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GRAVITATIONAL EFFECTS OF MATTER IN RAPID MOTION

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In surveying the possibilities for artificially influencing gravitation, the first concern is to decide on the physical theory of gravitation to be adopted. It is, of course, easy to invent new theories in terms of which the possibilities of influencing gravitation are rather promising, or to postulate the existence of material with properties that have never been observed, such as a negative mass. In the writer's opinion, such suggestions should be classified as science fiction, and serious survey of the possibilities must be based on Einstein's General Theory of Relativity in its original form (i.e. Unified Field Theories which might or might not lead to differences in the possibilities are also regarded as too speculative).

For the purposes of this essay it is assumed that the problem is to cancel or appreciably reduce the gravitational field in a small region near the surface of the earth. Although all schemes for affecting the gravitational field at a point seem to suffer from the fatal weakness that the effects are so small that they are difficult or impossible even to detect, it must be admitted that the Einstein theory provides in principle more promising possibilities than does the Newtonian. In the latter theory the gravitational field at a point is obtained by superposing the central fields due to each separate particle of matter, and the only possibility seems to be to accumulate an enormous amount of matter just above the point where the field is to be reduced. In the Einstein theory, however, the field is determined in a more complicated way that depends on the motion of nearby matter as well as on its momentary distribution, and in fact the gravitational effects due to nearby matter in rapid motion may be many orders of magnitude greater than that which the Newtonian theory would predict for the same mass.

To show this, we consider the following situation: a test particle at rest is affected by the gravitational field according to the equation of motion

$$\frac{d^2x^\mu}{ds^2} + \Gamma^{\mu}_{\nu\sigma}\frac{dx^\nu}{ds}\frac{dx^\sigma}{ds} = 0 \tag{1}$$

and our objective is to study the possibility of artificially influencing the force which it experiences. We assume further that the gravitational field as seen by the test-particle is static. Under these conditions, we have

$$\frac{dx^4}{ds} = \frac{\delta_{\mu 4}}{g_{44}}, \quad \frac{\partial g_{44}}{\partial t} = 0$$

and (1) reduces to

$$\frac{1}{c^2}\frac{d^2x^\mu}{dt^2} + \Gamma^{\mu}_{44} = 0 \tag{1a}$$

Thus, for $\mu = 1, 2, 3$, the component Γ^{μ}_{44} of the affine connection plays the role of a gravitational field intensity.

We have now to consider the manner in which this field is influenced by the presence of nearby matter. For this, the general rule is the field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\varkappa T_{\mu\nu}. \tag{2}$$

This is to be approximated in a way similar to that used in showing that the Einstein theory contains the Newtonian theory as a first approximation*, except that we do not assume small velocities for the matter giving rise to the field. We can, of course, assume that the metric differs only slightly from the Euclidean, since this would remain true even in gravitational fields that are enormous from the terrestrial point of view.
Thus, we put

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \gamma_{\mu\nu} \tag{3}$$

where

$$g^{(0)}_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & +1 \end{pmatrix}$$

is the Euclidean metric. The $\gamma_{\mu\nu}$ are then small compared to unity, and we neglect their squares and higher powers. From the general formula

$$\Gamma^{\mu}_{\sigma\tau} = \frac{1}{2} g^{\mu\nu}\left[\frac{\partial g_{\sigma\nu}}{\partial x^{\tau}} + \frac{\partial g_{\tau\nu}}{\partial x^{\sigma}} - \frac{\partial g_{\sigma\tau}}{\partial x^{\nu}}\right]$$

using (3) and the fact that the field is static, we get

$$\Gamma^{\mu}_{44} = \frac{\partial}{\partial x^{\mu}}\left(\frac{\gamma_{44}}{2}\right), \quad \mu = 1, 2, 3. \tag{4}$$

Comparing with (1a) we see that the gravitational potential is

$$\varphi = \frac{\gamma_{44} c^2}{2} \tag{5}$$

Einstein shows (MR, p. 87) that by expressing the contracted Riemann tensor $R_{\mu\nu}$ in terms of the metric and making a certain choice of the coordinate system, the solution of (2) may be written in a retarded potential form:

$$\gamma_{\mu\nu} = -\frac{\varkappa}{2\pi}\int \frac{T^{*}_{\mu\nu}(x', y', z', t' - \frac{r}{c})}{r}\, d^3x' \tag{6}$$

where

$$T^{*}_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^{\sigma}_{\sigma}.$$

** See, for example, A. Einstein, "The Meaning of Relativity", Princeton Press, 1946 (Hereafter referred to as MR); pp 79-90

If the matter near the test particle is described by a rest-density ρ and a four-velocity $\frac{dx^\mu}{ds}$, we have ($x^4 = ct$)

$$T^{\mu\nu} = \rho \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}$$

and, neglecting terms in $g_{\mu\nu}$,

$$T^\sigma_\sigma = \rho, \quad T^{44} = T_{44} = \frac{\rho}{1-\beta^2} \qquad \left(\beta = \frac{v}{c}\right)$$

$$T^*_{44} = \frac{\rho}{1-\beta^2} - \frac{\rho}{2} = \frac{\rho}{2}\left(\frac{1+\beta^2}{1-\beta^2}\right).$$

Thus, the gravitational potential seen by the test particle is given by

$$\varphi = -\frac{\kappa c^2}{8\pi} \int \left[\rho\left(\frac{1+\beta^2}{1-\beta^2}\right)\right] \frac{d^3x}{r}, \tag{6a}$$

the square brackets indicating retarded quantities. The quantity ($\kappa c^2/8\pi$) in (6a) is equal to the Newtonian gravitational constant $G = 6.67 \times 10^{-8}$ $cm^3\ gm^{-1}\ sec^{-2}$.

It is seen that as the velocity of nearby matter approaches that of light, its gravitational effects increase without limit. This increase is not just that due to the increased energy density, which is (kinetic + rest energy)

$$\frac{\rho c^2}{1-\beta^2}$$

In this expression, one factor of

$$\frac{1}{\sqrt{1-\beta^2}}$$

may be regarded as arising from kinetic energy, the other accomplishing the transformation of mass density from a coordinate system moving with the matter (in which the density is ρ) to the laboratory system. Because of the factor ($1 + \beta^2$) in (6a), a quasi-Newtonian theory in which we make use of the equivalence of mass and energy, but not any other relativistic effects, would not predict the correct gravitational effects. At high energies, this factor approaches two, and therefore the gravitational field due to matter in rapid motion is twice as great as one might expect from its mass. This corresponds to a famous factor of two occurring in the expression for the deflection of light in the gravitational field of the sun. There also, a quasi-Newtonian point of view in which one uses the velocity and mass of a photon but not relativistic effects proper, results in a prediction of just half the correct amount of deflection.

Regardless of what is in principle possible, the sad fact remains that the magnitude of these effects is so small that there is no prospect in sight of making any practical utilization of them, and little prospect even of detecting them in the laboratory with available techniques, so there is no point at present in speculating on the details of the gadgets that might be involved.

One loophole in the above arguments which has not been explored by the writer concerns the assumption of a static gravitational field at the test particle. If nearby masses are set into violent oscillation, new effects appear, some of which are discussed by Einstein in MR. However, the above remarks about numerical magnitude remain applicable, and we conclude that there is no prospect, within the limits of present physical theory and present techniques, of influencing gravitational fields to any practically useful extent.

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Summary A short calculation demonstrates that according to the Einstein theory of gravitation, the gravitational field produced by a given amount of matter may be increased by setting it in rapid motion. This is due to two causes; (1) the added kinetic energy increases its mass according to the equivalence of mass and energy, and (2) a further factor of two occurs at high velocities, which is of kinematic origin. Thus, in the Einstein theory the gravitational field produced by matter in rapid motion may be many orders of magnitude greater than that which the Newtonian theory would predict for the same rest-mass, and is in fact twice as great as would be predicted by a quasi-Newtonian theory in which the mass of kinetic energy is taken into account.

The numerical magnitude of these effects is, however, so small that there appears to be no prospect at present for any practical utilization.