Quantum gravity experimental physics?

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Abstract

Canonical quantum gravity theories predict a polymer-like structure for space time at the Planck size. This granularity can be probed using gamma ray burst observations. Quantum gravity effects typically amount to corrections of Planck length size per wavelength. Because the distance to gamma ray burst is very large as measured in the wavelength of gamma rays, the effects accumulate and are on the brink of being observable. These observations can constrain certain aspects of the quantum state underlying our universe.

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The quantization of the gravitational field has historically been stymied, in a significant part, due to the lack of experimental guidance. Never before in physics have people attempted to study the quantum mechanics of a fundamental system with so little experimental evidence to constrain the possible theories. The obstacles to face appear as monumental. Order of magnitude estimates suggest that even the most favorable quantum gravity effects from the experimental point of view are several —in most cases dozens— of orders of magnitude away from observation.

A situation where an order-of-magnitude estimate does not predict abysmally disappointing prospects was recently suggested by Amelino-Camelia et al. [1]. Consider the light that comes from a distant astronomical object and assume, as all theories of quantum gravity predict, that space-time has some sort of "granular" structure at the Planck-length level. Generically, one expects that propagation of light on such a space-time will exhibit departures from the usual propagation in a continuum. The kind of effects one gets will be related to the wavelength of the light, and at most will be of order $\frac{\text{Planck length}}{\text{Wavelenght}}$. The effect per wavelength is very small. But if one considers the number of wavelengths that occur between a distant astronomical object and the observer, the effects became plausibly observable. Concretely, consider a gamma-ray-burst. It is now widely accepted that these are events that occur at cosmological distances $L \sim 10^{10}$ light years. The wavelength of the gamma-rays observed by the BATSE detector is in the 200kev range. If one assumes an effect of the order $\frac{\ell_P}{\lambda}$, one gets for the effect a time shift in the waves of $\sim 10^{-5}s$. Gamma-ray-burst spectra have been observed with quite a fine time structure. For instance, in reference [2] features of about 1 ms have been reported in bursts of 0.1s width. This makes such effects almost observable with current data. The challenge now from the theoretical side is to come up with specific predictions for the effects to be observed. The more detailed the prediction, the likelier it will be to experimentally check the effect in noisy data.

Effects in string theory have been suggested [1] to predict a frequency dependent dispersion in the propagation of light. Here we will suggest that in the polymer-like nature of space-time predicted by canonical quantum gravity models, one could find birefringent

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effects. Loop quantum gravity [3] is usually formulated in the canonical framework. The states of the theory are given by functions of spin networks, which are a convenient label for a basis of independent states in the loop representation. This kinematic framework is widely accepted throughout various formulations of the theory, and has led to several physical predictions associated with the "polymer-like" structure of quantum space-time [4]. The dynamics of the theory is embodied in the Hamiltonian constraint, and consistent proposals are currently being debated [5]. To show the existence of the birefringent effect we will not need too many details of the dynamics of the theory. We prefer to leave the discussion a bit loose, reflecting the state of the art in the subject, since there is no agreement on a precise dynamics.

The term in the Hamiltonian constraint coupling Maxwell fields to gravity is the usual "$E^2 + B^2$" term, but in a curved background,

$$H_{\text{Maxwell}} = \frac{1}{2} \int d^3x \underset{ab}{g} \, (\tilde{e}^a \tilde{e}^b + \tilde{b}^a \tilde{b}^b). \tag{1}$$

where we have denoted with tildes the fact that the fields are vector densities in the canonical framework. Thiemann [6] has a concrete proposal for realizing the operator corresponding to the metric divided by the determinant, but we will only use some general features of that proposal.

Since we are interested in low-energy, semi-classical effects, we will consider an approximation where the Maxwell fields are in a state that is close to a coherent state. That is, we will assume that the Maxwell fields operate as classical fields at the level of equations of motion, however, we will be careful when realizing the Hamiltonian to regulate operator products. For the gravitational degrees of freedom we will assume we are in a "weave" state [7] $|\Delta >$, such that,

$$< \Delta | \underset{ab}{\hat{g}} | \Delta > = \delta_{ab} + O(\frac{\ell_P}{\Delta}), \tag{2}$$

where ℓ_P is Planck's length. Weave states [7], characterized by a length Δ, are constructed by considering collections of Planck-scale loops. They are meant to be semi-classical states

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such that that if one probes these states at lengths much smaller than Δ one will see features of quantum space-time, whereas if one probes at scales of the order of, or bigger than Δ one would see a classical geometry. The weave we will consider approximates a flat geometry for lengths larger than Δ.

Let us now consider the action of the Hamiltonian we proposed above on a weave state. We need a few more details of the regularization of $\underset{ab}{g}$ that was proposed by Thiemann [6]. It consists in writing $\underset{ab}{\hat{g}}$ as the product of two operators $\hat{w}_a(x)$, each corresponding to a commutator of the Ashtekar connection with the square root of the volume operator. The only feature we will need of these operators is that acting on spin network states they are finite and only give contributions at intersections. We now point split the operator as suggested in [6], (to shorten equations we only consider the electric part of the Hamiltonian, the magnetic portion is treated in the same way)

$$\hat{H}^E_{\text{Maxwell}} = \frac{1}{2} \int d^3x \int d^3y \hat{w}_a(x)\hat{w}_b(y)E^a(x)E^b(y)f_\epsilon(x-y) \tag{3}$$

where $\lim_{\epsilon \to 0} f_\epsilon(x-y) = \delta(x-y)$, so it is a usual point-splitting regulator, and we have eliminated the tildes to simplify notation, and as we stated above, treat the electric fields as classical quantities. The operators $\hat{w}_a$ only act at intersections of the weave, so the integrals are replaced by discrete sums when evaluating the action of the Hamiltonian on a weave state,

$$< \Delta|\hat{H}^E_{\text{Maxwell}}|\Delta > = \frac{1}{2} \sum_{v_i,v_j} < \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta > E^a(v_i)E^b(v_j) \tag{4}$$

where v_i and v_j are vertices of the weave and the summation includes all vertices within the domain of characteristic length Δ. We now expand the electric field around the central point of the Δ domain, which we call P, and get,

$$E^a(v_i) \sim E^a(P) + (v_i - P)_c\partial^c E^a(P) + \cdots, \tag{5}$$

and given the assumptions we made about the long wavelength nature of the electric fields involved, we will not need to consider higher order terms in the expansion at the moment.

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Notice that $(v_i - P)_c$ is a vector of magnitude approximately equal to Δ, whereas the partial derivative of the field is of order $1/\lambda$, that is, we are considering an expansion in Δ/λ. We now insert this expansion in the Hamiltonian and evaluate the resulting terms in the weave approximation. One gets two types of terms, one is given by the product of two electric fields evaluated at P times the sum over the vertices of the metric operator. Due to the definition of the weave state, the sum just yields the classical metric and we recover the usual Maxwell Hamiltonian in flat space.

We now consider the next terms in the expansion Δ/λ. They have the form,

$$\frac{1}{2}\sum_{v_i,v_i} < \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta > ((v_i - P)_c\partial_c(E^a(P))E^b(P) + (v_j - P)_cE^a(P)\partial_c(E^b(P)), \quad (6)$$

When performing the sum over all vertices in the cell we discussed above, we end up evaluating the quantity $< \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta > (v_i - P)_c$. This quantity averages out to zero in a first approximation, since one is summing over an isotropic set of vertices. The value of the quantity is therefore proportional to ℓ_P/Δ, the larger we make the box of characteristic length Δ the more isotropic the distribution of points is. We consider the leading contribution to this term, which should be a rotational invariant tensor of three indices, i.e., it is given by $\chi\epsilon_{abc}\ell_P/\Delta$ with χ a proportionality constant of order one (that can be positive or negative).

We have therefore found a correction to the Maxwell Hamiltonian arising from the discrete nature of the weave construction. It should be noticed that the additional term we found is rotationally invariant, i.e., it respects the original spirit of the weave construction. It is, however, parity violating. If one were to assume that the weaves are parity-invariant, the term would vanish. The term would also vanish —on average— if one assumes that the different regions of size Δ have "random orientations" in their parity violation. The fact that we live in a non-parity invariant universe suggests that parity invariant weaves might not necessarily be the most natural ones to consider in constructing a semi-classical state of cosmological interest. Another way to put this is to notice that parity non invariant weaves seem to be allowed by the theory and we are "experimentally constraining" this fact with

observations.

Assuming a non-parity invariant weave, the resulting equations of motion from the above Hamiltonian can be viewed as corrections to the Maxwell equations,

$$\partial_t \vec{E} = -\nabla \times \vec{B} + 2\chi \ell_P \Delta^2 \vec{B} \tag{7}$$

$$\partial_t \vec{B} = \nabla \times \vec{E} - 2\chi \ell_P \Delta^2 \vec{E}. \tag{8}$$

As we see the equations gain a correction proportional to the Laplacian Δ^2 of the fields, the correction is symmetrical in both fields, but is not Lorentz covariant. This already suggests that there will be modifications to the usual dispersion relation for light propagation. The lack of covariance is not surprising, since the weave selects a preferred foliation of space-time. If one now seeks solutions with a given helicity,

$$\vec{E}_\pm = \text{Re}\left((\hat{e}_1 \pm i\hat{e}_2)e^{i(\Omega_\pm t - \vec{k} \cdot \vec{x})}\right), \tag{9}$$

we get

$$\Omega_\pm = \sqrt{k^2 \mp 4\chi \ell_P k^3} \sim |k|(1 \mp 2\chi \ell_P |k|). \tag{10}$$

We therefore see the emergence of a birefringence effect, associated with quantum gravity corrections. The group velocity has two branches, and the effect is of the order of a shift of one Planck length per wavelength. This effect is distinct from other effects that have been discussed in string theory [1] which only imply a change in the dispersion relation. Here we in addition see a helicity-dependent effect. Being this a more detailed prediction, it might be easier to "dig out of the noise" of the received gamma-ray-bursts using statistical techniques, which probably makes the effect almost detectable with current technology.

Finally, can one expect similar effects for propagation of other types of waves? In string theory models, the corrections are all-encompassing (they can be viewed as modifications of quantum mechanics itself). In our case, if one considers scalar waves, the kinds of corrections we study in this paper appear to vanish. For Fermions the situation is more involved, since they couple in fundamental waves to the weave it is not immediate to develop a coherent

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state approximation as the one we considered here. An intriguing possibility is that dispersive effects could depend on the size of the elementary particles considered and one could therefore envisage terms that violate the equivalence principle, i.e. different kinds of particles would propagate in different ways in a semi-classical gravitational field. Gravitational waves could be a another promising place where effects like the one predicted here could be present, especially since space-based interferometers could detect waves coming from almost arbitrarily large distances [8]. Unfortunately, the low frequency of the kinds of gravitational waves likely to be detected in the near future make the effect that appear to be too small to be detected.

Summarizing, one expects that propagation of classical waves in a disordered medium will generically produce dispersive and possibly birefringent effects. This note can be considered as a first step towards a more exhaustive analysis of these effects. Given the possibility of experimental observation, this line of research should definitely be pursued.

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