ROTATION IN COSMOLOGY

Yu.N.Obukhov

Department of Theoretical Physics

Moscow State University

117234 Moscow, USSR

S u m m a r y

Cosmological models for a universe with expansion and rotation are considered. In particular, we analyse some effects of the universal rotation on observational cosmology. It is shown that pure cosmic rotation does not produce neither causality violations, nor parallax effects, nor anisotropy of the microwave background radiation. It can be detected by studying angular dependence of standard cosmological tests, and is directly measurable via polarization observations. The latter are used to obtain experimental estimates for the direction and value of the rotation of the universe.

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Dept. of Theoret. Physics, Moscow State Univ., 117234 Moscow

Noticing that most of physical objects (elementary particles, celestial bodies, stars, galaxies, etc) are rotating, one may wonder, why the largest physical system - the universe - should be an exception. Here we shall not enter into a discussion of philosophical significance of cosmic rotation (though, in our opinion, the analysis of its relation to the Mach's principle is of great interest). Instead, this paper will describe some effects on observational cosmology of the universal rotation.

Since the first studies of Lanczos, Gamow and Gödel [1,2], great number of rotating cosmological models were considered in the literature (these will not be reviewed here, see e.g. [3,4]). Nevertheless the full understanding of observational manifestations of cosmic rotation is still far from reach. Moreover, there is a general belief that rotation of the universe always is a source of many undesirable consequences, most serious of which are timelike closed curves, parallax effects, and anisotropy of the microwave background radiation. The aim of this paper is twofold: to show that the above phenomena are not inevitable (and in fact, are <u>not</u> caused by rotation), and to find <u>true</u> effects of cosmic rotation.

Our starting point are two fundamental assumptions of Kristian and Sachs [5] which underlie the general analysis of observations in arbitrary cosmology: (i) the universe is described by a Riemannian space-time with sufficiently slowly varying metric, (ii) the light travels along null geodesics according to the geometric optics laws. In order to obtain certain numerical estimates of effects, Kristian and Sachs use also the third assumption that metric satisfies the Einstein gravitational field equations for dust. However the latter will be omitted here: for the sake of generality we shall not restrict ourselves to a particular dynamical model as far as it is possible.

The spatial distribution of matter we observe at present is isotropic and homogeneous to a high degree. Therefore, it seems natural, so long as pure rotation effects are concerned, to consider a class of rotational shear-free spatially homogeneous models

$$ds^2 = dt^2 - 2R(t)\,n_i(x)\,dx^i dt - R^2(t)\,\gamma_{ij}(x)\,dx^i dx^j . \tag{1}$$

Homogeneity here means the existence of a three-dimensional group of motions which acts simply transitively on $t = \text{const}$ hypersurfaces; hence (1) describes Bianchi type space-times. These are classified according to the form of Killing vector fields $\xi^{\mu}_{(a)}$, $a = 1,2,3$, and their commutators $[\xi_{(b)}, \xi_{(c)}] = C^a{}_{bc}\,\xi_{(a)}$. Spatial metric components are then defined by

$$n_i = \mu_a e^{(a)}_i , \qquad \gamma_{ij} = \lambda_{ab}\, e^{(a)}_i e^{(b)}_j , \tag{2}$$

where μ_a, λ_{ab} are constant, and $e^{(a)} = e^{(a)}_i dx^i$ are relevant invariant one-forms, $L_{\xi_{(a)}} e^{(b)} = 0$. The explicit form of $C^a{}_{bc}$, $\xi_{(a)}$, $e^{(a)}$ is well known, see e.g. [4,6]. Let us choose the matrix λ_{ab} to be positive definite which ensures that $t = \text{const}$ slices are everywhere spacelike.

Space-times (1)-(2) are the simplest consistent cosmological models with rotation. Their kinematical properties are evident: the shear tensor is trivial $\sigma_{\mu\nu} = 0$, volume expansion is $\theta = 3\dot{R}/R$, and vorticity is given by

$$\omega_{0i} = 0, \qquad \omega_{ij} = -\frac{R}{2}\mu_a\left(\partial_i e^{(a)}_j - \partial_j e^{(a)}_i\right) . \tag{3}$$

The value of rotation $\omega = (\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu})^{1/2}$ decreases in expanding universe, $\omega \sim \frac{1}{R}$.

Cosmologies (1)-(2) are free from the above mentioned undesireable phenomena.

The absence of closed timelike curves is guaranteed by the positive definiteness of the matrix λ_{ab} . The proof is elementary [4] and is left to the reader. As an illustration, let us notice that in the old Gödel [2] model $\lambda_{ab} = \mathrm{diag}(1,-\frac{1}{2},1)$, and this is a loophole for closed timelike curves there.

As concerns parallax effects which a local observer interprets as a change in time of an angle between visible positions of any pair of distant optical sources, these were thoroughly analysed by Hasse and Perlick [7]. In particular, they have shown that the universe is parallax-free iff there exists a conformal Killing vector which is proportional to the velocity u^μ of comoving matter. One easily finds

$$\xi^\mu_{conf} = R u^\mu \tag{4}$$

to be the required conformal Killing vector for (1).

Finally, an extremely important question is how rotation affects the microwave background radiation. It is well known that (provided one assumes the black body spectrum) the temperature T of this radiation depends on the red shift [8] which reflects motion of cosmological matter with velocity u^μ ,

$$T_o = \frac{T_e}{1+z} = T_e \frac{(k^\mu u_\mu)_o}{(k^\mu u_\mu)_e} . \tag{5}$$

Here k^μ is a null vector field tangent to the light ray which connects observer and source, denoted by subscripts "o" and "e" respectively. In general, (5) shows that the observed temperature T_o depends on the direction of observation [9]. However, this is not the case for models under consideration. The crucial point is the existence of the conformal Killing vector (4) which yields that $k_\mu \xi^\mu_{conf}$ is constant along any null geodesics. Hence (5) reduces to

$$T_o = T_e \frac{R(t_e)}{R(t_o)} \quad . \tag{6}$$

Thus, the microwave background radiation is completely isotropic, exactly as in the standard Friedmann-Robertson-Walker (FRW) models.

Summarizing, we have shown that metrics (1)-(2) describe quite plausible rotating cosmological models which in many important respects are similar to the standard cosmologies. Namely, these are spatially homogeneous, completely causal, parallax-free, and have isotropic background radiation. As we see, pure rotation can be, in principle, large, contrary to the wide-spread prejudice that large vorticity confronts many crucial observations. In particular, the most popular claim that vorticity causes anisotropy of the microwave background radiation is apparently wrong. This, however, by no means contradicts with the earlier results [8,9] which were obtained for rotating metrics with non-trivial <u>shear</u>. It is shear, not rotation, which is the true (and only) source of anisotropy of the background radiation.

It is worth noticing that (1)-(2) is a sufficiently large set of cosmological models. It includes the Bianchi II - IX types with a rich choice of possible geometries and topologies: both spatially open and closed rotating universes can be described in this way.

Now let us proceed from general discussion (which revealed the features of expanding universe that are insensitive to pure rotation) to estimates of specific rotation effects on observational cosmology. In order to measure rotation quantitatively it is necessary to derive theoretical relations between various observable quantities in the universes (1)-(2). The list of such observables includes apparent magnitude m, red shift z, size and shape, polarization and other characteristics of distant optical and radio sources. Information about these quantities comes to us in the form of electromagnetic waves, and hence what one needs is the knowledge of the null geodesics structure in (1)-(2). This problem is simplified greatly due to existence of three Killing vectors $\xi_{(a)}$, and the conformal Killing (4). With the help of these, the null geodesics equations are exactly integrated in quadratures for any Bianchi type. The final formulas are rather complicated (see e.g. [4]) and will not be given here. Instead, it appears more illumiative to use the approach of Kristian and Sachs [5] who have derived general expressions for observables in a form of power series expansions with respect to red shift.

One of the most important relations in observational cosmology is the $m-z$ (apparent magnitude - red shift) test. As compared to the standard FRW models rotation introduces angular dependence of $m-z$ correlations. Straightforward calculation gives

$$m = m_{nonrot}(z) - \frac{5}{\ln 10} \ln(1 + p\cos\alpha) + O(z), \tag{7}$$

where the series $m_{nonrot}(z)$ is isotropic rotationless contribution, and α is an angle between the visible position of a source and a fixed direction defined by the three-vector p_i (with lenght p). Without the loss of generality (using spatial homogeneity) we assume that the observer is located at the origin of x^i coordinates on the $t = t_0$ hypersurface which corresponds to the present time of observation; for convenience, as usually, all direction angles are defined with respect to a local orthonormal basis of observer. Let h_a^i be 3x3 matrix such that $\lambda_{ab} = h_a^i h_b^j \delta_{ij}$, h_i^a is its inverse, and λ^{ab} is the inverse of λ_{ab}, then $p_i = \dfrac{\mu_a h^a{}_i}{1 + \lambda^{ab}\mu_a\mu_b}$, modulo an orthogonal rotation of observer's local basis. The term $O(z)$ represents further angular dependent contributions of rotation, proportional to z, z^2, and higher orders.

The $m-z$ relation (7) can be used for direct observation of a specific rotation effect for small enough z : the observer would see that the sources of equal absolute magnitude (intrinsic luminosity) look brighter in one hemisphere of the sky than in the opposite one. As far as we know, nobody searched such an asymmetry yet. Anyway, this effect would give an estimate of p, which is likely to be rather small.

Another important cosmological test is the $N-z$ relation which describes the number $dN(z)$ of sources observed in a given solid angle $d\Omega$ whose red shift is less than a given z. Direct calculation for (1)-(2) yields

$$\frac{dN}{d\Omega} = \frac{\left(\frac{dN}{d\Omega}\right)_{nonrot}(z)}{(1+p\cos\alpha)^3} + O(z) \quad , \tag{8}$$

where again the series $\left(\frac{dN}{d\Omega}\right)_{nonrot}(z)$ describes isotropic rotationless contribution, p and α are as above, and $O(z)$ contain higher order in z angular dependent rotational contributions.

The $N-z$ relation (8) presents another observational effect of rotation for small z : there would be an asymmetry in distribution of number of sources per angle over the sky. In particular, this yields inequality of total numbers N_1 and N_2 of sources in one hemisphere and another. The relative asymmetry is easily found to be

$$\frac{N_1-N_2}{N_1+N_2} = \frac{p(3-p^2)}{2} \ . \tag{9}$$

The position of hemisphere's poles is given (like in the $m-z$ test) by the direction p_i .

Both $m-z$ and $N-z$ tests strongly support "the importance of trying to observe angular variations in the various cosmological effects...", as emphasized by Kristian and Sachs [5]. However, these and analogous simple effects do not give the possibility of direct measurement of cosmic rotation, of its axis and value. Fortunately, this can be done in polarization observations, described below.

Recently Birch [10] has analysed correlations between position angles and polarization of 132 radio sources. He discovered highly organized asymmetry in distribution of angles of rotation of polarization vector over the sky, and suggested that this effect could be qualitatively explained by a cosmic vorticity. However Birch failed to support this by any theoretical model. Let us now use the important empirical data of [10] and obtain quantitative estimates of the universal rotation.

The crucial point is to make explicit the spin-orbital gravitational interaction which is characteristic for test particles in rotating space-times. In application to photons, one can verify that polarization vector rotates when light moves along any null geodesics, provided the latter is not orthogonal to the axis of vorticity (3). The relevant basic formula for (1)-(2) reads

$$\Phi = \omega r \cos\theta + O(z^2), \tag{10}$$

where Φ is a total angle of rotation of polarization seen by the observer for a source at an area distance r, whose visible position has an angle θ with the axis of cosmic vorticity. We omit rather lengthly derivation of (10), which can be proved, e.g. on the basis of knowledge of null geodesics structure for (1)-(2).

As one knows, there is a problem with determining distances for radio sources. Selecting from the catalogues those objects of Birch's data which have the known r, we finally obtained the sample of

51 radio sources with red shifts $0.0049 \leq z \leq 0.5610$. Neglecting the last term in (10) we then find, by the least mean square method, the direction (in galactic coordinates ℓ, b) and the value of cosmic rotation,

$$\ell = 295° \pm 25° \;, \qquad b = 24° \pm 20° \;, \tag{11.a}$$

$$\frac{\omega}{H} = 1.8 \pm 0.8 \;, \tag{11.b}$$

where $H = \left(\frac{\dot{R}}{R}\right)_{t_0}$ is the present value of the Hubble constant.

Although estimates (11) are not very accurate (the reason is evident: the formula $\Phi \approx \omega r \cos\theta$ is approximate, and available statistics is not too large), it is satisfactory to see that (11.a) coincides with direction of the large scale anisotropy of the Metagalaxy, observed by different methods in several groups [10-11]. As far as we know, (11.b) gives the first direct experimental estimate for the angular velocity of the universe. Notice that it turms out to be larger than it is usually assumed, $\omega \approx 10^{-10} \div 10^{-11}$ yr^{-1}.

To conclude, in this paper we have clearly demonstrated that pure cosmological rotation does not produce neither timelike closed curves, nor parallax effects and anisotropy of the microwave background radiation. It can be detected by studying angular dependence of observational tests (such as $m-z$ and $N-z$ relations) and is directly measurable via polarization observations of Birch's type. We therefore believe that models (1)-(2) describe physically reasonable cosmologies. As a final remark it seems worth noticing that all the results obtained are valid irrespectively of cosmological evolution of the scale factor $R(t)$. The reader may ask, however, are (1)-(2) dynamically realizable as solutions of gravitational equations. The answer is yes: e.g., in [12] relevant solutions are obtained for the Einstein's general relativity, and in [13] we discuss them in the framework of the Poincaré gauge theory of gravity.

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T a b l e 1

Polarization rotation and red shift for radio sources

Source	Φ	z	Source	Φ	z	Source	Φ	z
0336-35	-89	0.0049	2356-61	-25	0.0959	2318+23	-32	0.2680
1358-11	+83	0.0250	0938+39	+34	0.1075	1420+19	+59	0.2720
0300+16	-84	0.0324	0936+36	+07	0.1367	0605+48	-47	0.2771
0618-37	+80	0.0326	1441+52	+33	0.1410	0410+11	+13	0.3056
0518-45	90	0.0342	0114-47	+58	0.1460	1627+23	-35	0.3100
0427-53	-51	0.0392	1615+32	+32	0.1520	1158+31	-16	0.3610
2058-28	0	0.0394	1832+47	-24	0.1614	0125+28	-30	0.3952
2040-26	90	0.0406	1726+31	+29	0.1670	0903+16	+25	0.4110
1637-77	+49	0.0423	0958+29	+17	0.1846	1232+21	+04	0.4220
1350+31	+26	0.0450	1658+47	-12	0.2050	0133+20	-80	0.4250
0415+37	-09	0.0488	0453+22	-24	0.2140	1030+58	+29	0.4280
0043-42	90	0.0526	2141+27	-80	0.2145	0132+37	-20	0.4373
1845+79	-26	0.0569	0307+16	+06	0.2255	0824+29	-40	0.4580
1602-63	-41	0.0591	0651+54	-30	0.2387	1609+66	+50	0.5490
0I06+I3	-I4	0.0595	1308+27	-02	0.2394	1618+17	+10	0.5550
0802+24	-89	0.0599	0154+28	-28	0.2400	1241+16	+56	0.5570
1251+27	-74	0.0857	1545+21	+30	0.2640	1634+26	-57	0.5610

(Data of Birch [10])