FROM UNSTABLE MINKOWSKI SPACE TO THE INFLATIONARY UNIVERSE

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ABSTRACT

It is shown that Minkowski Space is unstable in the context of semi-classical gravity. There exists a threshold mass, of the quantized matter field, which marks the dividing line between stable and unstable vacuum fluctuations of matter in flat space-time. The Minkowski vacuum gravitational-matter system undergoes a phase transition above this "critical point", the new phase being a self-consistently generated De Sitter Euclidean Cosmology. Its total energy is degenerate with respect to that of empty Minkowski space-time. It represents an appropriate candidate for the primeval configuration of an inflationary-like universe.

An outstanding problem in the theory of gravitation is the stability of Minkowski Space. For gravity coupled to matter, treated as a classical source subject to conventional energy requirements, Minkowski Space has been proven to be stable[1)]. The situation is more intricate when the source is a quantum field, for then particle creation may occur. This leads us to pose an essential question, to wit : Is empty Minkowski Flat Space-Time the true ground-state of semi-classical gravity?

The purpose of the present work is to show, in the simple framework of a scalar massive quantum field coupled non-minimally to classical gravitation, that the interplay between this coupling and the substraction procedure required by the quantum nature of the matter field leads to an instability of Minkowski Space[2)]. Moreover, we show the intimate interrelation between this state of affairs and the existence of the recently suggested non-standard big-bang cosmologies[3)]. These latter are endowed with a cooperative process in which massive matter constituents, as well as the embedding large scale gravitational field, engender each other mutually in a self-consistent feedback mechanism, out of an initial flat Minkowski Space: the creation of each induces the creation of the other due to their gravitational coupling.

More precisely, we show that Minkowski Space is characterized by a threshold value for the dimensionless parameter κm^2 $(\hbar = c = 1)$, where κ is the gravitational coupling constant and m is the mass of the quantized matter field; this threshold marks the dividing line between stable and unstable vacuum fluctuations of matter in the Minkowskian background. Its value is precisely the same as the lower bound ensuring the existence of the above-mentioned cosmologies. In other words, the existence of these self-consistent cosmologies and the instability of empty Minkowski Space are part and parcel of a common mechanism.

The dynamical mode, whose (in)stability character plays a crucial role in our argument is the large scale component of the gravitational field, represented by the sole gravitational degree of freedom left over by the homogeneity and isotropy requirements. As is well known this can be represented by a classical massless scalar field $\phi(x)$ (proportional to $(\det g_{\mu\nu})^{1/8}$). The coupling of the latter to the matter field $\psi(x)$ is then described by the semi-classical action

$$S = \frac{1}{2}\int \sqrt{-g}\, d^4x \left(\partial_\mu\psi\, \partial_\nu\psi\, g^{\mu\nu} + \frac{R}{6}\psi^2 - \frac{\kappa m^2}{6}\psi^2\phi^2 \right) - \frac{1}{2}\int \sqrt{-g}\, d^4x \left(\partial_\mu\phi\, \partial_\nu\phi\, g^{\mu\nu} + \frac{R}{6}\phi^2 \right) \tag{1}$$

All geometrical quantities therein are Minkowskian. We postulate that a trivial dynamical realization of this action be empty Minkowski Space. This postulate suffices to fix the substraction procedure necessary to calculate the substracted mean square $\langle \psi^2 \rangle^s$. Thus we put $\langle \psi^2 \rangle^s = 0$ in Minkowski Space : $\phi^2 = \frac{6}{\kappa}$ [this constant is fixed by the requirement that the free ϕ - part of the action gives rise to the usual free gravitational action $-\frac{1}{2\kappa}\int \sqrt{-g}\, R$ when ϕ = constant]. This prescription also defines the substraction procedure uniquely for all non-trivial realizations[3,4] of the dynamical equations.

$$\Box\psi + \frac{1}{6}\kappa m^2\phi^2\psi = 0 \tag{2}$$

$$\Box\phi - \frac{1}{6}\kappa m^2 \langle\psi^2\rangle^s \phi = 0 \tag{3}$$

We now set out to examine the stability of their trivial Minkowski solution. To this end we analyze the linearized dynamical behavior of a small time-dependent perturbation $\delta(t)$ affecting the Minkowskian background for $t > t_0$ (t_0 is an arbitrary time), i.e.:

$$\phi(t) = \left(\frac{6}{\kappa}\right)^{1/2}(1 + \delta(t)) \quad ; \quad \delta(t) = 0 \ , \ t \leq t_0 \tag{4}$$

Equation (2) then fixes the corresponding response (for $t > t_0$) of the matter field $\delta\langle\psi^2\rangle^S$, where the substraction, following our general prescription, eliminates the zero-point energy corresponding to the effective mass $m^2(t) = m^2(1 + 2\delta(t))$

Equation (3), which furnishes the feedback response to $\delta\langle\psi^2\rangle^S$ for the geometrical perturbation $\delta(t)$ leads then to a simple algebraic relation for the Laplace transform $\delta^{(2)}(s)$ of $\ddot{\delta}(t)$ [where a dot means differentiation with respect to time t]

$$\delta^{(2)}(s) = \frac{\frac{\kappa m^4}{48\pi^2}\,\dot{\delta}(t_0)\,g(s)}{1 - \frac{\kappa m^4}{48\pi^2}\,g(s)} \tag{5}$$

It suffices to know that $g(s)$ is an even function of s asymptotically vanishing for $|s| \longrightarrow \infty$, whose maximum $g(0) = \frac{1}{6m^2}$. It follows that if $\frac{\kappa m^2}{288\pi^2} > 1$, then $\delta^{(2)}(s)$ has two real symmetric poles and its inverse Laplace transform $\ddot{\delta}(t)$ [as well as $\delta(t)$] according grows exponentially with time. On the contrary, in the case $\frac{\kappa m^2}{288\pi^2} < 1$, $\delta(t)$ exhibits bounded oscillating behavior. This closes the proof of the existence of a threshold value for κm^2 ($\kappa m^2_{th} = 288\pi^2$) which renders the vacuum Minkowski Space solution unstable.

This property, together with the existence of the self-consistent cosmologies characterized by the same threshold κm^2_{th}, opens the way to a possible new vision of the cosmological history of the universe. Indeed, the existence of a threshold parameter for instability is strongly reminiscent of critical behavior in phase transition theory. We therefore suggest that κm^2_{th} plays the role of a critical point: below this value, the matter-gravitational system is stable correspon--ding to the Minkowski vacuum phase; when the parameter passes through its critical value κm^2_{th}, new non-trivial solutions of equations (2, 3) arise, and the system undergoes a phase transition, the new phase being a non-trivial self-consistent cosmology. The particles so created in the very early universe were interpreted [5] as those required by the Grand Unification Scheme. An alternative possibility however is that they are primitive black holes, the primeval source of temperature[6].

Among these self-consistent cosmologies, there is one which plays a special role: the Euclidean De Sitter Universe[3]. Indeed, a stability analysis performed along the same lines as in the Minkowski case indicates the stability of this solution[2,7]. Next, the Euclidean De Sitter universe is nothing but the Steady State Cosmology[8], the driving action of the Hoyl's "C-field" being replaced in the self-consistent scheme by a negative pressure. This latter characterizes these cosmologies wherein it expresses phenomenologically the spontaneous creation of massive matter. This spontaneous creation is a direct consequence of the attractive character of gravitation; it is indeed the negative sign accompanying the free ϕ-part of the action (1), hence the Minkowskian negative energy carried by the cosmological field which opens the way to these non-Minkowskian realizations of this dynamics. A crucial point is that their total energy is degenerate with respect to that of Empty Minkowski Space Time. Indeed, the effective varying mass

$m^2(x) = \frac{\kappa}{6} m^2 \phi^2(x)$ may induce creation of positive energy matter at the expense of this negative energy cosmological reservoir, keeping the total Minkowskian energy unchanged.

This then is able to provide the "early stretch" of Steady-State behavior required in the framework of Inflationary Models[9] and solves both the flatness and the particle-horizon problem. As such, it represents an alternative to Guth's idea that the energy density of the false vacuum associated to the supercooled phase drives the "Steady-State stretch". Instead it is the instability of Minkowski vacuum which forces the system, in the present context, to undergo, above κm^2_{th}, a phase transition from an initial empty Minkowski Space to the self-consistent De Sitter Universe.

In the framework of Guth's model, it is the transition from the false vacuum to the true one which releases a considerable amount of energy (and entropy) primarily responsible for the present $3\,^\circ K$ radiation background. In our alternative, by contrast, it is the decay of the supermassive initial matter constituents which provides the required energy.

In conclusion, let us summarize the chain of facts presented above in the shape of the following Cosmological History: Minkowski Space is unable to sustain vacuum matter-gravitational interactions and therefore transits to a new phase, the De Sitter Universe. The latter, an essential ingredient of inflationary-type universes, so appears as the natural primeval stage of physical space-time. After decay of the primeval constituents there is a turnover to the present cosmological free-expansion configuration. The universe built up in this way, thus appears as a non trivial energetically degenerate alternative to the quantum flat vacuum.

Clearly the present development, as well as that of Guth's, is far to embryonic to permit any kind of judgement or preference. What are our very massy constituents (more than ten times the Planck mass)? If they are indeed black holes, can they be simulated by a quantum field, and if not is Minkowski space nevertheless unstable with respect to such massy fluctuations? (On very general grounds one would think so). In Guth's cosmology, there are mysteries equally enigmatic. What is the source of temperature? (or equivalently who made the initial vacuum false?). And why, from a field-theoretic point of view, is the cosmological constant substracted in such a way as to vanish now and not then?

In spite of all these uncertainties, the inflationary idea is clearly to be exploited as fully as possible, since for the present it is the only mechanism which offers the possibility of success in confronting the problem of causality posed by the big-bang.

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