The Breakdown of Quantum Mechanics in the Presence of Time Machines

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Summary: General Relativity allows for the existence of closed timelike curves. Various attempts have been made to exploit this possibility and build a "time machine," that is a spacetime that has closed timelike lines inside some compact domain. We examine a simple model of a time machine, and construct the quantum-mechanical propagator for free particles in the vicinity of the causality violating domain. We discover that it is impossible for such propagators to be consistent with the law of conservation of probability. We speculate on the possible deeper consequences of our calculations.

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Causality is the keystone of physics. One rather loose statement of what is meant by causality is that cause precedes effect. We would like to be much more precise however, and the usual statement of causality in physics is that if we specify appropriate initial data, then it is possible to make certain predictions about the future. In classical mechanics, the future is completely determined by the initial conditions, whereas in quantum mechanics it is only partly determined in a way consistent with the uncertainty principle.

General Relativity has built into it causality at a microscopic level. In an infinitesimal neighbourhood of any point p, spacetime is isomorphic to Minkowski space, and so the lightcone of p provides all the structure needed to ensure that causality is enforced. However, on large scales the issue is not so simple. Many solutions of the Einstein equations have closed timelike curves. Such curves are characterized by the property that if one starts from the point p and moves along a future-directed timelike or null curve, one returns to the point p. Such spacetimes violate causality, and lead to well known paradoxes in classical mechanics, which are widely regarded as being unphysical. It is worthy of some thought as to how one should go about treating a physical theory where such things are apparently allowed.

The first example of causality violation is the Gödel Universe [1]. This spacetime is axisymmetric and rotating, the rotation induces frame dragging to the extent that the closed curves around the symmetry axis become timelike. Such curves are not geodesic, and Gödel [2] suggested that any paradox would be resolved by virtue of the fact that it is impracticable to follow such paths. However, it was realized [3] that the van Stockum solution [4] contains closed timelike geodesics and numerous other examples have been found, for example, the Taub-NUT solution [5], [6] and more recently the Gott spacetime [7]. In all of these cases, the spacetime has *always* possessed closed timelike curves and so does not contain any partial Cauchy surface. None of these examples, which we call eternal time machines, resemble our apparently orderly universe.

The Kerr Solution [8], also has closed timelike curves, but hidden inside an event horizon close to a spacetime singularity. One might hope that this is a generic state of affairs. Indeed, Tipler [9], [10] has proved that any spacetime with a partial Cauchy surface, where matter obeys the weak energy condition, and where closed timelike lines develop, must be null geodesically incomplete and thus singular. If one assumes the cosmic censorship hypothesis, then one would imagine that all regions with closed timelike curves will always be enclosed by event horizons, [11]. However, if one drops the requirement that the weak energy condition is obeyed, then it is perfectly possible to find non-singular spacetimes that contain partial Cauchy surfaces in the past, but develop closed timelike curves in the future. We refer to such spacetimes as "time machines." There appears to be nothing in the laws of general relativity forbidding such solutions to the Einstein equations. One might worry about the fact that we demand violations of the weak energy condition, however quantum mechanical matter certainly does not obey this requirement. Examples from quantum field theory are provided by both the Casimir effect [12] and the Hawking effect [13], and arise from violations of Poincaré invariance of the vacuum state. We are thus forced to face up squarely to the physical issues posed by causality violation.

A concrete model of a time machine has been proposed by Morris, Thorne and Yurtsever [14]. It consists of spacetime that is flat outside some compact region, from which at some instant one manufactures a wormhole by removing the interior of two spheres on a spacelike surface, and connecting their boundaries with a tube. By moving the mouths of the wormhole relative to each other, one can construct a spacetime with closed timelike curves which pass through the wormhole. It is easy to manufacture numerous paradoxes in this spacetime. By studying the kinematics of billiard balls, Echeverria, Thorne and Klinkhammer [15] show that for each case where a paradox arises, there are infinite number of consistent classical solutions to the equations of motion arising from fixed initial conditions. It is in this way that the classical Cauchy problem is not well-defined. Thorne [16] has suggested that quantum mechanics may somehow rescue one from the difficulties encountered in classical treatment of time machines. Our aim now is to check if quantum mechanics can be formulated in the presence of time machines.

We study the quantum mechanics of a single free non-relativistic particle using the path integral [17]. Suppose that in the past, at time t_i, the particle is in some eigenstate of position $\mid i, t_i\rangle$ and in the future in a state $\mid j, t_j\rangle$. The propagator is given by the path integral

$$\Delta_{ji} = \langle j, t_j \mid i, t_i\rangle \sim \sum e^{iS(j,i)} \tag{1}$$

where the sum is taken over all paths joining i to j, and $S(j,i)$ is the classical action of that path. Δ_{ji}, is physically reasonable if it obeys the completeness condition

$$\Delta_{ji} = \sum_k \Delta_{jk}\Delta_{ki} \qquad t_j > t_k > t_i \quad , \tag{2}$$

and the unitarity condition

$$\sum_k \Delta^*_{ki}\Delta_{kj} = \begin{cases} \delta_{ij}, & \text{if} \quad t_i = t_j < t_k \\ \Delta_{ji}, & \text{if} \quad t_i < t_j < t_k \\ \Delta^*_{ij}, & \text{if} \quad t_k > t_i > t_j. \end{cases} \tag{3}$$

Let us construct a very simplified model of a time machine. Consider a flat spacetime labeled by spatial coordinates x and a time coordinate t. One then identifies a point (x^+, t^+) with the point (x^-, t^-) in such a way that a future-directed timelike line arriving at (x^+, t^+) emerges at (x^-, t^-) again traveling toward the future. Similarly, a future-directed timelike line arriving at (x^-, t^-) will emerge at (x^+, t^+). This picture, despite its simplicity, contains all the relevant features of a time machine. A more realistic example could be envisaged as a superposition of many such identifications.

We now evaluate the propagator, G_{ji}, in our model spacetime in terms of the ordinary flat space propagator, K_{ji}. K_{ji} obeys (2) and (3) and for a particle of mass m in three dimensional space is given explicitly by

$$K_{ji} = \left(\frac{m}{2\pi i\hbar(t_j - t_i)}\right)^{3/2} \exp\left[\frac{im(x_j - x_i)^2}{2\hbar(t_j - t_i)}\right] \quad \text{for} \quad t_j > t_i \tag{4}$$

and

$$K_{ji} = 0 \quad \text{for} \quad t_i > t_j. \tag{5}$$

This last equation comes about as a consequence of requiring our non-relativistic particles to propagate only towards the future. G_{ji} is made up of *all* future-directed paths from i to j, as illustrated in figure 1. There are a number of topologically distinct cases. Those paths that do not travel through the wormhole (see Fig. 1a) give a contribution of $K_{ji} - K_{j+}K_{+i} - K_{j-}K_{-i}$. This includes all paths from i to j except those that travel via either $+$ or $-$, which would then go through the wormhole. The contribution from paths that travel the wormhole a single time from $+$ to $-$ is $(K_{j-} - K_{j+}K_{+-})K_{+i}$ (Fig. 1b). Similarly, those paths that travel through the wormhole n times from $+$ to $-$ give a contribution of $(K_{j-} - K_{j+}K_{+-})K_{+-}^{(n-1)}K_{i+}$. Finally, one can go through the wormhole from $-$ to $+$ once and only once, giving a contribution of $K_{j+}K_{-i}$ (Fig. 1c). Summing up all these contributions leads to

$$G_{ji} = K_{ji} + \frac{(K_{j-} - K_{j+})K_{+i}}{1 - K_{+-}} + (K_{j+} - K_{j-})K_{-i} \quad . \tag{6}$$

Using the fact that K_{ji} satisfies (3), (2), and (5) one can easily show that G_{ji} obeys the completeness condition (2) except when $t_i < t_- \leq t_k \leq t_+ < t_j$, that is when the time machine is between the initial and final points and the intermediate surface is in the causality violating domain. It fails on these intermediate surfaces because the particle can cross such a surface any number of times.

A similar calculation reveals that unitarity (3) fails unless *all* of t_i, t_j and t_k are either to the future or to the past of the causality violating region. From this we conclude that probability is not conserved for particles propagating in the neighbourhood of the time machine. Despite the simplicity of our model we believe that this is a generic feature of any spacetime that involves a time machine. Furthermore, although we used a non-relativistic model for our particles, it should be apparent that precisely the same properties would hold had we used a relativistic model, or had we introduced interactions. Therefore we believe that it is impossible to construct any conventional probability-conserving quantum mechanical interpretation of events involving time machines ¶.

One possible modification of quantum mechanics is to suppose that the time evolution of this system would require pure states to evolve into mixed states [19], which would violate unitarity. It would seem that this is perhaps inevitable whenever there is a failure of global hyperbolicity, as is found in black hole physics [20]. It may be the case that this is sufficient to resolve the difficulties that we have encountered here.

We are thus led to an intriguing speculation. If the laws of nature are adequately described by quantum mechanics, since time machines are inherently associated with a breakdown of the unitarity condition, we conjecture that it will be impossible to construct time machines. On the other hand, if the laws of physics have to be modified so as to encompass evolution that allows for pure states to evolve into mixed states, (as is suggested by black hole evaporation) then the obstacle for the construction of time machines that we raise here is removed.

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¶ Note that our result does not contradict that of Friedman and Morris [18] who considered only eternal time machines and were forced to use boundary conditions at past timelike infinity.

Figure Captions

Fig. 1: A schematic picture of different routes from i to j. Each line between two points denotes all possible paths of this kind between them. Solid lines describe paths in "regular spacetime", dashed lines stand for paths that have to be removed and dotted lines stand for paths inside the wormhole.

Fig. 1a: Paths that have to be deleted from the free space propagator between i and j.

Fig. 1b: Paths that pass through the wormhole from $+$ to $-$. The inner loop from $+$ to $-$ and back can be repeated n times.

Fig. 1c: Paths that pass through the wormhole from $-$ to $+$.

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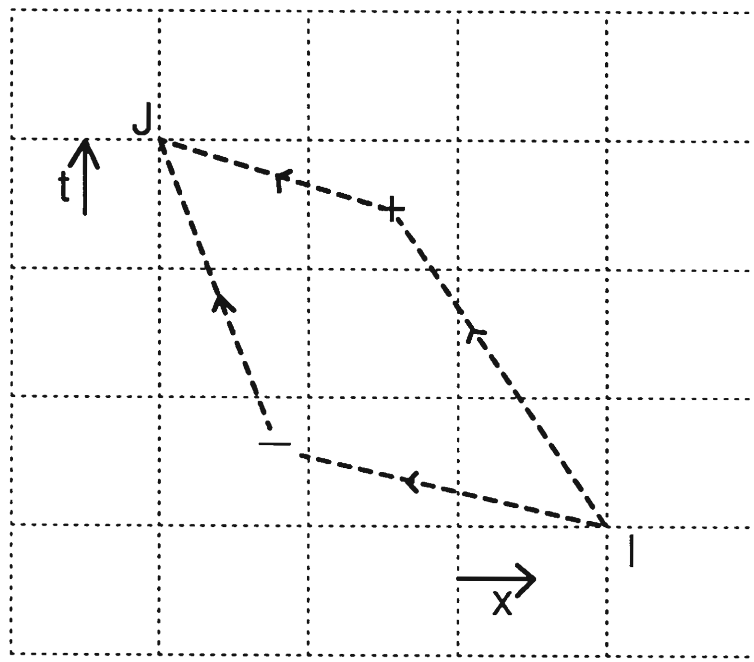

Fig. 1.a

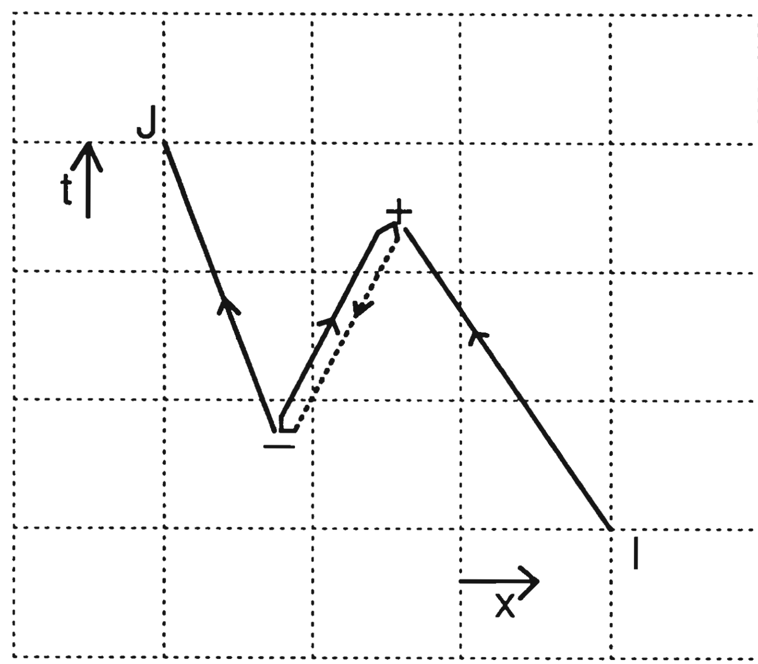

Fig. 1.b

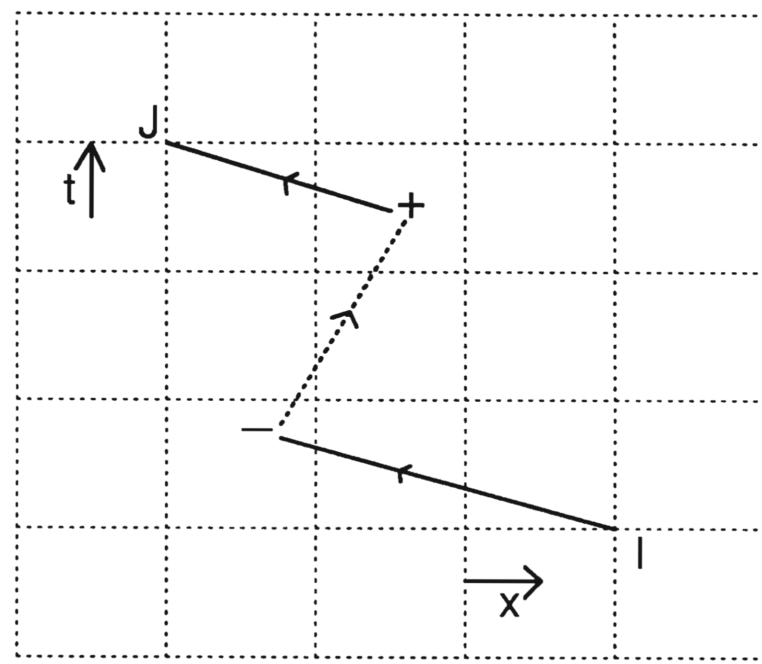

Fig. 1.c