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[illegible] OF THE NOON-MIDNIGHT RED SHIFT

by

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Abstract

A terrestrial atomic clock at noon is 8000 miles nearer the sun than an antipodal clock at midnight. The difference in gravitational potential due to the sun causes a red shift of order 8×10^{-13}, but this is almost exactly cancelled by the centrifugal effect. However, any significant shielding or focussing of solar gravity by the earth would affect the gravitational but not the centrifugal red shift, and so might be detected. This would be a test of gravitational shielding for a non-mechanical phenomenon and thus a new test of the principle of equivalence, among other things.

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[illegible] atomic clocks at antipodal points on the equator at the time of an equinox, so that when one clock is at local noon with the sun directly overhead the other clock is at local midnight. Then the noon clock will be some 8000 miles closer to the sun. Therefore, since the gravitational potential due to the earth is the same for both clocks, the noon clock will be at a lower gravitational potential than the midnight clock.

A quick calculation shows that this difference of gravitational potential corresponds to a red shift of the noon clock relative to the midnight clock of order

$$\frac{\Delta\nu}{\nu} = 8 \times 10^{-13}$$

Atomic clocks are already accurate to better than one part in 10^{12}, and we can look forward to significant increases in the precision of time measurement. Thus we are emboldened to propose the experiment of comparing the rates of the noon and midnight clocks.

But reflection shows that the above calculation was too hasty. For the noon clock, being closer to the sun, has a smaller orbital velocity than the midnight clock. This causes a change in the relative frequencies of the clocks that may be attributed to the second order Doppler effect or, more interestingly, to the difference in the "centrifugal potential" of the two clocks. When we calculate the centrifugal effect we find that it almost exactly cancels the above purely gravitational red shift, leaving a residual shift of the order 10^{-21}. Nor does taking account of the diurnal rotation of the earth significantly alter the situation.

This result is so disappointing that we are tempted to abandon the proposed experiment forthwith.

But on further reflection we see that the experiment can yield highly significant information about gravitation; and indeed that it can do so irrespective of whether the solar gravitational and centrifugal effects almost cancel or not.

For in the above calculations we assumed there was no gravitational shielding — we use the word in its broadest possible meaning. Yet 8000 massive miles of earth are interposed between the midnight clock and the sun. There is no *a priori* reason why gravitational shielding should be impossible. On the contrary, the fact that the field equations of the general theory of relativity are non-linear shows that, according to that theory, there must be gravitational

"shielding" effects — either positive or negative — in the sense that the gravitational field of the sun and the earth together is not equal to the sum of their individual fields. Admittedly, these non-linear general relativity effects are extremely small and, in the situation envisaged, well below present limits of measurability; and they are of a different nature from electromagnetic shielding effects which arise from the existence of charges of both signs and do not depend on non-linearity. But they show there is nothing inherently impossible about the idea of gravitational shielding of some sort, even if negative gravitational mass does not exist. If significantly larger shielding effects than the general relativistic ones were experimentally detected there would be important repercussions, not the least of which would be the need to revamp the general theory of relativity, and perhaps even to replace it.

The success of the Newtonian theory ~~and the null result of the Eotvos experiment~~ suggests that gravitational shielding effects are extremely small, ~~Yet we cannot conclude that they are negligible. The Eotvos experiment, which weighs gravitational against centrifugal forces, does place an upper limit on the amount of gravitational shielding~~ for purely mechanical phenomena. But the essence of the principle of equivalence is Einstein's bold extrapolation from the purely mechanical equivalence of uniform acceleration and uniform gravitation to their complete equivalence.

The red shift is not a purely mechanical effect. This is particularly clear in Einstein's early derivation of the red shift on the basis of the principle of equivalence. For there, in the accelerational case, the shift arises as a first order Doppler effect in the light transmitted from the top to the bottom of Einstein's hypothetical elevator.

If the principle of equivalence failed for non-mechanical phenomena, we could no longer rely on our previous result that the solar gravitational and centrifugal effects on the noon and midnight clocks almost exactly cancelled; for neither the Newtonian nor the Einsteinian theory would be a safe guide.

The non-mechanical aspects of the principle of equivalence have been tested by observation of the solar and other red shifts and of the bending of light rays by the sun. But neither of these effects has been verified really satisfactorily: thus the solar red shift has the expected value only at the edge of the sun's disc; and while a deflection of light definitely occurs, its dependence on $1/r$ is by no means established. Moreover — and this is significant — none of these experiments is a test of gravitational shielding. Nor are the recent terrestrial red shift measurements using the Mössbauer effect; nor, to any significant extent, the proposed satellite experiments.

Suppose that the "shielding" were of such a nature as to cause a focussing effect — of gravitation, not light. In the case of the deflection of light rays by the sun the effect of the sun's self-focussing might well be negligible. But with the spherical earth focussing the locally almost uniform gravitational field of the sun onto the midnight clock the effect could be significantly different — just as the optical effect of a glass sphere is different on radial rays leaving it from what it is on parallel rays traversing it.

We already know that light has unexpected gravitational properties, as witness the 1/2-factor anomalies discovered by Tolman and described in his book Relativity, Thermodynamics and Cosmology. Suppose there were a significant gravitational shielding or focussing effect on the optical, non-mechanical part of the noon-midnight red shift experiment, amounting to, say, one percent of the calculated gravitational effect on the midnight clock. Such an effect would not essentially alter the centrifugal contribution to the frequency shift. Therefore there would be an effect of order 10^{-14} and this would not be hopelessly beyond the range of future detection.

The experiment would not be easy. Signalling between antipodal points would have to be by means of coaxial cable, or, failing that, by radio ground wave, to avoid time distortion due to reflections of the air wave. Since the gravitational effect of the moon (which is almost exactly cancelled by its centrifugal effect) is of order 4×10^{-15}, any terrestrial shielding of the moon's gravitational field will yield only about one hundredth of the red shift produced by terrestrial shielding of the solar gravitational field. Differences in the heights of the clocks above the geoid — an equipotential surface closely approximating the shape of the earth — can be corrected by performing the experiment at 12-hour intervals, and other extraneous effects can be allowed for experimentally by comparing results with those for the $6A_0M_0$-$6P_0M_0$ case, where there is no difference in the terrestrial shielding of the two clocks. Tidal distortions of the earth's shape can be neglected since not only are they small but they act to raise (or lower) both clocks by substantially the same amount.

Even if the experiment were to yield no measurable red shift it would still be significant since it would demonstrate that no gravitational shielding or focussing effects had been observed in a non-mechanical experiment. But if a definite red shift were observed that could not be accounted for by other means, the experiment would assume transcendental importance as evidence for the existence of gravitational shielding.

The great difficulty with experiments on gravity is that the gravitational force is intrinsically extremely weak. But the earth is a massive body that generates relatively large gravitational fields, and the proposed experiment shows how we can use it as a piece of experimental apparatus in an investigation of the possibility of gravitational shielding or focussing for non-mechanical phenomena.