

A New Test of the Equivalence Principle: A Null Phase-Delay Experiment

T. P. Krisher

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109

Summary

We propose a new test of the Einstein Equivalence Principle (EEP) called a null phase-delay experiment, in which the phase-delay of a signal propagated over a coil of optical fiber is monitored as the gravitational field at the coil is varied. Any variation of the phase-delay would signal a violation of the EEP. An interesting test of the EEP in the solar gravitational field can be performed in the laboratory under carefully controlled conditions. With presently available technology, we show that such an experiment could provide a 0.01% test.

Submitted to Gravity Research Foundation 1993 Essay Contest.

A New Test of the Equivalence Principle: A Null Phase-Delay Experiment

T. P. Krisher

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California 91109

Submitted to Gravity Research Foundation 1993 Essay Contest.

Abstract

The Einstein Equivalence Principle (EEP) provides the fundamental basis for any metric theory of gravity, such as general relativity. It implies that the only observable effects of gravitation are those mediated by the spacetime metric. Therefore, at the origin of a local freely falling frame there should be no observable effects of gravity. Null redshift tests provide an interesting way to test this prediction. In this test, the frequencies of two oscillators of a different type are compared at the same location in a gravitational field as the field is varied. Any variation in the frequency would signal a violation of the EEP. Here we shall propose a new test of the EEP called a null phase-delay experiment, in which the phase-delay of a signal propagated over a coil of optical fiber is monitored as the gravitational field at the coil is varied. An interesting test of the EEP in the solar gravitational field can be performed in the laboratory under carefully controlled conditions. With presently available technology, such an experiment could provide a 0.01% test.

The Einstein Equivalence Principle encompasses three invariance principles for gravitation: 1) the equality of free-fall, 2) local Lorentz invariance, and 3) local position invariance [1]. The third principle (LPI) can be tested in a null gravitational redshift experiment [2]. If we denote the frequency of an oscillator infinitely far from a gravitating body by f_0 , then its frequency when a distance r from the body is given by

$$f(r) = f_0[1 - \alpha U(r)], \quad (1)$$

to first-order in the Newtonian potential $U(r)$, defined positive, and where α is a parameter. (For convenience, we use units in which $G = c_0 = 1$). If LPI is valid, then α should be the same for any oscillator ($\alpha = 1$ in general relativity). This prediction can be tested by comparing the frequencies of two different oscillators side-by-side in a gravitational field. The difference in their frequencies is given by

$$\Delta f(r) = f_2(r) - f_1(r) = f_0[1 - (\alpha_2 - \alpha_1)U(r)], \quad (2)$$

where we assume that each oscillator has the same frequency f_0 at infinity.

Equation (2) has been tested in the gravitational field of the Sun by monitoring the frequencies of two oscillators in a laboratory [3]. Variation of the solar gravitational potential was provided by both the rotation of the Earth and its motion in an eccentric Keplerian orbit. The accuracy of the experiment was limited by the long-term frequency stability of the oscillators. By using a hydrogen maser and a superconducting cavity stabilized oscillator (SCSO), Turneaure and his colleagues obtained the limit $|\Delta\alpha| < 1.7 \times 10^{-2}$, or slightly better than a 2% test.

Recently, a group at the Jet Propulsion Laboratory (JPL) has been developing a new test of local Lorentz invariance (LLI) involving time and frequency technology available in the NASA Deep Space Network [4]. In the experiment, two widely separated hydrogen masers (several kilometer baseline) are used to monitor the phase-delay of a signal propagated directly from one maser to the other along a fiber optic link. The velocity of the system in space is varied by letting the Earth rotate. A diurnal variation in the phase-delay would signal a possible violation of LLI. Because of its small temperature coefficient, optical fiber provides a highly stable link between the masers [5].

This type of instrumentation, differently configured, could also be used to perform a new test of LPI: a null phase-delay experiment. The schematic in Fig. 1 shows how to measure the phase-delay of a signal propagated over a certain length of optical fiber, which has been wrapped into a tight coil. Only a single frequency standard is necessary, which we have assumed to be a hydrogen maser oscillator. The 100 MHz output frequency of the maser is split into two signals. One signal is fed directly into a network analyzer, or an equivalent device to measure phase. The other signal is fed into a fiber optic transmitter and used to modulate a laser carrier which is propagated along the fiber to a fiber optic receiver, which feeds the signal into the other port of the network analyzer. For a length L of optical fiber, the measured phase-delay is given by $\Delta\phi = fl/c$, where f is the output frequency of the maser and we have defined $l = nL$ for a fiber with an index of refraction n (typically $n = 1.43$).

If the EEP is valid, then in a local freely falling frame (LFFF) we would expect $\Delta\phi$ to be independent of gravitational potential. In fact, in any metric theory of gravity each of the quantities f , l , and c would depend upon gravitational potential in such a way that it completely cancels-out when $\Delta\phi$ is computed in a LFFF. A

nonmetric coupling to the physical laws governing the frequency of the maser, or the length of the fiber optic cable and its index of refraction, could disturb this precise dependence and lead to locally observable effects on $\Delta\phi$. (This can be demonstrated explicitly in the $\text{TH}\epsilon\mu$ formalism, for example [6].) We can parametrize this dependence by expressing f , l , and c in terms of $U(r)$ according to

$$f(r) = f_0[1 - \alpha_f U(r)], \quad (3a)$$

$$l(r) = l_0[1 - \alpha_l U(r)], \quad (3b)$$

$$c(r) = c_0[1 - \alpha_c U(r)]. \quad (3c)$$

Using these relations to compute $\Delta\phi$, we obtain

$$\Delta\phi(r) = [1 - (\alpha_f + \alpha_l - \alpha_c)U(r)]\Delta\phi_0, \quad (4)$$

to first-order in U , where $\Delta\phi_0 = f_0 l_0 / c_0$. If the EEP is valid, then $\alpha_f + \alpha_l - \alpha_c = 0$. This can be tested by monitoring $\Delta\phi$ as $U(r)$ is varied.

This experiment can be performed in the laboratory under carefully controlled conditions to minimize unwanted phase errors. It is possible to coil several kilometers of thin optical fiber onto a spool, which can be kept in a thermally controlled chamber to minimize the effect of temperature variations on the length of the fiber. For 10 kilometers of fiber and a 100 MHz signal, we obtain a phase-delay of $\Delta\phi_0 = 1.7 \times 10^6$ degrees. Phase measurements with a resolution of 1 microdegree would provide a limiting sensitivity of 6×10^{-13} for a 10 km fiber. The diurnal variation in the solar gravitational potential is of order $|\Delta U| = 10^{-13}$. However, we can obtain a larger

variation in the potential as the Earth travels in its eccentric orbit ($e = 0.0167$). Between perihelion and aphelion, the variation in $U(r)$ is nearly linear and is of order $|\Delta U| = 3 \times 10^{-12}t$ for t in solar days. In slightly over a month, we would have $|\Delta U| = 10^{-10}$. At our limiting sensitivity, this would provide better than a 1% test of Eq. (4). For 100 km of fiber, there would result a 0.1% test. By increasing the signal frequency to 1 GHz we could push this limit to 0.01%.

A possible source of systematic error is an uncontrolled temperature drift affecting the length of the fiber, despite attempts to maintain the fiber at constant temperature. However, we can measure small temperature variations and then correct for their effects on the phase-delay. For a fiber with a temperature coefficient of delay (TCD) of 10^{-7} per degree Celsius and thermometry to a precision of 1 microdegree Celsius, we could keep the error below the assumed limiting sensitivity of the experiment.

A more serious error source is linear drift of the maser frequency, which would produce a phase error of $\delta\phi = (\delta f/f)\Delta\phi_0$. A typical drift-rate for the hydrogen maser is $\delta f/f = 10^{-14}$ per day. By comparing this to the rate of change of the solar gravitational potential, 3×10^{-12} per day, we see that we would be limited to a 0.3% test. This could be improved by using a trapped-ion frequency standard, which has better long-term stability [7]. It is possible to keep the drift-rate below $\delta f/f = 3 \times 10^{-16}$, which would provide our goal of a 0.01% test.

To summarize, we have shown that a test of the Equivalence Principle to 0.3% could be performed by monitoring the phase-delay of a coil of optical fiber for 1 month in the laboratory, assuming 1) phase resolution of 1 microdegree, 2) 100 km of fiber, 3) 100 MHz signal, and 4) maser frequency-drift less than 1 part in 10^{14} per day. A 0.3% test would provide a factor of 5 improvement upon the result of

Turneaure *et al.* However, a 0.01% test could be achieved by increasing the signal frequency to 1 GHz and reducing frequency drift to less than 3 parts in 10^{16} per day with a trapped-ion frequency standard. A 0.01% test would exceed the result of Vessot's 1976 spaceborne maser experiment [8], which was limited by a transmitter failure to a 0.02% test.

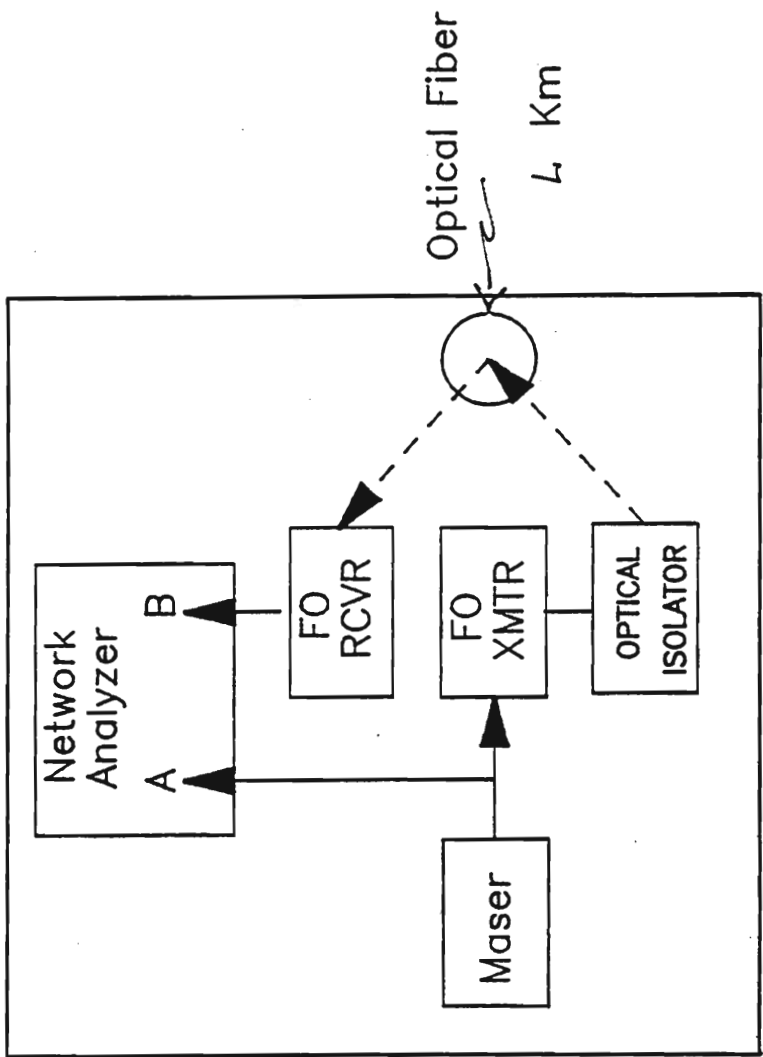
The research described in this report represents one phase of research performed at the Jet Propulsion Laboratory of the California Institute of Technology, which is under contract to the National Aeronautics and Space Administration.

References

- [1] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge Univ. Press, Cambridge, England, 1981).
- [2] C. M. Will, “Gravitational Red-Shift Measurements as Tests of Nonmetric Theories of Gravity”, *Phys. Rev. D* 10, 2330–2337 (1974).
- [3] J. P. Turneaure *et al.*, “Test of the Equivalence Principle by a Null Gravitational Red-Shift Experiment”, *Phys. Rev. D* 27, 1705–1714 (1983).
- [4] T. P. Krisher *et al.*, “Test of the Isotropy of the One-Way Speed of Light Using Hydrogen Maser Frequency Standards”, *Phys. Rev. D* 42, 731–734 (1990).
- [5] L. E. Primas, R. T. Logan, and G. F. Lutes, “Applications of Ultra-Stable Fiber Optic Distribution Systems”, in *43rd Ann. Freq. Control Symp.* (1989).
- [6] T. P. Krisher, to be published.
- [7] J. J. Bollinger, J. D. Prestage, W. M. Itano, and D. J. Wineland, “Laser-Cooled-Atomic Frequency Standard”, *Phys. Rev. Lett.* 54, 1000–1003 (1985); J. D. Prestage, G. R. Janik, G. J. Dick, and L. Maleki, “Linear Ion Trap for Second-Order Doppler Shift Reduction in Frequency Standard Applications”, in *IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control*, Vol. 37, 535–542 (1990).
- [8] R. F. C. Vessot *et al.*, “Test of Relativistic Gravitation with a Space-Borne Hydrogen Maser”, *Phys. Rev. Lett.* 45, 2081–2084 (1980).

FIGURE CAPTIONS

Fig. (1) Schematic for measurement of the phase-delay of an optical fiber of length L km, which has been wrapped into a tight coil.



CAN GRAVITY BE A THERMAL RESERVOIR?

Carlos E. Laciana

Instituto de Astronomía y Física del Espacio

Casilla de Correo 67-Sucursal 28, 1428 Buenos Aires, Argentina

E-mail: laciana@iafe.edu.ar.

It is shown, in an isotropic universe, that the quantum conformal fluctuations of the metric can play a rol analogous to the fictitious field of Thermo Field Dynamics (TFD) of Takahashi-Umezawa [1], when a massless scalar field minimally coupled to gravity is regarded as matter content. Moreover, by means of an extremum condition on the entropy and the energy, a thermal spectrum for the created particles is obtained.

In the black body problem, the thermalization of photons in the cavity is produced by the interaction with the quantum oscillators of the wall. In the TFD of Takahashi-Umezawa the effect of the walls is replaced by the quantum fluctuations of the reservoir [2]. This theory allows to us calculate mean values of physical observables as vacuum expectation values of quantum operators. The vacuum used is a superposition of states of an extended Fock space. This new Fock space is a direct product of the physical states and fictitious states representatives of the thermal reservoir. Israel [3] found a paralelism between this approach and the problems where an event horizon is present, as for example the Rindler observer or the black hole radiation. In both cases there are hidden modes in the other side of the horizon. This hidden modes play a rol analogous to the fictitious modes of TFD. We can ask about why the hidden states can affect the distribution in the observable side of the universe. The answer, as is remarked by Israel, is related

to the analytic extension of the metric, performed outside the horizon, which is related to the symmetry between the two kinds of modes. Similarly from in ref. [4] it is shown for the case where there is not a horizon, and anyway we can separate the modes symmetrically and construct thermal states as in TFD. The case studied in that reference is the scalar field in an isotropic curved space time. There we have neither fictitious nor hidden degrees of freedom, but anyhow again it is possible to get a thermal distribution, as in TFD. In ref.[4] a fictitious separation of modes is made. The observer can see the other side of the space but not 'look' there in the observation. In the present work we have another situation, there are neither fictitious nor hidden modes and all the modes of the matter field are observed, the role of the fictitious modes of TFD is played by the quantum fluctuations of the gravitational field. The gravitational field modes act as the oscillators of the cavity in the black body problem.

In the context of TFD, the operators of the thermal Fock space, are introduced. We will call these operators a^\dagger_β ($\beta = 1/k_B T$), which act on the thermal ket (at temperature T). The operator a^\dagger_β is written as a linear combination (Bogoliubov transformation) of the creation-annihilation operators at $\beta = \infty$ ($T = 0$), i.e.

$$a_{\mathbf{k}\beta} = a_{\mathbf{k}} \cosh \theta(\mathbf{k}, \beta) - \tilde{a}_{\mathbf{k}}^\dagger \sinh \theta(\mathbf{k}, \beta) \quad (1a)$$

$$\tilde{a}_{\mathbf{k}\beta} = \tilde{a}_{\mathbf{k}} \cosh \theta(\mathbf{k}, \beta) - a_{\mathbf{k}}^\dagger \sinh \theta(\mathbf{k}, \beta) \quad (1b)$$

where $T = 0$ corresponds to $\theta = 0$ and the operators satisfy:

$$a_{\mathbf{k}}^\dagger |n_{\mathbf{k}}\rangle = (n_{\mathbf{k}} + 1)^{1/2} |n_{\mathbf{k}} + 1\rangle$$

$$a_{\mathbf{k}} |n_{\mathbf{k}}\rangle = n_{\mathbf{k}}^{1/2} |n_{\mathbf{k}} - 1\rangle \quad (2)$$

with $n_{\mathbf{k}} = 0, 1, \dots$, and analogously the $\tilde{a}_{\mathbf{k}}^\dagger$ and $\tilde{a}_{\mathbf{k}}$ operators that act on the states $|\tilde{n}_{\mathbf{k}}\rangle$, also $a_{\mathbf{k},\beta}^\dagger$ and $a_{\mathbf{k},\beta}$ on $|n_{\mathbf{k}}, \beta\rangle$ and $\tilde{a}_{\mathbf{k},\beta}^\dagger$ and $\tilde{a}_{\mathbf{k},\beta}$ on $|\tilde{n}_{\mathbf{k}}, \beta\rangle$. In this approach the following extended Fock space is introduced

$$|n, \tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle$$

which we will call simply $|n\rangle$.

As it is shown in ref.[2] the following relation is valid

$$a_{\mathbf{k},\beta}^\dagger |0, \beta\rangle = -n_{\mathbf{k}}^{-1/2} \tilde{a}_{\mathbf{k}} |0, \beta\rangle = (n_{\mathbf{k}} + 1)^{-1/2} a_{\mathbf{k}}^\dagger |0, \beta\rangle = |0 + 1, \beta\rangle \quad (3)$$

In this context the average number of particles with momentum \mathbf{k} and temperature T is

$$n_{\mathbf{k}} := \langle 0 | a_{\mathbf{k}\beta}^\dagger a_{\mathbf{k}\beta} | 0 \rangle = \sinh^2 \theta \quad (4)$$

From eq. (3) we can interpretate the creation of one thermal state as related to the annihilation of one ‘hole’, as we can see from the term with operator $\tilde{a}_{\mathbf{k}}$. The total hamiltonian for free fields, as is proved, in TFD has two quantum contributions, one of them corresponds to the matter field, and the other one to the reservoir. The total hamiltonian is equal to

$$\hat{H} = H - \tilde{H} + C \quad (5)$$

with C a c-number (in the terminology of ref.[2], we can say that \hat{H} is weakly equal to $H - \tilde{H}$).

The explicit form of the two operators of eq.(5) are

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \quad (6a)$$

$$\tilde{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \tilde{a}_{\mathbf{k}}^\dagger \tilde{a}_{\mathbf{k}} \quad (6b)$$

The total hamiltonian \hat{H} is invariant under the transformation given by eq.(1).

As an example of system plus reservoir, we will considered a scalar field minimally coupled to the gravity. The quantum behaviour of the reservoir will be described by the quantization of the conformal fluctuation of the metric. Why only the conformal degree of freedom?. Firstly by simplicity. Moreover, as it is known, for a general fluctuation the causal relation between two space-time points is not invariant because the light cone structure is not preserved under the fluctuation, this does not occur when the fluctuation is restricted to the conformal one [5].

Similarly as in reference [6] we can represent the metric $g_{\mu\nu}$ as a conformal perturbation of the background $g^0_{\mu\nu}$ by means of the transformation

$$g_{\mu\nu} = g^0_{\mu\nu} \exp u, \quad g^{\mu\nu} = g^{0\mu\nu} \exp(-u) \quad (7)$$

Using the calculations of ref.[7] with the convention used by Birrell and Davies [8].

The Einstein equation, for the metric $g_{\mu\nu}$, with matter source is

$$G_{\mu\nu} = -8\pi G T^M_{\mu\nu} \quad (8)$$

When the transformation given by eq.(4) is performed, we get [6]:

$$G^0_{\mu\nu} = -8\pi G T^{ef}_{\mu\nu} \quad (9)$$

with $T^{ef}_{\mu\nu} = T^M_{\mu\nu} + T^G_{\mu\nu}$, $G^0_{\mu\nu}$ the Einstein tensor for the 'metric' $g^0_{\mu\nu}$, and $T^G_{\mu\nu}$ is given by

$$T^G_{\mu\nu} = (4\pi G)^{-1/2} D_{\mu\nu}\phi - (\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^0_{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi) \quad (10)$$

with $D_{\mu\nu} := \nabla_\mu\nabla_\nu + g^0_{\mu\nu}\nabla^\lambda\nabla_\lambda$ and the rescaling $\phi := (16\pi G)^{1/2}u$. As is shown in ref.[6] the field ϕ satisfies the equation

$$\nabla^\mu\nabla_\mu\phi(x) = J(x) \quad (11)$$

with

$$J(x) \simeq -\frac{2}{3}(16\pi G)^{1/2}R^0(x)$$

(in the last equation the term of order G^2 respect to $G^{1/2}$ is neglected)

R^0 is the scalar curvature of the metric $g^0_{\mu\nu}$. Then the general solution of eq.(11) can be written in the usual form

$$\phi(x) = \phi^0(x) + \int d^4x' G^{ret}(x, x')J(x') \quad (12)$$

where each part satisfies

$$\nabla^\mu\nabla_\mu\phi^0(x) = 0 \quad (13a)$$

$$\nabla^\mu\nabla_\mu G^{ret}(x, x') = \delta^4(x - x') \quad (13b)$$

We can now make the usual in-out second quantization of ϕ^0 field, with the condition that the source J is a classical one with zero limit at infinity. Then we can introduce the operator

$$\hat{T}^G_{\mu\nu} := T^G_{\mu\nu}[\hat{\phi}^0] \quad (14)$$

In the particular case that ϕ^0 represents a homogeneous perturbation of an isotropic, Friedmann-Robertson-Walker (F-R-W) metric, we can expand ϕ^0 in spherical harmonics $\{Y_{\mathbf{k}}(x), Y^*_{\mathbf{k}}(x)\}$ when the metric is closed, and as plane waves when the metric is flat:

$$\hat{\phi}^0(x, t) = \int d^3k [\hat{a}_{bfk}\phi_{\mathbf{k}}(t)Y_{\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}}\phi^*_{\mathbf{k}}(t)Y^*_{\mathbf{k}}(x)]$$

We can introduce moreover the operator

$$\hat{\phi} = \hat{\phi}^0 + \phi^1 \hat{I} \quad (15)$$

with $\phi^1 = \int d^4x' G^{ret}(x, x') J(x')$. By substituting eq.(15), in eq.(14) we get

$$\hat{T}_{\mu\nu}^G = C_{\mu\nu} - \hat{T}_{\mu\nu}^{0G} \quad (16)$$

where $C_{\mu\nu}$ is a c-number with the form

$$C_{\mu\nu} = (4\pi G)^{-1/2} g^0_{\mu\nu} J(x) + (4\pi G)^{-1/2} \nabla_\mu \partial_\nu \phi^1 - (\partial_\mu \phi^1 \partial_\nu \phi^1 - \frac{1}{2} g^0_{\mu\nu} \partial^\lambda \phi^1 \partial_\lambda \phi^1) \quad (17)$$

and

$$\hat{T}_{\mu\nu}^{0G} = \partial_\mu \hat{\phi}^0 \partial_\nu \hat{\phi}^0 - \frac{1}{2} g^0_{\mu\nu} \partial^\lambda \hat{\phi}^0 \partial_\lambda \hat{\phi}^0 \quad (18)$$

It is interesting to note that eq.(18) is equal to the one corresponding to a scalar field minimally coupled to gravity (given by $g^0_{\mu\nu}$) and the field ϕ^0 satisfies the same equation (13a) that a minimally coupled massless scalar field. The effective energy-momentum tensor operator can be written as

$$\hat{T}_{\mu\nu}^{ef.} = \hat{T}_{\mu\nu}^M - \hat{T}_{\mu\nu}^{0G} + C_{\mu\nu} \quad (19)$$

The equation for the metric hamiltonian (which is in this case equal to the canonical one), is then

$$\hat{H} = H - \tilde{H} + H_I \quad (20)$$

where $\hat{H} := V \hat{T}_{00}$, $H := V \hat{T}_{00}^M$, $\tilde{H} = V \hat{T}_{00}^{0G}$, $H_I = V C_{00}$.

In F-R-W metrics $V = 2\pi^2 a^3$, (with a the scalar factor of the metric). If $T^M_{\mu\nu}$ corresponds to a minimally coupled massless scalar field, then it has the same functional form as $T^{0G}_{\mu\nu}$. When we choose the base of solutions that diagonalizes the hamiltonian, the functional form given by eq.(6) (with $\epsilon_k = \frac{k}{a}$) is obtained. Moreover in terms of a base of solutions of the field eq.(13a), the condition of the diagonalization of the hamiltonian selects the modes with periodicity in the conformal time, which is analogous to the condition used in the black body problem, when the periodicity of the field on the walls of the cavity holds. Then we have a complete identification between the fictitious modes of TFD and the conformal quantum fluctuations of the metric. The Fock space related to the modes of the matter field is independent of the one related to conformal fluctuations of the metric. Then it is possible to construct the thermal states in a way analogous to TFD. But

in our case the modes $\tilde{}$ have a physical meaning, because they are related with the creation-annihilation operators of the quantum fluctuations of the metric.

Analogously as in ref.[4], we can impose the extremum condition on the thermodynamic potential, respect to the parameter θ , which labeled the Bogoliubov transformation that connects two in-out Cauchy surfaces;

$$\frac{\delta\Lambda}{\delta\theta} = 0 \quad (22)$$

with

$$\Lambda = \langle 0 | -\frac{1}{\beta} K + H | 0 \rangle \quad (23)$$

The vacuum $\langle 0 |$ is at $\theta = 0$. β is a Lagrange multiplier. K is the entropy operator introduced in ref.[1], as a function of the operators $a_{\mathbf{k}\beta}$

$$K = -\sum_{\mathbf{k}} \{ a_{\mathbf{k}\beta}^\dagger a_{\mathbf{k}\beta} \log \sinh^2 \theta - a_{\mathbf{k}\beta} a_{\mathbf{k}\beta}^\dagger \log \cosh^2 \theta \} \quad (24)$$

The variation of Λ gives us a planckian spectrum of created particles of the matter field, i.e.

$$n_{\mathbf{k}} = \frac{e^{-\beta \frac{k}{a}}}{1 - e^{-\beta \frac{k}{a}}} \quad (25)$$

where $n_{\mathbf{k}}$ satisfies also eq.(4) and represents the average number of created particles between the states $|0\rangle$ and $|0, \beta\rangle$, due to the interaction with the gravitational field. If we want to reobtain the thermal radiation contribution to Einstein equation (with units such that $c = 1, \hbar = 1 (h = 2\pi), k_B = 1$), as in equilibrium eq.(25) can not depend on the scale factor of the universe a , and moreover h would have to appear in the exponentials then $\beta = 2\pi a$. But $\beta = \frac{1}{T}$ where T is the temperature at the equilibrium. This value of the temperature is coincident with the one coming from the standard phenomenological, radiation-dominated Friedmann cosmology [9].

The result given by eq.(25) is coincident with the one obtained in ref.[4]. But with a conceptual difference. In ref.[4] the two Fock subspaces correspond to two disjoint sets of particles with momentum \mathbf{k} and $-\mathbf{k}$ respectively. In order to compare with the result of ref.[4] we can associate to each subspace one space of conformal modes \mathbf{k} of the gravitational field and one with $-\mathbf{k}$ respectively. Then we would have two thermal spectra, one related with the particles with momentum \mathbf{k} and the other with the ones with $-\mathbf{k}$.

It is important to note that in the last process the problem of infinite particle creation was bypassed. In our case we use the diagonalization of the hamiltonian as

a criterium to choose the particle model, although there is a hard criticism in the literature about that (see for example ref.[10]). But in all the cases where infinites appear perturbative expansions are made. Probably some terms, important in order to make the series convergent, are lost. However in our calculation the determination of the particle creation number it is exact because it is obtained by means of eq.(22). Our procedure is also very diferent, because eq.(22) gives us an extremum condition for the entropy and the energy.

A physical consequence of TFD to cosmology is the interplay between the energy of the matter field and the gravitational field. As we can see from eq.(20) the hamiltonian is weakly equal to the diference between the hamiltonians of matter and gravity. Therefore the eigenvalue of the total hamiltonian is weakly null, and this result is invariant respect to the Bogoliubov transformation given by eq.(1). Moreover as we can see from eq.(3) to the creation of one particle of the matter field corresponds the annihilation of one conformal graviton. Therefore the energy gained by the matter field is obtained from the gravitational field.

A comment about the entropy: the entropy operator, given by eq.(24), for the gravitational field (\tilde{K}) has the same functional form than K , but with the operators \tilde{a} instead of a (see ref.[1]). Therefore, taking into account again eq.(3), the entropy of the gravitational field is complementary to the one of the matter field. Then we hope that reversibility may be themodynamically valid in this model.

It is an interesting attempt, for the future, to study the transition to thermal equilibrium starting from a non-equilibrium TFD formalism (see ref.[11]), taking into account other orders in the perturbative expansion of the conformal fluctuation of the metric.

Acknowledgements

This work was supported by the European Community DGXII and by the Departamento de Física de la Fac. de Ciencias Exactas y Nat. de Buenos Aires.

References

- [1] *Y.Takahashi and H.Umezawa; Collective Phenomena (1975) 2. pp.55-80.*
- [2] *H.Umezawa, H.Matsumoto and M.Tachiki; 'Thermo Field Dynamics and Condensed States', (1982), North-Holland Publishing Company Amsterdam, New York, Oxford.*
- [3] *W.Israel; Phys. Lett., 57A,Nº2, (1976), pp 107-110.*
- [4] *C.E.Laciana; 'Quantum Field Theory in Curved Space Time as Thermo Field Dynamics', submitted to J. of Math. Phys.*

- [5] *J.V.Narlikar and T.Padmanabhan, 'Gravity, gauge theories, and quantum cosmology', (1986), D.Reidel Publishing Company.*
- [6] *M.Castagnino and C.Laciana; Prog. of Theor.Phys., 84, N°4, pp. 616-632, (1990).*
- [7] *S.W.Hawking and G.F.R.Ellis, 'The large scale structure of space-time'. Cambridge Monographs on Math.Phys.(1973).*
- [8] *N.D.Birrell and P.C.W.Davies, 'Quantum Fields in Curved Space'. Cambridge Univ. Press., London/N.Y., (1982).*
- [9] *A.Linde, 'Particle Physics and Inflationary Cosmology', Harwood Academic Publishers (1990).*
- [10] *S.A.Fulling, Gen.Rel.Grav. 10, (1979), 807.*
- [11] *T.Aritmitsu and H.Umezawa; Prog. of Theor.Phys. 77, N°1, (1987).*