CU-TP-560
IASSNS-HEP-92/17
March 1992

The Fate of Magnetically Charged Black Holes

Kimyeong Lee, V.P.Nair

Physics Department, Columbia University
New York, New York 10027

and

Erick J. Weinberg

Physics Department, Columbia University *
New York, New York 10027
and
School of Natural Sciences
The Institute for Advanced Study
Princeton, New Jersey 08540.

Abstract

The magnetically charged Reissner-Nordström black hole solutions of Maxwell-Einstein theory cannot evaporate completely, because their Hawking temperature tends to zero as their mass to charge ratio approaches unity. This situation changes when these solutions are considered in the context of a non-Abelian gauge theory containing nonsingular magnetic monopoles. If the horizon is sufficiently small, the Reissner-Nordström solution develops a classical instability and evolves into a new type of magnetically charged black hole solution. The temperature of these new solutions increases monotonically as the horizon contracts, so that there is no obstacle to the complete evaporation of a magnetically charged black hole.

* Permanent Address

The Reissner-Nordström solutions of the Maxwell-Einstein equations describe black holes carrying electric or magnetic charge. These objects can radiate via the Hawking process [1], as do the uncharged Schwarzschild black holes. However, their Hawking temperature, while inversely proportional to their mass for large mass, falls to zero as the black hole approaches the extreme Reissner-Nordström solution, for which the mass to charge ratio is unity in Planck units. As a result, instead of evaporating completely, as uncharged black holes apparently do, these appear to be eventually stabilized by the conserved charges which they carry.

The existence of light electrically charged particles drastically changes this picture for electrically charged holes [2]. Long before such a hole reaches the extreme solution, the electric field outside the horizon becomes strong enough to copiously produce electron-positron pairs. The particles with the same charge as the hole are then repelled, while those of opposite charge fall into the hole. This process eventually neutralizes the hole, thus removing the obstacle to complete evaporation. One might imagine that a similar process effect could lead to the discharge of magnetically charged holes, provided that the magnetic monopoles — if any such exist — were light enough that pair production could take place, but that for larger monopole masses the extreme magnetically charged hole would be stable. However, as we will describe in this paper, this is not the case.

To sensibly discuss magnetically charged black holes, one should do so in the context of a theory containing magnetic charges. The simplest such theory is a non-Abelian gauge theory with an $SU(2)$ gauge symmetry which is spontaneously broken to the $U(1)$ of electromagnetism when a scalar field ϕ acquires a vacuum expectation value of magnitude v. The elementary particles of this theory include the massless photon, two spin-1 gauge bosons with electric charges $\pm e$ and mass $m_W = ev$ and a neutral spin-0 particle, corresponding to the ϕ field, with mass m_H. This theory also contains magnetic monopoles [3] with magnetic charges $\pm \hbar/e$ and mass $M_{mon} \sim \hbar v/e$. These arise as solutions of the classical field equations. Within a core of radius $\sim \hbar/(ev)$ these solutions have nonzero values for all of the gauge fields while the magnitude of ϕ is not equal to its vacuum value. Outside this core, there is a Coulomb magnetic field, but, except for exponentially falling tails, the massive gauge fields and the scalar field take on their vacuum values. Because the

Compton wavelength $\hbar/M_{mon}$ is much smaller than the radius of the monopole core (for $e^2/\hbar$ much less than unity, which we will assume throughout), quantum fluctuations do not significantly change this picture and the monopoles survive as particles in the quantum theory.

The Reissner-Nordström solution is readily incorporated into this theory. We recall that the magnetically charged solution of the Einstein-Maxwell equations is described by the metric

$$ds^2 = -C(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \tag{1}$$

with

$$C(r) = A^{-1}(r) = 1 - \frac{2MG}{r} + \frac{4\pi G Q_M^2}{r^2} \tag{2}$$

and has a radial magnetic field with magnitude

$$B(r) = \frac{Q_M}{r^2} \tag{3}$$

There is a true curvature singularity at $r = 0$. This is hidden within a horizon at $r = r_+ = GM + \sqrt{G^2M^2 - 4\pi G Q_M^2}$ provided that the mass M is greater than the critical mass $M_{cr} = \sqrt{4\pi Q_M^2/G}$ of the extreme black hole; for $M < M_{cr}$ there is a naked singularity. With $Q_M = \hbar/e$, this is also a solution to the full non-Abelian gauge theory, provided that the scalar field ϕ is set equal to v everywhere and that the fields corresponding to the charged gauge bosons vanish identically.

It is instructive to analyze the stability of this solution [4]. This can be done by considering small fluctuations about the static solution and looking for modes which grow with time. In the linearized approximation the equations separate into three decoupled sets. The first, involving the perturbations of the metric and of the electromagnetic field, are the same as the ones encountered in studying the pure Maxwell-Einstein charged black hole, which has been shown to be stable [5]. Next are the equations governing the fluctuations of ϕ; it is rather easy to show that these never have growing solutions. Finally, there are the equations for the fluctuations in the charged boson fields. When the horizon distance is greater than $\hbar/(ev)$ the stability of these modes is manifest. However, growing modes appear as the horizon becomes smaller than this. Specifically, there is an instability if $r_H < b\hbar/(ev)$, where $b > 0.32$ for $M \gg M_{cr}$ while $b \to 1$ as $M \to M_{cr}$. (For a multiply charged black hole with $Q_M = n\hbar/e$, the instability sets in for horizon sizes smaller than $\sqrt{n}b\hbar/(ev)$.) Physically, the instability is due to the formation of a charged boson field configuration just outside the horizon with its magnetic moment aligned along the magnetic field of the monopole. For small enough horizons, this is energetically favorable, since the energy gained from the cancellation of the magnetic Coulomb field at small r is more than that needed to create the charged boson fields.

It should be stressed that this instability is purely classical, as compared to discharge by quantum mechanical pair production of magnetic monopoles. One finds, for example [6], that the magnetic fields outside the horizon are strong enough to produce such pairs [7] only when the horizon is smaller by a factor of $e/\sqrt{\hbar}$ than the value where the instability sets in.

This instability drives the Reissner-Nordström solution to a new type of magnetically charged black hole solution which may be best described as a black hole inside a magnetic monopole [8,9]. In these solutions the various matter fields have a nontrivial behavior outside the horizon which resembles that in the corresponding regions of the flat-space monopole solution. Although one might expect that such behavior would be forbidden by a no-hair theorem, none of these theorems apply to this case. However, similar methods can be used to show that for solutions of this type to exist the horizon distance r_H must be less than the greater of $\hbar/(ev)$ and $\hbar/m_H$ [8]. The area of the horizon for these solutions is always larger than the area of the Reissner-Nordström solution of the same mass and charge. Hence the instability of the Reissner-Nordström solution and its evolution to these new solutions are consistent with the law of increase of area for classical evolution. Because the usual relationship between entropy and horizon area holds for these new solutions, this evolution also gives an increase in entropy.

The horizon and singularity structure of these new solutions resemble those of the Schwarzschild metric rather than the Reissner-Nordström metric. There is only one horizon, and a physical singularity at $r = 0$. As with the Schwarzschild solution, this singularity is spacelike. More importantly for the present discussion, the Hawking temperature, given by

$$T_H = \frac{\hbar}{4\pi} \frac{C'}{\sqrt{AC}}\bigg|_{r=r_H} \tag{4}$$

increases monotonically as the horizon distance is decreased, and goes to ∞ as $r_H \to 0$.

We are now in a position to describe the history of a magnetic black hole. Such an object might form by the collapse of a massive object containing a magnetic monopole. After collapse the system eventually settles down as a Reissner-Nordström black hole. Gradually, Hawking radiation lowers the mass and causes the horizon to move inward, with the solution remaining Reissner-Nordström. As the horizon falls below $\hbar/(ev)$, the classical instability causes the first glimpse of a monopole core to appear outside the horizon, and the solution begins to deviate from the Reissner-Nordström form. With continued evaporation the horizon moves inward, the Hawking temperature increases, and more of a monopole core is revealed. Eventually the horizon shrinks to within a Planck length, and the black hole presumably evaporates, leaving behind a nonsingular monopole indistinguishable from the one which fell within the emerging horizon long before. Thus, contrary to previous belief, possession of a magnetic charge does not save a black hole from eventual evaporation.

Acknowledgements

This work was supported in part by the U.S. Department of Energy (VPN and EJW), by the Monell Foundation (EJW), and by an NSF Presidential Young Investigator award and the Alfred P.Sloan Foundation (KL).

References

1. S.W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

2. G.W. Gibbons, Commun. Math. Phys. **44**, 245 (1975).

3. G. 't Hooft, Nucl. Phys. **B79**, 276 (1974); A.M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)].

4. K. Lee, V.P. Nair and E.J. Weinberg, Phys. Rev. Lett. **68**, 1100 (1992).

5. S. Chandrasekhar, Proc. Roy. Soc. **A365**, 453 (1979).

6. W.A. Hiscock, Phys. Rev. Lett. **50**, 1734 (1983).

7. I.K. Affleck and N.S. Manton, Nucl. Phys. **B194**, 38 (1981).

8. K. Lee, V.P. Nair and E.J. Weinberg, Columbia-Fermilab Preprint, CU-TP-539, FERMILAB-Pub-91/312-A&T (1991) (to be published in Phys. Rev. D).

9. P. Breitenlohner, P. Forgacs and D. Maison, Max Planck Institute Preprint, MPI-91-91 (1991).