COSMIC CENSORSHIP, BLACK HOLES, AND PARTICLE ORBITS

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ABSTRACT

Perhaps one of the main reasons for believing in the cosmic censorship hypothesis is the disquieting nature of the alternative: the existence of naked singularities, and hence loss of predictability, the possibility of closed timelike lines, etc. The consequences of assuming the cosmic censorship hypothesis can also be somewhat strange and unexpected. In particular, we apply Hawking's black hole area theorem to the study of particle orbits near a Schwarzschild black hole. If the cosmic censorship hypothesis (and hence the area theorem) is true, then there exist stable near-circular orbits arbitrarily close to the horizon at $r = 2M$.

Determining the validity of the cosmic censorship hypothesis is widely regarded as the most important task within classical general relativity today. While nothing approaching a proof of the hypothesis has yet been offered, it is widely accepted as true owing to the failure of numerous attempts to construct counterexamples(see, e.g. Refs. 1 and 2). Philosophically, it is comforting to believe in cosmic censorship because of the bizarre physical nature of the alternative: the existence of naked singularities. The existence of naked singularities destroys predictability, allows the existence of closed timelike lines[3], and in general wrecks havoc with the causal structure of spacetime.

The purpose of this paper is to show that assuming the cosmic censorship hypothesis also leads to rather unexpected and somewhat strange consequences. In particular, we will examine a class of test particle orbits in the Schwarzschild geometry for which the geodesic equation predicts grossly different behavior than the cosmic censorship hypothesis (via Hawking's area-increase theorem)[4]. Since cosmic censorship is manifestly the worst assumption of the area theorem, we shall consider the cosmic censorship hypothesis and the area theorem to be equivalent.

Few topics in general relativity have been studied as thoroughly as particle motion in the Schwarzschild geometry. All previous efforts, however, seem to have merely studied the geodesic equations of motion and ignored the possibility of using Hawking's theorem to chose which particle motions are physically acceptable. It is well known, for example, that the innermost stable circular orbit is located at $r = 6M$. However, we shall see that is (if) the cosmic censorship hypothesis is correct, then there must exist stable orbits at arbitrarily small distances from the horizon, i.e. at $r = 2M+\varepsilon$, $\varepsilon \ll M$.

We will consider the motion of an axisymmetric ring of test particles in the equatorial plane of a Schwarzschild black hole. Axisymmetry is necessary to avoid the possibility of radiating any angular momentum in the form of gravitational waves. This way the orbital angular mometum of the test particles is truly a conserved quantity, and not merely approximately conserved. The geodesic equations of motion for the Schwarzschild geometry are then[5]:

$$\frac{dt}{d\tau} = \frac{\gamma}{(1-2M/r)} \quad , \qquad (1)$$

$$\frac{d\phi}{d\tau} = \frac{\ell}{r^2} \quad , \qquad (2)$$

$$\left(\frac{dr}{d\tau}\right)^2 = \gamma^2 - V^2(\ell,r) \quad , \qquad (3)$$

$$V^2(\ell,r) = (1 - 2M/r)(1 + \ell^2/r^2) \quad , \qquad (4)$$

in the usual Schwarzschild coordinates, where

$$\gamma = E/\mu = \text{energy/unit rest mass} \qquad (5)$$

and

$$\ell = L/\mu = \text{angular momentum/unit rest mass.} \qquad (6)$$

We will be concerned with the question of whether or not the test particles enter the black hole. Thus, Eqn.(3) merits our attention. The test particles must have $(dr/d\tau)^2 \geq 0$, hence particle motion is restricted to regions where $\gamma^2 \geq V^2(\ell,r)$. The range in r-coordinate of a particle's motion is thus completely determined by the shape of $V^2(\ell,r)$, and the proper energy γ.

Although it is possible to construct many interesting conflicts between the area theorem and the geodesic equation, we will focus on a particularly simple example: We chose

$$\ell = 2\sqrt{3}\, M \quad . \qquad (7)$$

In this case, $V^2(\ell,r)$ has no maxima except a infinity: its value decreases monotonically as r decreases, reaching zero when r = 2M (see Fiq. 1). Thus, it seems that any test particle, sent inwards from infinity if $\gamma \geqq 1$, or started inward from the outer turning point $r = r_o$, determined implicitly by $\gamma = V(2\sqrt{3},r_o)$ if $\gamma < 1$, will inevitably enter the black hole. There is no "angular momentum barrier" to repulse an incoming particle.

We now turn to the area theorem. The initial state of the black hole has mass M, and zero angular momentum (Kerr parameter a = 0). If the axisymmetric ring of test particles enters the black hole, it will gain angular momentum L and not more than an energy E in mass (the actual mass gain could be less than E due to gravitational radiation). Thus, the final state will have mass

$$M' \leqq M + E \tag{8}$$

and proper angular momentum

$$a' \geqq L/(M+E) \quad . \tag{9}$$

Now, the black hole area theorem tells us that the final area of the black hole must be greater than or equal to the initial area. The area of a Kerr black hole is given by:

$$A = 4\pi(r_+^2 + a^2) = 8\pi M[M + (M^2 - a^2)^{1/2}] \quad . \tag{10}$$

The area theorem then states that

$$2M^2 \leqq (M + E)^2\{1 + [1 - L^2/(M + E)^4]^{1/2}\} \quad . \tag{11}$$

Recalling that we have chosen $L = 2\sqrt{3}\,\mu M$, Eqn. (11) can be reduced to

$$6\mu^2 \leqq 2ME + E^2 \quad . \tag{12}$$

Thus, although the geodesic equations say that all test particles, regardless of the value of γ, enter the black hole, the area theorem tells us that only those satisfying Eqn. (12) can enter.

We must now pause and consider this conflict of predictions more carefully. First, the area-increase theorem applies to all black hole interactions, while only particles with $\mu \ll M$ fulfill the test particle approximation. The particles we are interested in are those which fulfill the test particle approximation ($\mu \ll M$), but at the same time violate Eqn.(12). If Eqn.(12) is violated, then

$$E \leqq M[-1 +(1 + 6\mu^2/M^2)^{1/2}] \quad , \qquad (13)$$

or, if $\mu \ll M$,

$$E \leqq 3\mu^2/M \qquad \text{or} \qquad \gamma \leqq 3\mu/M \quad . \qquad (14)$$

Clearly, if $\mu \ll M$, any test particle violating Eqn. (12) will have $\gamma \ll 1$. This means that the test particle will have its motion entirely confined to the region just outside the event horizon. More specifically, if $\gamma \ll 1$, then the outermost turning point at r_o is found to be approximately at:

$$r_o \simeq 2M + 3\mu/2 \quad . \qquad (15)$$

Since the particle cannot enter the black hole(or else the area theorem, and cosmic censorship, would be violated), it must orbit between the turning radii:

$$2M < r < 2M + 3\mu/2 \quad . \qquad (16)$$

There is one possible objection that must be disposed of . The outermost accessible radius of these orbits (Eqn. (15)) is only $3\mu/2$ outside the horizon, in r-coordinate length. Whatever the test particles are, they must have a finite size, in particular a radius of at least 2μ, their own Schwarzschild radius. The orbits would clearly not be of much interest if at the furthest separation of the black hole and the test particle, the test particle were already partially inside the horizon. We can easily show that this is not the case: at the outermost point of its orbit (r_o), the proper distance to the horizon is given by:

$$s = [r(r - 2M)]^{1/2} + 2M \ln\left|(r/2M - 1)^{1/2} + (r/2M)^{1/2}\right| \quad . \quad (17)$$

It is then easy to see that there is always "enough room" for the test particle outside the horizon. Ignoring the second term in Eqn. (17), which is always positive outside the horizon, we find:

$$s > [r_o(r_o-2M)]^{1/2} \geqq 2\mu \quad , \quad (18)$$

which will always be satisfied if

$$M > 7\mu/12 \quad , \quad (19)$$

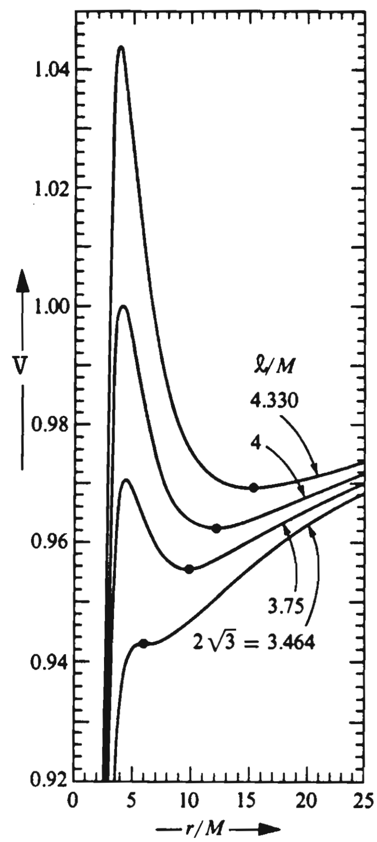
which, of course, will always be true in the test particle approximation.

Thus we see that the test particle equations, combined with Hawking's area theorem, predict the existence of orbits just outside the horizon of Schwarzschild black holes. The orbits are clearly stable, since the particles can't enter the black hole without violating the area theorem, and can't escape to large radii because of their ultra-low energies, constrained by Eqn. (14).

One cannot claim that the test particle approximation is invalid, since one can choose μ/M to be arbitrarily small, and hence the test particle approximation arbitrarily good, as long as E satisfies Eqn.(14). It seems then that either cosmic censorship must be incorrect, and the particles do enter the black hole, or else these rather peculiar, stable, almost circular orbits exist an infinitesimal distance outside the horizon. We see that even if we accept the cosmic censorship hypothesis in order to avoid the dilemma of naked singularities, we are then forced into strange new conclusions, namely the existence of stable almost-circular orbits in the Schwarzschild geometry at any radius down to $r = 2M$, rather than the classic result that the innermost stable orbit is at $r = 6M$.

Finally, I should perhaps point out that these curious orbits are probably not of great importance astrophysically. While the outermost radius of the orbits is large enough that the test particles could be small black holes (recall we assumed a test particle dimension of $R \simeq 2\mu$), it is not large enough that they could be ordinary matter. As an example, consider an object of mass 1 kg in orbit around a solar mass black hole. The proper distance from the horizon to r_o is then $s_o \simeq 10^{-19}$cm, which is much larger than $2\mu = 10^{-25}$cm, but much smaller than any 1 kg rock might be. Only in the encounter of a neutron star with a supermassive black hole is it conceivable that "ordinary" matter could be trapped in these orbits.

Fig. 1



This graph plots V(ℓ,r) vs. r/M for various values of ℓ. Note the lack of any angular momentum barrier near the black hole for $\ell = 2\sqrt{3}$ M.

Figure reproduced from C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation, Freeman, San Franscisco, (1973), page 622.

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BIOGRAPHICAL DATA

William A. Hiscock was born Oct. 31, 1951, in Santa Monica, California. He received his B. S. degree in June , 1973 from the California Institute of Technology, and his M. S. in May, 1975 from the University of Maryland. He is married and currently working towards the Ph. D. degree in physics at the University of Maryland.