Non-Classical Hair on Black Holes

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Abstract

Black holes can have non-classical hair, i.e. they can be characterized by observable quantum numbers associated with discrete charges which are not coupled to long range propagating gauge fields. This blurs the apparent distinction between small black holes and elementary particles, as well has having important implications for wormhole physics.

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It is commonly held that black holes can be characterized entirely by quantum numbers which correspond to those quantities coupled to long range gauge fields, including gravity. That is, they can be characterized completely by their mass, angular momentum, and electric charge. The heuristic reasoning behind this assumption is clear. It is only via long range fields that classical observers located sufficiently far from the event horizon---so that they will not be swallowed up by the hole during the measuring process---can probe the structure of black holes.

However, this reasoning is purely classical. It is well known that in quantum mechanics situations exist where non-zero cross sections for scattering occur even when no classical forces or long range fields exist. The most famous example is that first elucidated in the classic paper of Aharanov and Bohm [1] in which they described scattering of charged particles off an infinitesimally thin solenoid. In this case, even though the electromagnetic field strength outside the solenoid is exactly zero, charged particles whose wave-functions vanish identically at the solenoid can still scatter with a non-geometric cross section per unit length:

$$\frac{d\sigma}{d\theta} = \frac{\sin^2(\pi\alpha)}{2\pi k \sin^2(\theta/2)} \qquad (1)$$

where θ is the scattering angle, k is the momentum in the scattering plane (i.e. perpendicular to the solenoid) and α is the phase, modulo 2π, induced as a particle with charge e travels in a loop around a solenoid carrying flux Φ, i.e $\alpha = e/2\pi\,\Phi$.

In this case, the scattering results because, while the field strength may be zero outside of the solenoid, the vector potential is not. As Aharonov and Bohm stressed, it is the path ordered integral of the vector potential which enters into the global phase of the wavefunction of a charged particle travelling in the vicinity of the solenoid. Beams travelling on opposite sides around the solenoid will therefore in general interfere if the loop integral of the phase factor is not a multiple of 2π. In this case, the scattering cross section in (1) is non-vanishing.

The above example is not directly relevant to the point in question, namely the existence of extra "charges" associated with black-holes or wormholes, where the charges are not associated with currents which couple to massless gauge fields. After all, the vector potential which governs the phenomenon described in this example is that associated with a massless gauge field, even if in this particular example the field strength itself vanishes. Nevertheless, one might question whether an analogous phenomenon might occur in a more general framework. Namely, are there circumstances where no massless propagating gauge fields exist, yet some fields are nevertheless not strictly periodic around arbitrary closed non-trivial loops? If this is the case, then Aharonov-Bohm type arguments suggest that there are topological scattering effects which would allow one to determine that black-holes might have swallowed up "charges" which are not coupled to long range gauge fields. i.e. black holes can have non-classical "hair".

In this essay, I discuss how a general class of field theories [2] can allow for fields to be multi-valued around closed nontrivial loops, without massless long range gauge fields inducing the relevant phase rotations. The multi-valued nature of the fields is a manifestation of a discrete symmetry which is present in the theories at low energies. The key point which allows for observable phases is the fact that this discrete symmetry is *local* [2], not global, in a sense which I shall now describe.

Local discrete symmetries in continuum field theories are not widely discussed precisely because they lack that most important dynamical consequence of their continuous cousins, namely a gauge field. In the continuous case such fields are necessary to formulate covariant derivatives. In the case of a discrete symmetry, this is not necessary, because the ordinary derivative already transforms simply. How then can one tell the difference between local and global continuous symmetries? The form of the local Lagrangian alone is not enough. Consider a local Z_p symmetry generated from a broken U(1) gauge theory. Imagine two scalar fields, η and ξ, carrying charge *pe* and *e*, respectively. If η condenses at some very high mass scale M, while ξ does not, then the effective theory below the scale M will simply appear to be the theory of a single complex scalar field ξ, which is invariant under the seemingly global discrete transformation:

$$\xi \rightarrow \exp(2i\pi/pe)\ \xi.$$

What distinguishes this theory from a theory possessing a global discrete symmetry if the same local interaction terms are allowed in both theories? It is the fact that the local theory from which the former is derived is a gauge theory, so that only gauge-invariant quantities are physically meaningful. Terms which violate the remaining local discrete gauge symmetry will therefore involve operators which are not well defined physically. In this case, no process, not even those associated with black holes or wormholes, can induce such operators.

The original motivation for considering discrete symmetries in this way was to address an apparent embarrassment associated with the recent proposal that wormhole tunneling might determine the parameters of low energy physical theory and in particular explain why the cosmological constant is zero today [3]. It has been argued that wormhole tunneling induces all local interactions consistent with *continuous* gauge symmetries (motivated by the argument that the "baby universes" which may be produced by such tunneling events cannot be closed and at the same time carry non-zero charges with respect to gauge interactions). Even if the wormhole physics is relegated to the Planck scale, this can have disastrous effects on low energy physics. For example, in models with low-energy supersymmetry, renormalizable interactions which allow proton decay are usually forbidden by the presence of discrete symmetries in the models. The argument given above suggests that global discrete symmetries are not immune to violation by wormholes--i.e. fields carrying discrete "charges" may disappear down a wormhole with impunity.[4] If dimension four operators are thus induced by wormhole physics, unsuppressed proton decay can occur, in clear conflict with observation! If these symmetries are local however it was suggested [2] that such operators would not be induced by wormholes, or anything else.

For the purposes of this discussion however, we can state the difference between global and local discrete symmetries in another way. In the local case, additional topological information is necessary to specify the theory---whereas in the global case, fields will be strictly periodic around nontrivial closed loops, in the local case they need be only periodic up to a discrete symmetry transformation.[5] Thus, a low energy observer could distinguish between the two cases by performing an Aharanov-Bohm type experiment.[1] As has been argued above, however, this is precisely what is required to generate non-classical black hole hair.

I give here two examples of non-trivial observable topological phases which can exist in the simple theory described above where, by construction, there are no long range propagating gauge fields.

(a) Consider the observable associated with the operator:

$$\exp(2\pi i Q/pe). \quad (2)$$

While Q itself is ill defined in the broken phase, because the scalar field in the condensate can screen charges, it can only screen charges modulo *pe*. Hence the Hermitian operator in (2) is well defined [6,7], and thus should be associated with an observable. By Gauss' law, however, (2) can be expressed in terms of a surface integral, which can be defined on an arbitrary surface enclosing the charge. Hence, it should be insensitive to what happens near the surface of a black hole which captures such a charge.

(b) One can imagine a thought experiment which replicates exactly the Aharonov-Bohm analysis. In this case the solenoid of infinitesimal width can in fact exist as a stable string solution in our model, if it is threaded by magnetic flux, $2\pi/pe$. A particle with charge *e* (which is therefore not an integer multiple of *pe*) will therefore scatter off the string [8] with a cross section identical to that given in (1) earlier, with $\alpha=1/p$. Hence a black hole accompanied by a charge e arbitrarily close to its event horizon will scatter off such a cosmic string located arbitrarily far away in such a way as to allow a unique determination of the charge e, modulo pe. Since the cross section involves long distance and large time behavior, it should not vary discontinuously depending upon the exact moment the charge e crosses the event horizon. Hence the final asymptotic black hole state should be characterized by such discrete "hair".

Both these examples demonstrate that charges associated with discrete local gauge symmetries should be measurable arbitrarily far from a collapsing black hole, and therefore should represent an intrinsic feature of the asymptotic black hole state. [9] They also indicate why the non-classical hair in this case obviates rather than vitiates the original no-hair theorem. The long-range scattering in this case is in fact due to the existence of the underlying gauge field. In the case of scattering off a cosmic string, a non-zero vector potential exists outside the string to compensate for the scalar field expectation value, even if the gauge field associated with this vector potential is massive. Thus, even though no propagating field exists, the gauge field insidiously works its own topological magic.

Nevertheless, one wonders whether in general the local discrete symmetry can be implemented directly in the continuum, without the intermediate step of breaking a continuous local symmetry. After all, such a possibility seems to arise naturally in the limit that the mass scale M of the symmetry breaking is made arbitrarily large. In this case, the gauge fields should decouple from the theory. Nevertheless, the observable phases should remain, since nowhere in the above argument does the mass scale enter explicitly. However, without the gauge field as an intermediary it is hard to imagine how a non-zero phase can build up. One may require the introduction of topological terms in the theory induced by the different prescription for carrying out the functional integrals which define the theory in the local versus the global case. Such terms could then feed back into topological scattering effects which would give black holes hair. These interesting possibilities are currently under investigation, and relate to ideas being discussed in string theory [10] and in quantum gravity. [11]

In any case, it seems clear that once one extends one's analysis beyond the purely classical domain, to include at least the possibility of discrete gauge symmetries, black holes may have plenty of additional "discrete hair". If black holes can then be characterized by their quantum numbers under such discrete symmetries, as well as under continuous gauge symmetries, then the distinction between small black holes and familiar elementary particles begins to blur. Perhaps someday it shall vanish altogether.

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