THE VALUE OF SINGULARITIES

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The Value of Singularities

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Abstract

We point out that spacetime singularities play a useful role in gravitational theories by eliminating unphysical solutions. In particular, we argue that any modification of general relativity which is completely nonsingular cannot have a stable ground state. This argument applies both to classical extensions of general relativity, and to candidate quantum theories of gravity.

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General relativity provides an accurate description of a wide range of gravitational phenomena. However it is not a complete theory of gravity since it exhibits spacetime singularities. This would not be a serious limitation if singularities were rare, but the theorems of Hawking and Penrose [1] show that they are ubiquitous, arising in large classes of solutions to Einstein's equations. More precisely, these theorems state that under rather generic conditions, solutions will be geodesically incomplete. In many explicit examples, *e.g.*, gravitational collapse to form a black hole, the incompleteness occurs when geodesics terminate in a region of diverging curvature. Yet while general relativity reaches an end, physics must continue. Thus the description provided by general relativity breaks down in a domain where the curvature is large, and a proper understanding of such regions requires new laws of physics.

In a domain of Planck scale curvatures, the character of gravity will change radically since its quantum nature will become manifest. The true physics of curvature singularities may only be revealed in the fully quantized theory. The widespread expectation[1] is that singularities will be "smoothed out" or "resolved" in this theory.

Alternatively, the physics required to understand curvature singularities may arise at a classical level. Quite possibly, our classical description of gravity must be modified before quantization. For example, classical string theory modifies the equations of motion from those of general relativity [2]. Typically these modifications can be understood in the context of a generally covariant extension of the Einstein action with new higher curvature interactions

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + F(g_{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma}) \right] \tag{1}$$

where F is an arbitrary scalar function of the metric, the curvature and its derivatives.[2] In

[1] A notable exception is Roger Penrose, who has argued that since the early universe was very special, the Big Bang singularity must remain in some form in the ultimate theory.

[2] We assume the cosmological constant vanishes, although the following argument should extend to nonzero Λ (as well as higher dimensions and/or the inclusion of matter fields).

the modified equations of motion, the effect of these new terms will be negligible for modest gravitational fields, so these theories are still consistent with all of the usual experimental tests. In regions of large curvature, though, these terms can greatly affect the nature of the solutions. In particular, the contributions from the higher curvature interactions spoil certain local energy conditions required to prove the singularity theorems. Hence such theories evade these theorems, and one might hope to construct a singularity-free extension of general relativity [3].

In this essay, we argue that on physical grounds any reasonable theory will not "resolve" certain classes of timelike singularities. The elimination of these singularities would lead to a theory without a stable ground state. Thus some form of singularity is *required* for the theory to be well-behaved. This argument applies to the full quantum theory of gravity as well as classical extensions of general relativity, with which our discussion begins.

Since we want (1) to reduce to general relativity for long distances and weak curvatures, the first term (with the fewest derivatives) is the scalar curvature. Now consider the negative mass Schwarzschild metric. Asymptotically, the curvature is small, so the higher order terms in the equation of motion are negligible and this metric provides an approximate solution for (1). What happens as we extend the solution in toward $r = 0$? Since the field equations differ significantly from general relativity in regions of large curvature, it is certainly possible that the metric remains completely nonsingular. However, the theory would then have a regular negative energy solution, and so Minkowski spacetime would not be stable. In fact, if we want the theory to have any stable lowest energy solution, it must have singularities in order that one may discard what would otherwise be pathological solutions. Even if the theory claims to have a ground state with $E < 0$, we can always start with the Schwarzschild metric with $M < E$ and argue that it must be singular. This does not contradict the positive energy theorem because the higher curvature terms violate the local energy condition required by this theorem. Thus one has no guarantee that a ground state exists. In fact we see that removing all singularities yields states with arbitrarily negative energy.

This simple observation is a powerful constraint on attempts to construct a singularity-free extension of general relativity. Notice that it is not necessary to define a singularity in terms of geodesic incompleteness in order to apply this result. For example, string theory is a modification of general relativity in which singularities are defined in terms of the motion of (quantum) test strings. It has been argued that several geodesically incomplete solutions (*i.e.,* singular spacetimes from the viewpoint of general relativity) are nonsingular by this criterion [4]. Unless the string solution which approaches the negative mass Schwarzschild metric at large distances is singular in the string sense, and hence unphysical, no stable ground state will exist.

A similar result must also exist in quantum gravity. If the classical theory has regular solutions with arbitrarily negative energy, then it is not likely to lead to a quantum theory with a stable ground state. Note that these states constitute a new instability beyond those usually considered in higher derivative theories [5]. Also, unlike the case of the classical hydrogen atom in which the negative energy orbits are confined to a compact region of phase space [5], here we expect a large volume of negative energy solutions since one can superpose arbitrary numbers of them (at wide separation) with independent positions and velocities.

Alternatively, suppose the negative mass solutions are classically singular, but the quantum theory of gravity "smooths out" these singularities. Then there will again be states of arbitrarily large and negative energy. In particular, there have been frequent suggestions that spacetime is essentially discrete in quantum gravity at the Planck scale. This possibility has been considered in string theory [6], and the nonperturbative canonical quantization program initiated by Ashtekar [7], as well as other approaches [8]. If spacetime is fundamentally discrete, it is difficult to see what will prevent a state which resembles the negative mass Schwarzschild solution from existing in the theory.

There is one caveat to the above result which should be mentioned. If a theory has negative energy configurations one usually cannot just "throw them out" since one would expect them to be dynamically produced. However, it may be that such production is prohibited, and the negative energy states simply decouple from the theory causing no instability. An illustration of this is provided by Kaluza-Klein theory. It is known that if one allows nontrivial topology, Kaluza-Klein theory admits nonsingular initial data sets with arbitrarily negative energy [9]. However if fermions are included, the theory has two noninteracting "superselection sectors", in which the spinors are periodic or antiperiodic about the compact dimension. One can show that the energy cannot be negative for the periodic sector. Thus the pathological configurations would simply not be a part of the stable and presumably physical sector of the theory.

However, it is difficult to imagine how such a superselection argument could be implemented in the case discussed here. By just considering geodesics in the asymptotic region or the scattering of gravitons in these configurations, we know that they do indeed act as localized negative mass objects which couple to gravity in the usual way. Hence conventional wisdom would indicate that they would be created in gravitational interactions, *e.g.*, the collision of gravitational waves. Note that a four dimensional negative mass solution would destabilize even the periodic sector of a Kaluza-Klein theory.

To summarize, we have argued that rather than being an undesirable feature of a theory, singularities play a useful role – they enable a stable ground state to exist. Of course our argument only requires the existence of timelike singularities that persist for all time. Of greater concern are the singularities which form from the evolution of nonsingular initial conditions. It is possible that there exists a theory with a stable ground state in which these singularities are removed. However most of the attempts to eliminate singularities consist of brute force approaches which do not distinguish between singularities resulting from collapse and those existing for all time. The lesson we should draw is that if we wish to find a more complete theory which prohibits the formation of singularities from regular initial conditions, we must find a more subtle mechanism which distinguishes time-independent and time-dependent strong curvature regions.

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