GRAVITATIONAL RADIATION AS A STELLAR ENGINEERING DEVICE

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SUMMARY

Gravitational radiation losses, by continuously reducing the scale size of the system, induce mass transfer in the cataclysmic variables, or dwarf novae of type U Geminorum. The mass transfer proceeds at a rate that is equal to that observed, and equal to that apparently required by theoretical studies of mass accretion as the cause of dwarf nova outbursts. An origin is suggested for the recently discovered binary AM CVn (= HZ 29) of unusually low period ~1051 seconds. In our model the secondary is a degenerate helium star of mass ~0.04 $M_\odot$, the remnant core of a main sequence star.

I. INTRODUCTION

During the past year, we have shown[1] that gravitational radiation provides the dominant mechanism required to explain the evolution of dwarf novae (explosive binary variables) from those with periods ~10 hours down to ~81.6 minutes (WZ Sge). Until a month ago the latter was the shortest known binary star period. However, Warner and Robinson have now shown[2] that AM CVn (=HZ 29) is a binary with period P ~1051 secs. (~17.5 minutes)

In this essay we will briefly discuss the relevant data, describe our general theory, and show how AM CVn fits naturally into our scheme.

II. THE DWARF NOVAE

Spectroscopic evidence suggests[3,4] that dwarf novae generally consist of a white dwarf and a sub-luminous late-type companion. The existence of one component is sometimes inferred indirectly. The few white dwarfs visible have very broad absorption lines. If somewhat shallower, they would go undetected, undoubtedly resulting in lack of identification in other cases. Longer period systems exhibit dG or early dK spectra, becoming later and fainter as P approaches ~5 hours. Below this, the cool component is sometimes inferred from eclipses of the primary. The spectra also contain rapidly varying emission features which arise in a "hot spot" at a reasonably well-defined point in the system, as shown by the high-speed eclipse photometry of Warner and Nather[5].

These conclusions are summarized in Figure 1. A cool, main sequence star fills its Roche lobe, ejecting matter towards its white dwarf companion. Eventually an orbiting ring of material is formed. The hot spot occurs where infalling material impinges on the ring, flickering being attributed to unevenness in mass flow. Viscous effects in the ring slowly force the orbiting material to be accreted at a rate which ultimately leads to a thermonuclear runaway as sub-surface hydrogen is ignited. This "nova outburst" may recur after intervals of decades or more.

Little of this is new. Clearly, however, the most important question is "What causes the secondary to lose mass to the primary?" We have answered this question by showing that gravitational radiation appears to be the long sought after mechanism.

III. THE EFFECTS OF GRAVITATIONAL RADIATION

Because of the changing quadrupole moment, gravitational radiation inexorably removes both energy and angular momentum from the system. Without mass exchange, the physical scale of the system would always decrease. We may imagine the Roche equipotential surface encroaching within the cool component, imposing a reduced volume on the star. The latter, having time to relax, begins to eject excess matter. This tends to reexpand the lobe, while the stellar radius tends to a new equilibrium value corresponding to the slightly reduced mass. Given appropriate dependances, one can calculate the mass transfer required to produce a valid new situation consistent with the angular momentum radiated away in the time interval under consideration.

Under fairly reasonable assumptions (e.g. circular orbits satisfying Kepler's law, mass conservation except possibly in nova outbursts, rotational angular momentum small compared with orbital angular momentum, etc.), the evolutionary equation governing the system is formally

$$\frac{dJ}{dt}_{\mathrm{mech}} = \frac{-dJ}{dt}_{\mathrm{grav}} = \frac{-1}{\omega}\,\frac{d\mathcal{E}}{dt}_{\mathrm{grav}} \qquad (1)$$

where J is the angular momentum (mechanical or gravitational), $d\mathcal{E}_{grav}/dt$ is the formula for energy loss[6], and $\omega = \frac{2\pi}{P}$ is the orbital angular frequency. In greater detail, we have

$$\frac{d}{dt}\left(\frac{G^{1/2}\, M_1\, M_2 D^{1/2}}{(M_1 + M_2)^{1/2}}\right) = \frac{-32\ G}{5\ c^5}\left(\frac{M_1\, M_2}{M_1 + M_2}\right)^2 D^4\, \omega^5 \qquad (2)$$

and

$$\omega^2\, D^3 = G(M_1 + M_2) \qquad (3)$$

Some variables are eliminated from equation (2) by relating the size of the lobe to the separation D and the mass ratio of the stars, and by making the lobe size consistent with the secondary mass from stellar structure considerations. Power law approximations are surprizingly good. By setting (or defining) quantities as follows

$$R_2 = \mathcal{R} M_2{}^n\ , \quad \frac{R_L}{D} = B\mu^a\ , \quad R_2 = R_L\ , \quad \mu \equiv \frac{M_2}{M_1 + M_2}\ , \qquad (4)$$

measuring masses and radii in solar units, and introducing a dimensionless time τ, where

$$\tau = {}^t/T\ , \quad T = \frac{5\ c^5}{32\ G^3}\left(\frac{\mathcal{R}}{B}\right)^4 R_\odot^4\ M_\odot^3\ (M_1 + M_2)^{4n-3}\ , \qquad (5)$$

the evolutionary equation (2) reduces to:

$$\frac{d\tau}{d\mu} = \frac{\mu - \frac{1}{2}\,[(n-a) + 2]\ (1-\mu)}{(1-\mu)^2\ \mu^{2-4\,(n-a)}} \qquad (6)$$

The appearance of the parameter (n-a) is readily understood. It represents the competition between the secondary adjusting its size from stellar structure considerations, and the lobe size responding to the changing mass fraction.

Values B = 0.459 and a = 1/3 give R_L/D to better than three percent for $0 < \mu \lesssim 0.5$. These figures are assumed in what follows.

IV. THE STANDARD MODEL

Figure 2 shows the lower end of the main sequence[7]. Clearly n = 1 gives a good fit from $M_2 \sim 1\,M_\odot$ to $\sim 0.07\,M_\odot$ where hydrogen burning ceases. This gives our "standard model", with $\mathcal{R}$ in the range $0.87 \lesssim \mathcal{R} \lesssim 1$.

The resulting equations and some applications have already been published[1], and we highlight the main conclusions before passing to our new work. Two interesting results concern the period and the maximum lifetime T_{max} (assuming equal initial masses) for M_2 to be reduced to zero.

$$P = 3.22 \times 10^4\, M_2\, \mathcal{R}^{3/2} \text{ seconds} \tag{7}$$

$$T_{max} = 6.04 \times 10^9\, (M_1 + M_2)\mathcal{R}^4 \text{ years} \tag{8}$$

The gravitational radiation lifetime is clearly less than the nuclear lifetime if $M_1 \sim M_2 \lesssim 0.8\,M_\odot$ at the start of this phase, but long enough to permit the continuous hydrostatic readjustment that we assumed. The result $P \propto M_2$ agrees well with the period-spectral type correlation.

Eventually, when hydrogen burning ceases, a transition (indicated schematically in Figure 2) must occur to the cold, degenerate equation of state. In the equilibrium solution, the secondary expands and retreats as it continues losing mass. For low masses where non-relativistic degeneracy occurs (Pressure $\propto \rho^{5/3}$), P becomes proportional to M_2^{-1} . Doubts about the stability of this type of equilibrium solution appear to have been resolved by the observations of AM CVn (see § VI).

V. APPLICATIONS

The standard model is applied to two systems in Table 1. WZ Sge has been extensively investigated[8,9]. The mass ratio is unknown, but the values suggested give a fair range. For Z Cam (P = 6 hours 57.5 minutes), μ is known from spectroscopy[10].

The WZ Sge mass transfer rate agrees with observations[9]. The figures for the two, quite different systems suggest a canonical mass transfer rate of ~10^{-10} $M_\odot$/year, a figure of considerable interest. Giannone and Weigert[11], studying accretion onto a 0.5 $M_\odot$ white dwarf, found that a rate of 10^{-9} $M_\odot$/year produced dramatic thermal runaways, whereas 10^{-11} $M_\odot$/year could be absorbed without ever reaching ignition temperatures. It may be no coincidence that our mass transfer rate, and the behaviour of the dwarf novae, lie between these two extremes.

For Z Cam, the value of $M_2 \sin^3 i$ is known[10], and we deduce the inclination $i \sim 53° - 59°$, suggesting the possibility of at most grazing eclipses, as Warner and Nather have occasionally observed.

VI. THE REMARKABLE CASE OF AM CVn (= HZ 29)

HZ 29, having once been a suspected quasar, is now a confirmed binary with P ~1051 seconds . It shows a white dwarf spectrum containing only helium lines, sometimes doubled, presumably by "hot spot" emission filling them in.

Assuming only Kepler's law and the relation $R_L/D = 0.459\ \mu^{1/3}$, we may deduce a lower limit for the mean density of the secondary, since, if it fills the Roche lobe, we find

$$\bar{\rho}_2 = \frac{3M_2}{4\pi R_L^{\ 3}} = \frac{3\pi}{(0.459)^3}\ \frac{1}{GP^2} \tag{9}$$

For AM CVn, this gives $\bar{\rho}_2 \sim 1.32 \times 10^3$ gm cm^{-3}, far too high for a hydrogen burning main sequence star (the highest mean densities reached in the latter case give $P_{min} \geq 30$ minutes). Alternatively, as a degenerate hydrogen body it would require $M_2 \sim 0.25\ M_\odot$ - but one cannot assemble this much hydrogen together without it igniting! The lack of hydrogen emission, in marked contrast with classical dwarf novae, agrees with our arguments ruling it out from the small value of P.

We now recognize, belatedly, that it is quite possible for objects like WZ Sge to contain a helium core from their earlier nuclear evolution, a core which will ultimately be revealed following removal of its hydrogen envelope. Two possible helium configurations may have $\bar{\rho}_2 \sim 1.32 \times 10^3$, (i) a degenerate object with $M_2 \sim 0.04\ M_\odot$, or (ii) a helium-burning object with $M_2 \sim 0.5\ M_\odot$. The latter can be ruled out as it would dominate the spectrum, and also give huge radial velocity variations to the white dwarf for any reasonable value of M_1. A spectrum of AM CVn shows there may be some variation at the limit of detectability (~50-60 km/sec?)[12]. The value $M_2 \sim 0.04\ M_\odot$ leads to primary velocities of ~70 km/sec ($M_1 \sim 0.28\ M_\odot$) or ~30 km/sec ($M_1 \sim 1.04\ M_\odot$), and therefore appears to be consistent with this eyeball estimate. Since main sequence stars produce helium cores of order ~0.08 - 0.12 $M_\odot$ before evolving off the main sequence, our value of M_2 fot the remnant of such a core is quite reasonable. By comparison, any other production mechanism would seem contrived.

VII. FURTHER WORK

Work is now in progress on understanding the prior evolution of these systems from WUMa main-sequence binaries. It calls for a type of cooperative nuclear and gravitational evolution hitherto uninvestigated. The relativistic limit in which a realistic star orbits a black hole has also been studied. Such a star is ripped apart by tidal forces prior to reaching $R = 3M$ - but that's another story. The field is clearly ripe for further investigations, and it is hoped that the results reported above will stimulate the interest of many other workers in these problems.

TABLE 1

The standard model applied to WZ Sge and Z Cam

	WZ Sge				Z Cam	
μ	0.25		0.10		0.427	
$\mathcal{R}$	1.00	0.87	1.00	0.87	1.00	0.87
$M_2/M_\odot$	0.15	0.19	0.15	0.19	0.78	0.96
$M_1/M_\odot$	0.46	0.56	1.37	1.69	1.04	1.29
$-dM_2/dt$ (10^{23} grams/year)	1.38	2.41	2.50	4.36	1.26	2.19
$d\mathcal{E}_{grav}/dt$ (10^{31} ergs/sec)	0.73	1.45	3.54	7.11	0.21	0.42
$-dP/dt$ (10^{-13})	0.71	1.00	1.28	1.82	0.64	0.91
Age* (10^9 years)	2.35	1.66	8.48	5.99	1.87	1.32
Expectation* (10^9 years)	0.96	0.76	0.50	0.38	8.95	6.34

*"Age" is calculated from a hypothetical initial configuration with $\mu = 0.50$ and "expectation" assumes that the main sequence phase ends when $M_2 = 0.07\ M_\odot$. Evolution is assumed to occur without mass loss from the systems.

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FIGURE CAPTIONS

Figure 1. The system envisaged. As pointed out in the text (§ VI), the main-sequence component may contain a small helium core, which will be revealed after sufficient mass transfer. In later stages of evolution, the main sequence component is replaced by a degenerate system, mass transfer still proceeding as shown.

Figure 2. Bodenheimer's zero-age radius-mass relationship, the degenerate relationship for the same composition (hydrogen content by mass = 0.66), and a schematic transition region.

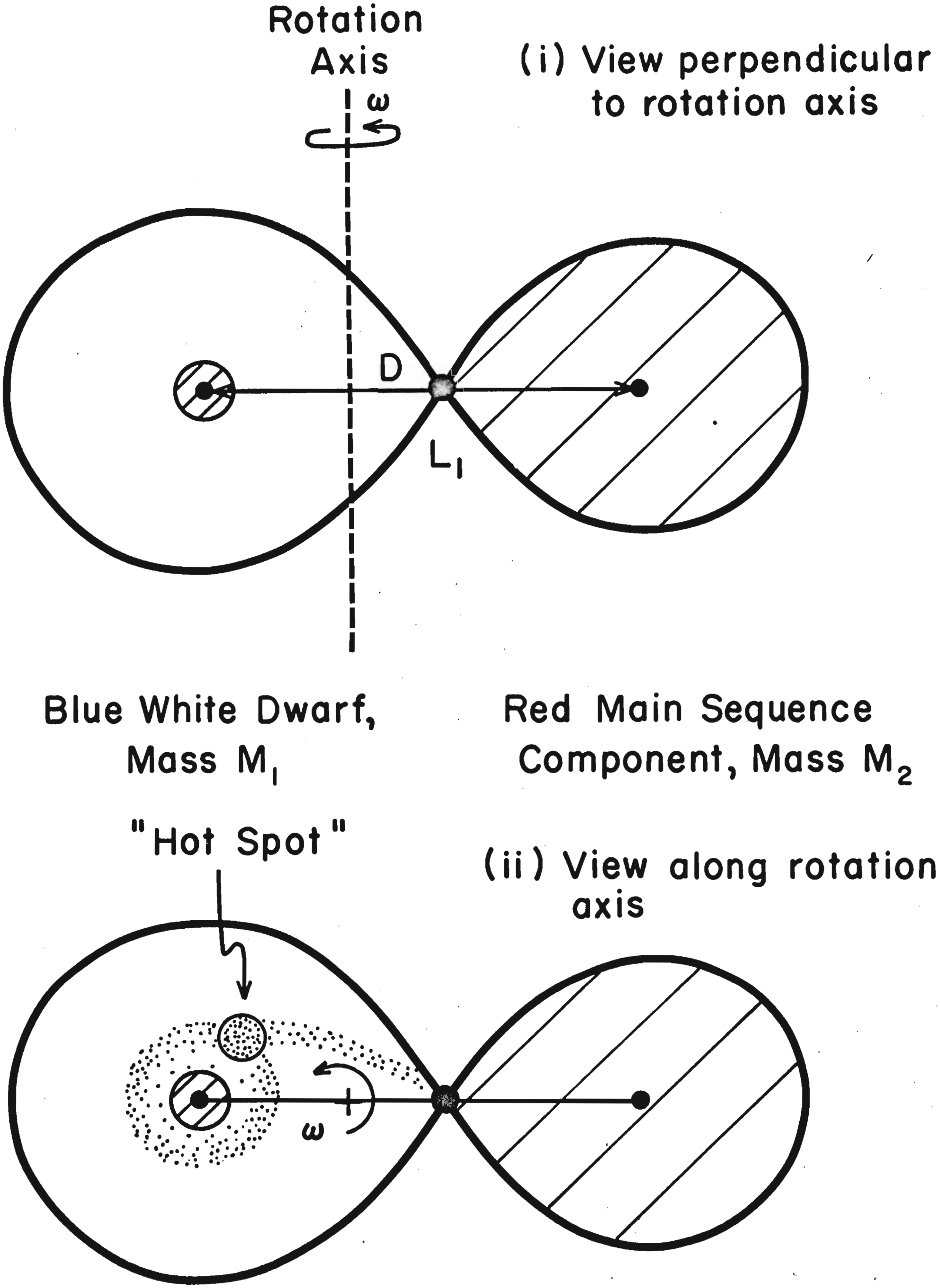
Rotation Axis
ω
(i) View perpendicular to rotation axis
D
L₁
Blue White Dwarf, Mass M₁
Red Main Sequence Component, Mass M₂
"Hot Spot"
(ii) View along rotation axis
ω

Fig. 1

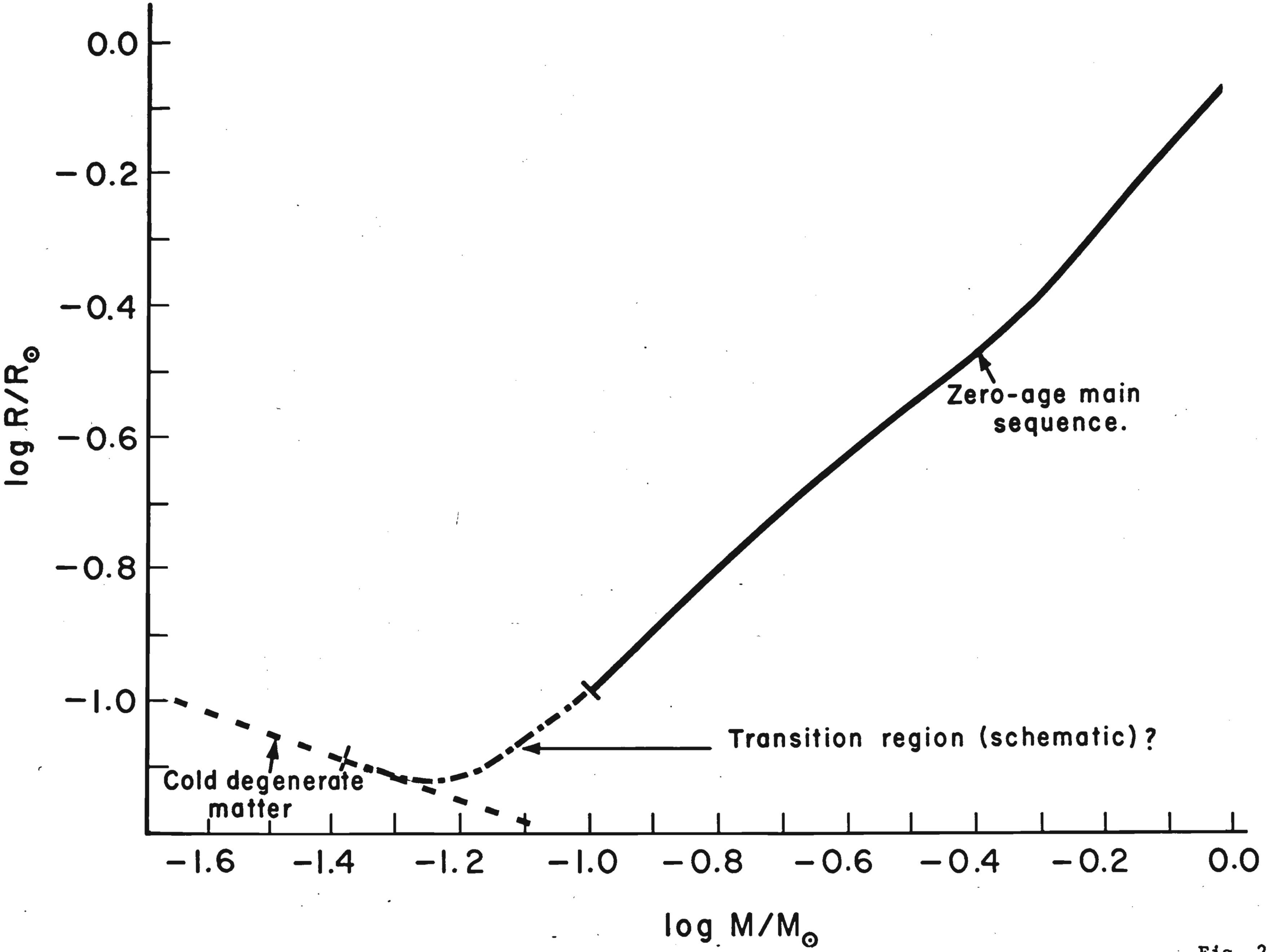
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
log R/R$_\odot$
Zero-age main sequence.
Transition region (schematic) ?
Cold degenerate matter
-1.6
-1.4
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
log M/M$_\odot$

Fig. 2

BIOGRAPHICAL SKETCH

John Faulkner. Born in Middlesex, England, 29 April 1937. Educated at the University of Cambridge, England. Wrangler (i.e. gained First Class Honours), Mathematics Tripos, Part II, 1959. Distinction, Mathematics Tripos, Part III, 1960. Ph. D. (Applied Mathematics and Theoretical Physics), 1964, for computations in the field of stellar structure. Research Assistant to Professor F. Hoyle, 1963-64. Research Fellow in Physics, Kellogg Radiation Laboratory, California Institute of Technology, 1964-66. Visiting Research Associate, Massachusetts Institute of Technology, 1965-67. William Stone Prize Research Fellowship, Peterhouse, Cambridge, 1965-68. Graduate Staff Member, Institute of Theoretical Astronomy, Cambridge, England 1967-69. Associate Professor of Astronomy and Astrophysics, Lick Observatory, University of California, Santa Cruz, 1969- . Chairman, Board of Studies in Astronomy and Astrophysics, UC Santa Cruz, January 1972- .

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