling constants at high curvature. We have so far obtained the form of the gravitationally significant coupling constants for SU(5) grand unified theories.[2,3] We have worked with the unbroken symmetric phase, so that our results are relevant to the era between the phase transition at the GUT scale and the Planck time.

When the renormalization group equations are solved for the effective cosmological constant $\Lambda(s)$ and its present value is set equal to zero, we find that $\Lambda(s)$ in the GUT era can be expressed in terms of the Planck and Higgs masses as

$$\Lambda(s) \sim (m_{Planck})^{-2}(m_{Higgs})^{4} \ , \qquad (5)$$

which is of the same order of magnitude and sign as the value of Λ obtained from the vacuum energy in the inflationary model of the universe.

Because of the present large value of G^{-1}, the solution of the renormalization group equation for $G^{-1}(s)$ is dominated by the constant of integration. Consequently, the effective gravitational constant $G(s)$ has essentially its present value through the GUT era. Only at the Planck scale is its value significantly affected.

For the coefficients α_i of terms in the action quadratic in the curvature tensor, integration of the renormalization group equations gives results of the form

$$\alpha_i(s) = b_i \ln s + \text{constant} \ , \qquad (6)$$

where the b_i are numbers that depend on the particular grand unified theory under consideration. To obtain an order of magnitude estimate of the $\alpha_i(s)$ in the GUT era, we can take the α_i corresponding to the present mean value of $R^{\mu\nu}R_{\mu\nu}$ as zero and evaluate s, the dimensionless scale factor, from $s^4(R^{\mu\nu}R_{\mu\nu})_{NOW} = (R^{\mu\nu}R_{\mu\nu})_{GUT}$. In this way, we find that the $b_i \ln s$ term is dominant in Eq. (6) during the GUT era.

If the scalar field coupling constants ξ approach their conformal values of 1/6, or if no scalar fields are present, then the values of the α_i are such that the terms quadratic in the curvature in the gravitational Lagrangian take the form

$$L_{quad} = A\ C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta} + B\ E\ . \tag{7}$$

Here $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor and E is the integrand of the Euler characteristic, a topological invariant. The surprising absence of an independent term proportional to R^2 in Eq. (7) implies that the contribution of these quadratic terms to the gravitational field equations is conformally invariant at high curvature. The values of A(s) and B(s) in Eq. (7) depend on the numbers of different fields in the particular GUT under consideration. We find that, regardless of these numbers, B is positive and A is negative. Lagrangians of the form of Eq. (7) have been considered as candidates for a possible fundamental gravitational Lagrangian in theories of induced gravitation.[8,9] If Eq. (7) were to be adopted as a fundamental gravitational action, then the negativity of A would insure convergence of the generating functional for metrics of Riemannian signature.

Finally, the effective coupling constants ξ_ϕ and ξ_H, which appear in terms such as $\xi RH^\dagger H$ linking the Higgs scalars to the scalar curvature, are found to approach the value of 1/6 in the fully asymptotically free theories. Thus, at high curvature the effective gravitational and matter equations of motion appear to be dominated by conformally invariant terms. This behavior will suppress particle creation by isotropically expanding universes.[10] In other SU(5) GUT's in which the Higgs self-couplings are not asymptotically free, a detailed analysis of the behavior of the various couplings in the theory will be necessary before one can determine if the effective ξ coupling constants approach the value of 1/6.

References

1. L. Parker and D. J. Toms, Phys. Rev. D, in press.
2. H. M. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
3. N. Chang, A. Das, and J. Perez-Mercader, Phys. Rev. D 22, 1829 (1980).
4. R. Utiyama and B. S. DeWitt, J. Math. Phys. 3, 608 (1962).
5. B. Nelson and P. Panangaden, Phys. Rev. D 25, 1019 (1982).
6. D. J. Toms, Phys. Lett. B 126, 37 (1983).
7. A. Guth, Phys. Rev. D 23, 347 (1981).
8. S. L. Adler, Rev. Mod. Phys. 54, 729 (1982).
9. A. Zee, in Grand Unified Theories and Related Topics, edited by M. Konuma and T. Maskawa (World Scientific, Singapore, 1981).
10. L. Parker, Phys. Rev. 183, 1057 (1969).