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EXPERIMENTAL TEST FOR MASS ANISOTROPY BASED ON NUCLEAR MAGNETIC RESONANCE

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Generally, in physics, the inertial mass of a body is assumed to be a scalar quantity.[1] Hence, for example, Newton's law of motion for a particle is taken to be:

$$\vec{F} = m\vec{a} \tag{1}$$

in which the force $\vec{F}$ and acceleration $\vec{a}$ are vector quantities and the mass m is a scalar quantity. However, we can ask, What is the experimental evidence against inertial mass being a tensor quantity? If this were the case, Newton's law would be:

$$F_i = \sum_j m_{ij} a_j \tag{2}$$

in which F_i and a_j are the vector components of the force and acceleration, respectively, and m_{ij} are the elements of the inertia mass tensor. It is clear that if equation (2) applies, the magnitude of the acceleration depends on the direction as well as on the magnitude of the force. If the inertial mass were a tensor quantity, it would then have a directional dependence, and the mass could be called anisotropic. According to the equivalence principle of general relativity[1] the gravitational mass is equal to the inertial mass, so that gravitational mass would then also be a tensor quantity with a directional dependence or spatial anisotropy.

A plausible physical principle which suggests that mass may be anisotropic is Mach's principle in one of its many forms.[2,3] Mach proposed that an inertial frame of reference is determined by the mass distribution in the universe, that the inertial force on a body is the gravitational interaction of distant matter on the body, and that the inertial mass of a body is determined by all the matter in the universe. From this viewpoint it is reasonable to suppose that the inertial mass of a body may have a directional dependence or be anisotropic, if the mass distribution in the universe is anisotropic about the body.

A specific model for mass anisotropy has been proposed by Cocconi and Salpeter,[4,5] which allows the computation of observable effects of mass anisotropy on energy levels and provides a useful formulation for comparison of the sensitivities of various experimental searches for mass anisotropy. The model specifies that the contribution dm to the anisotropic part of the inertial mass of a body due to a mass M at a vector distance $\vec{r}$ is given by the proportionality:

$$dm \propto \frac{M}{r^{\nu}} P_2(\cos\theta) \qquad (3)$$

in which θ is the angle between $\vec{r}$ and the direction of acceleration of the body, $P_2(\cos\theta)$ is the Legendre polynomial of 2nd order, and ν is a constant with $0 < \nu < 1$. A dependence of dm on higher even order Legendre polynomials is also possible. The total anisotropic mass Δm of a body is obtained by integrating equation (3) over all the mass in the universe. If we consider that Δm is due only to a mass M--for example, the mass in our galaxy located at the center of our galaxy--, then the kinetic energy term ΔT corresponding to Δm is given by:

$$\Delta T = -\frac{\Delta m}{m_0} T_0 P_2(\cos\theta) \tag{4}$$

in which m_0 is the isotropic mass of the body (the usual scalar mass) and T_0 is the usual kinetic energy associated with m_0.

In a quantum mechanical problem of a particle bound in an atom or a nucleus the contribution ΔE to the binding energy due to the term ΔT of equation (4) is given by:

$$\Delta E = \frac{\Delta m}{m_0} \overline{T_0 P_2(\cos\theta)} \tag{5}$$

in which first order perturbation theory is used and the bar indicates the expectation value of the operator. This term ΔE has the same angular dependence as that of an electric quadrupole interaction.

The most sensitive test for mass anisotropy (and the central theme of this essay) is that based on a nuclear magnetic resonance experiment.[6,7,8,9] Inspection of equation (5) indicates that the greatest sensitivity for detecting the anisotropic mass term $\Delta m/m_0$ is obtained with small ΔE and large $\overline{T}_0$ Conventional nuclear magnetic resonance (NMR) spectroscopy[10] provides the most favorable opportunity discovered thus far, since an energy shift ΔE as small as 0.01 cps (in frequency units) can be detected, and the kinetic energy term $\overline{T}_0$ is the characteristic large value of 10 MeV for a nucleon in a nucleus. The sensitivities available in an atomic spectroscopy experiment[6] or in an experiment using the Mössbauer effect[11] are many orders of magnitude less.

The principle of the NMR experiment is to study the line shape of an appropriate NMR signal as a function of the orientation of the magnetic field in space. In order to test for the presence of the $P_2(\cos\theta)$ angular dependence in equation (5), it is necessary to have a nucleus with spin $I \geq 1$. The nucleus studied was Li^7 which has a spin $I = 3/2$, and for which the shell model of nuclear structure[12] predicts that there is a single $1p_{3/2}$ proton outside of closed shells. In a magnetic field H_o there are four energy levels corresponding to the magnetic sublevels $M_J = +3/2, +1/2, -1/2$ and $-3/2$. If the term ΔE of equation (5) is neglected, the energy spacings between all adjacent levels which differ by 1 in the M_J value are equal to ΔW:

$$\Delta W = \mu_N g_{Li^7} H_o \tag{6}$$

in which μ_N = nuclear magneton and g_{Li^7} is the g-value of Li^7. A single resonance line should be observed. If the term ΔE is considered, then each of the M_J levels is shifted by the amount $\Delta E(M_J)$:

$$\Delta E(M_J) = \begin{cases} +\frac{1}{5}\frac{\Delta m}{m_o}\overline{T}\,P_2(\cos\beta) & \text{for } M_J = \pm 3/2 \\ -\frac{1}{5}\frac{\Delta m}{m_o}\overline{T}\,P_2(\cos\beta) & \text{for } M_J = \pm 1/2 \end{cases} \tag{7}$$

in which β is the angle between the direction of the magnetic field $\vec{H}_0$ and the direction from the nucleus to the mass M. Three separate resonance lines or a single broadened, unresolved line should be observed.

The method of the experiment was a modern version of a standard NMR absorption experiment. A Varian 12 inch high-resolution magnet was operated at a field near 12 kilogauss. It was powered by a current regulated supply and further stabilized using a superregenerative field tracking oscillator.[13] Magnetic field homogenizing coils and sample spinning were employed to improve the resolution of the NMR spectrometer. The detection system employed a twin-tee radio frequency bridge driven at a frequency of about 19 Mc/sec, which was derived from a digital frequency synthesizer and compared to a standard frequency of a cesium atomic frequency standard. The sample was a saturated aqueous solution of LiCl. Resonance line widths of 0.5 cps were obtained by sweeping the frequency with a fixed magnetic field. Resonance lines were observed for periods of 24 hours with a fixed orientation of the magnetic field in the laboratory. Such data were obtained for several different orientations of the magnetic field.

The data can be analyzed for a contribution due to mass anisotropy. Data obtained over a period of a day provide in general a variation of β. The range of variation of β will depend on the direction to the mass M and on the orientation of the magnetic field in the laboratory. The results we quote below assume that M is in the direction to the center of our own galaxy.

The results for the Li^7 experiment are that only a single resonance line was observed and its width of about 0.5 cps is due to magnetic field inhomogeneity and other instrumental effects. The coefficient of a diurnal variation in the linewidth of the form $P_2(\cos\beta)$ is less than about 0.01 cps. Use of equation (7) with $\overline{T}$ = 10 MeV gives the result:

$$\frac{\Delta m}{m_o} < 5 \times 10^{-23}$$

in which a 95% confidence level is employed.

In order to test for a $P_4(\cos\theta)$ dependence in an equation similar to equation (3), similar NMR studies were made with the nucleus Rb^{85}, which has a spin $I = 5/2$. Due to the larger linewidth of the Rb^{85} resonance arising from its electrical quadrupole interaction, a less sensitive test is obtained for a $P_4(\cos\theta)$ term. For this case

$$\frac{\Delta m}{m_o} < 3 \times 10^{-20}$$

Although the experiment discussed above on the anisotropic component of inertial mass was motivated by the model of Cocconi and Salpeter, its null result can be regarded as independent of the model. The implications of this null result have been discussed extensively by Dicke[14,15] and others.[16] Dicke points out that this null result "constitutes an extremely severe test of the local isotropy of space and thus imposes a strong condition on any theory of gravitation."

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Summary

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A highly sensitive experimental test for an anisotropic component of inertial mass is described, which is based on a study of nuclear magnetic resonance. If the interpretation of the experiment is based on the model of Cocconi and Salpeter, the limit to the anisotropic component Δm of inertial mass is:

$$\frac{\Delta m}{m_o} < 5 \times 10^{-23}$$

in which m_o is the usual scalar mass. This null result constitutes an extremely severe test of the local isotropy of space.

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