23

11

IS GRAVITATIONAL COLLAPSE IRREVERSIBLE?

Werner Israel

Department of Mathematics
University of Alberta
Edmonton, Canada

Series A: Preprints of Research Papers
No. 10, Vol. 2

April, 1966

ABSTRACT

The current doctrine of the irreversibility of gravitational collapse is subjected to a critical examination. It is concluded that irreversibility is not, as is generally believed, a consequence of general relativity, but arises from an implicit ad hoc assumption. A more natural interpretation of general relativity leads to a picture of gravitational pulsation which is much more in accord with the observed features of quasi-stellar sources.

1. Quasi-stellar Sources

If the cosmological interpretation of their large red-shifts is accepted, the quasi-stellar sources lie near the outskirts of the visible universe and close to its origin in time - "sparks of the primeval fireball". A lower bound to the masses which must be involved (10^8 solar masses) is obtainable from the enormous energies stored in the relativistic electrons which gyrate in their magnetic fields [1], whereas the observed optical and radio variability [2,3] place an upper limit of about a light-week (10^{16} cm) on the size of the central emitting nucleus. A non-rotating ball of 10^8 solar masses becomes gravitationally unstable if its radius gets smaller than 4.7×10^{17}cm [4]. It seems difficult to escape the conclusion that the typical quasi-stellar source is in a state of gravitational collapse approaching free fall.

The predicament to which this leads is well-known. According to the picture of gravitational collapse which has become traditional [5], the collapse proceeds irreversibly to zero volume. An external observer sees the body asymptotically shrinking to its Schwarzschild radius. The observed time of collapse is thus infinite; but progressive dimming due to the Doppler and gravitational red-shifts, and to the falling back of light rays sent out obliquely from the surface, will render the object invisible to the external observer in a time comparable to the Newtonian free-fall time of 25 years [6]. This picture is obviously incompatible with the life-times of 10^6 years inferred from the linear dimensions of the sources ($\sim 10^6$ light-years), and the clear evidence (from the extended life-times and the variability) of repeated injections of relativistic particles.

It has been known for some years that the continued activity in the supernova remnant of 1054, the Crab nebula, confronts us with a similar difficulty [7].

In the face of this apparent impasse, it is very curious and perhaps suggestive that a naive application of Newton's theory will account in a perfectly natural way for all the major observed features of quasi-stellar sources. Imagine the Newtonian collapse of a ball of 10^8 solar masses. If support from gas and radiation pressure fails, the collapse continues until, at zero radius, there is a violent "self-collision" and rebound, with the ejection of high-speed streams of ionized gas and particles. For a radius at maximum of 10^{17} cm, the resulting pulsations have a period of about 20 years. (Observations of the quasar 3 C 273 over the past 70 years suggest cycles of about 10 years for major fluctuations [2].) We can elaborate this model a little by imagining the ball deformed into an irregular lumpy mass. Each lump, while sharing in the oscillation of the mass as a whole, executes its own quicker pulsation. This reproduces the "light flashes" and irregular variations observed in many quasi-stellar sources.

At this point I should make it clear that I am not arguing for the reinstatement of Newton's theory. However, the above model, which is based essentially on the reversibility of gravitational collapse, fits the observed facts so naturally that one cannot help asking whether, on this issue, Einstein's theory is - not wrong - but misinterpreted. Is it really true that general relativity predicts the irreversibility of gravitational collapse? The remainder of this essay is devoted to a critical examination of this question.

2. Radial Motion in the Schwarzschild Field.

The history of a particle on the outer boundary of a collapsing ball is a radial time-like curve of the exterior Schwarzschild field. Thus, many of the essential features of gravitational collapse are brought into focus by studying the radial motion of particles in the Schwarzschild vacuum manifold.

In terms of Schwarzschild's familiar r, t co-ordiantes, the paths of incoming and outgoing radial light-pulses have the equations u = const. and v = const. respectively, where

$$4\,mu^{-1}\,du = (1-2m/r)^{-1}\,dr + dt\,,$$
$$4\,mv^{-1}\,dv = (1-2m/r)^{-1}\,dr - dt\,.$$

These null curves are shown in Fig. 1, together with the future light cones which they define.

The time-like path ABC of a radially moving test particle can now be traced out qualitatively if we remember that it must always be directed into the future light-cones. The particle reaches $r = 2m$ at a perfectly definite event C, attained in a finite proper time. But, because of failure of Schwarzschild's t co-ordinate at $r = 2m$, the event C cannot be mapped onto the finite part of the r, t plane. We can remedy this deficiency by straightening the null lines, i.e. adopting u, v as new co-ordinates. The new form of Schwarzschild's line-element is [8]

$$ds^2 = \frac{32m^3\,du\,dv}{r e^{r/2m}} + r^2\,d\Omega^2\,, \qquad (1)$$

with $r(uv)$ defined by

$$\left(\frac{r}{2m} - 1\right) e^{r/2m} = uv\,,$$

and this is regular down to $r = 0$.

3. Possible Interpretations of the Extended Manifold.

In the new map (Fig. 2), we can follow the career (ABCD) of the falling particle down to $r = 0$.

To proceed further, we have first to decide how to interpret the peculiar duplication which occurs in the uv plane. Mathematically, this corresponds to the symmetry of the line-element (1) under the transformation

$$u \to -u, \qquad v \to -v . \qquad (2)$$

It may be shown [9] that this symmetry is preserved even when the manifold is disturbed by the presence of other bodies.

The first idea that comes to mind is that the duplication is merely a duplication of the <u>mapping</u> - that the line-element (1) represents an "elliptic" space in which the events (u,v) and $(-u, -v)$ are physically identical.

Actually, until very recently [10,11] this idea was not followed up. The major discouraging factor appears to have been this: Let us assume that future light cones in quadrant I of the uv plane have a south-easterly direction, as indicated in Fig. 2. If we accept the identification just proposed, then the future light cones assigned to quadrant III must point north-west. Since this assignment cannot be extended continuously into II and IV, we would be forced to admit a breakdown of the global distinction between past and future in the domain with $r < 2m$. This raises the spectre of possible causal violations.

Consequently, it has been traditional to escape these worries by the (implicit) introduction of Postulate X: Past and future can be globally distinguished throughout the extended Schwarzschild manifold.

4. Postulate X and Irreversibility

Postulate X permits us to mark future light cones into the four quadrants of the uv plane, all pointing (say) south-east.

On this basis, let us return to consider the fate of the particle which has reached $r = 0$ at event D. Can it rebound from the center or pass through the center into the radially opposite space? Fig. 2 shows that every time-like path which emerges from the branch PDQ of $r = 0$ is directed into the past defined globally in quadrant IV, and this is forbidden. We have no alternative to the conclusion that a particle which has fallen to $r = 0$ is irreversibly swallowed up by the singularity. Correspondingly, a spherical body (boundary ABCD) which has collapesed under its own weight to zero radius can never re-expand: its subsequent history is the space-like singular curve DQ.

It is clear from this argument that the irreversibility of gravitational collapse is a consequence - not of general relativity - but of the ad hoc Postulate X.

Irreversibility is not required by the elliptic interpretation. Since future and past are not globally distinguished in quadrant IV, it is possible - and, indeed, most natural - to assume a "bounce" at $r = 0$.

5. The Difficulties with Postulate X.

Since the traditional picture of gravitational collapse hinges critically on the validity of Postulate X, it is important to test the soundness of this postulate. Examination reveals a number of disturbing features. I list a few of them.

(i) The imposition of a global past-future arrow onto the uv plane breaks the symmetry, since the arrow is not invariant under (2). To this extent, Postulate X is alien to the geometry and runs counter to the clear indications of general relativity.

(ii) It is unprecedented and very strange that a postulate based essentially on the statistical coarse-grained concept of a past-future distinction should need to be inserted into the foundations of the more primitive subject of dynamics.

(iii) Every slice $t = \text{constant}$ divides the extended manifold into two sheets with $r > 2m$, joined by an Einstein-Rosen bridge. These sheets are in all respects geometrically identical, even in the presence of external disturbances. But - if Postulate X is accepted - only one is the ordinary space of our experience; the other is a physically distinct space in which each of us has an identical twin. This seems uncanny. Even more uncanny is the fact that, while no signal can pass between us, my twin appears to be in some kind of instant telepathic contact with me: when I crook my little finger he simultaneously (at the same "time" t) crooks his.

(iv) The global past-future arrow defined by Postulate X cannot agree with the direction of increasing $t = 2m \ln (u/v)$ in both quadrants I and III. This has the following curious consequence. Suppose that a sequence of slices $t = 1, 2, 3, \ldots$ shows me aging. My twin, identical with me on each slice, will go through the same sequence of physical transformations. But, according to Postulate X, he would have to describe himself as getting younger! This drains most of the meaning from the words "past" and "future" as used in Postulate X.

(v) According to the traditional picture based on Postulate X , the final history of a collapsed star is a (singular) space-like curve. Relativistic principles would lead us to expect the history of any material body to be time-like.

6. Conclusion.

Individually, the foregoing objections are perhaps not fatal, but together they add up to a formidable indictment.

The elliptic interpretation is free of these objections. Moreover, detailed investigation [11] shows that the causality bogey does not materialize under actual physical conditions. There is thus good reason to expect that the most natural interpretation of the Schwarzschild manifold is, in addition, self-consistent and concordant with the astronomical evidence.

References.

[1] Jesse L. Greenstein and Maarten Schmidt: Ap. J. 140, 1 (1964).

[2] Harlan J. Smith and Dorrit Hoffleit: Nature 198, 650 (1963).

[3] W. A. Dent: Science 148, 1458 (1965),
P. Maltby and A. T. Moffet: Science 150, 63 (1965).

[4] S. Chandrasekhar: Ap. J. 140, 417 (1964).

[5] J. R. Oppenheimer and H. Snyder: Phys. Rev. 56, 455 (1939),
W. A. Fowler: Rev. Mod. Phys. 36, 545 (1964),
Ya. B. Zeldovich and I. D. Novickov: Soviet Physics (Uspekhi) 7, 763 (1965),
B. K. Harrison, K. S. Thorne, M. Wakano and J. A. Wheeler: Gravitation Theory and Gravitational Collapse (University of Chicago Press, Chicago 1965), Chap. 11.

[6] Ya. B. Zeldovich and I. D. Novikov, loc. cit.

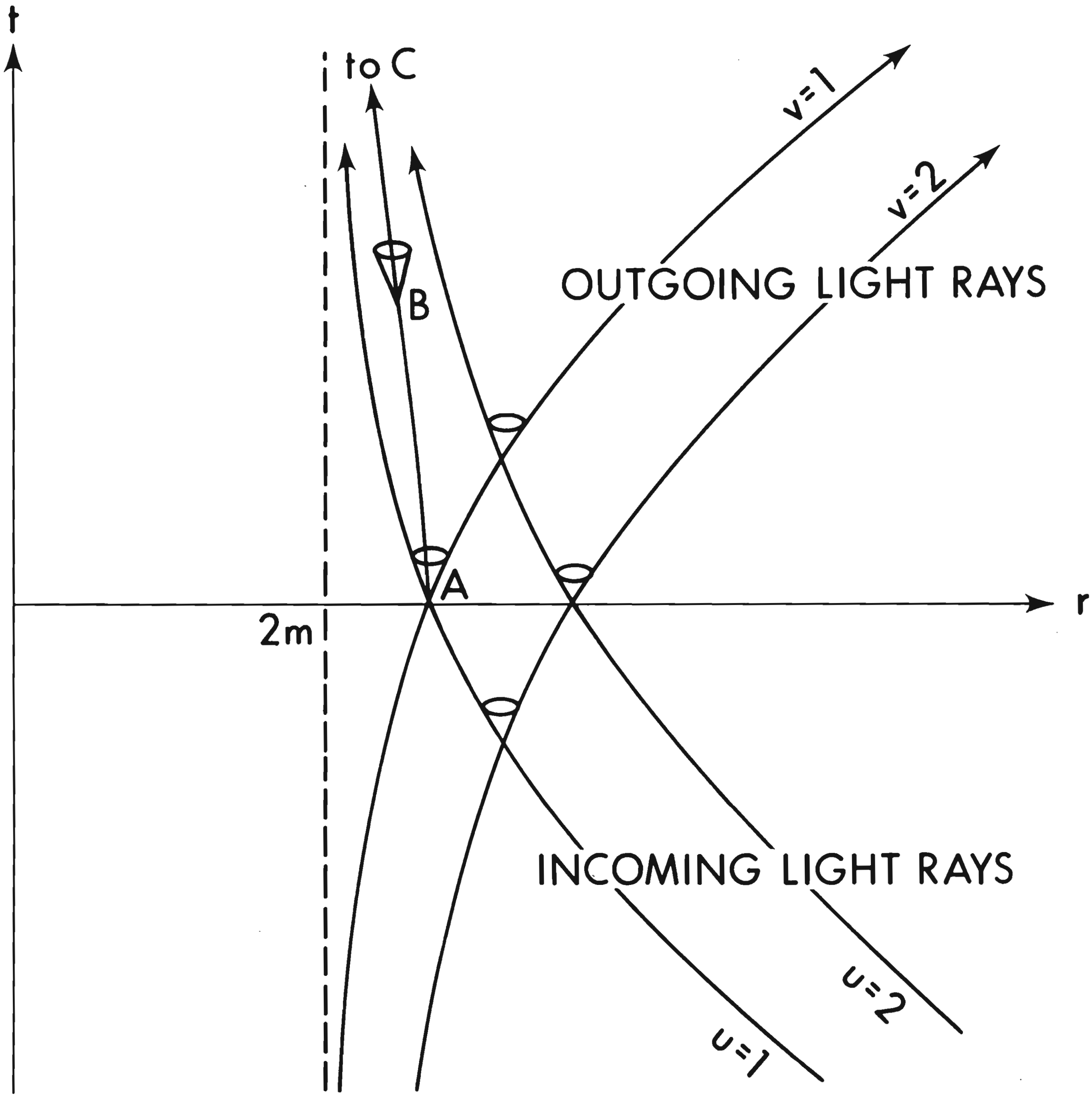
[7] See, for instance,
J. Shklovsky: Nature 201, 588 (1964),
F. Hoyle, W. A. Fowler, G. R. Burbridge and E. M. Burbridge: Ap. J. 139, 909 (1964).

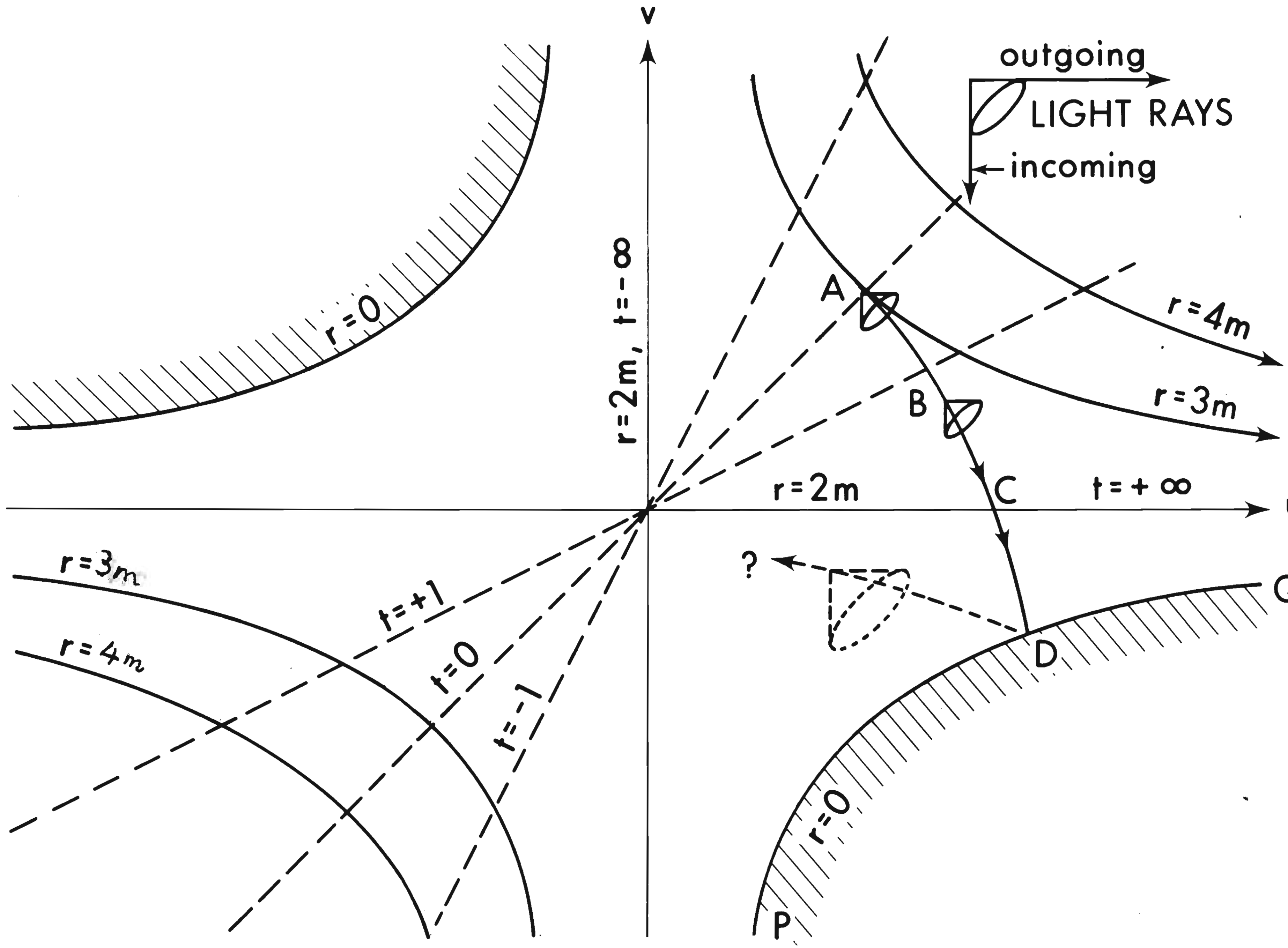
[8] Compare
M. D. Kruskal: Phys. Rev. 119, 1743 (1960)
G. Szekeres: Publ. Math. Debrecen 7, 285 (1960).

[9] L. A. Mysak and G. Szekeres: Can. J. Phys. 44, 617 (1966),
W. Israel and K. A. Khan: Nuov. Cim. 33, 331 (1964).
These papers treat static disturbances only. A general proof has recently been obtained by the author (W. Israel: Physics Letters, submitted for publication). Issue of March 25

[10] F. J. Belinfante: Physics Letters 20, No. 1 (1966),
W. Israel: Nature 209, 66 (1966), Phys. Rev. .... (1966).
J. L. Anderson and R. Gautreau: Physics Letters 20, No. 1 (1966),
W. Rindler: Phys. Rev. Letts. 15, 1001 (1965).

[11] W. Israel: Physics Letters 20, No. 7 or No. 8 (1966) (to appear),
: Nuov. Cim. (submitted for publication).


t
to C
v=1
v=2
B
OUTGOING LIGHT RAYS
A
r
2m
INCOMING LIGHT RAYS
u=2
u=1


v
u
r=0
r=2m, t=−∞
r=2m
t=+∞
outgoing
LIGHT RAYS
incoming
A
B
C
D
?
r=4m
r=3m
t=+1
t=0
t=−1
P
Q

DEPARTMENT OF MATHEMATICS

EDMONTON, ALBERTA
CANADA

Werner Israel

Born: Berlin, Germany, 4 Oct. 1931.

Attended: Cape Town High School;
University of Cape Town (B.Sc. 1951,
M.Sc. 1954);
Trinity College, Dublin (Ph.D. 1960).

Positions: Lecturer in Mathematics, University of Cape Town, 1955-56;

Scholar, Dublin Institute for Advanced Studies 1956-58;

Assistant Professor (1958-[illegible]),
Associate Professor ([illegible]-66)
Mathematics Department, University of Alberta, Edmonton;

Visiting Professor, Dublin Institute for Advanced Studies (1966-67, on sabbatical leave from University of Alberta).

Married [illegible], 2 children.

Research Interests: Relativity, Astrophysics, Statistical Mechanics, Hydrodynamics.