Gravitation and mass through gauge invariance

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Abstract

We explore the meaning and consequences of "gauge invariance of the second kind" as applied to the theory of gravitation. This generalizes the usual Lorentz invariant theories to theories which are covariant in the sense of general relativity as well as scale invariant. We are led to a formulation which on a macroscopic scale leads to Einstein's theory but with a self consistently determined coupling constant. It is also possible that the origin of the masses of the elementary particles is founded in this formulation of the theory.

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The two branches of field theory which have been crowned by complete quantitative success are the theories of electrodynamics and gravitation. In this essay we wish to emphasize two important features of electrodynamics which can be incorporated in a fundamental way into the theory of gravitation. These are gauge invariance of the second kind and invariance under scale length. These principles in addition to leading to the usual covariance of general relativity also imply the vanishing of the cosmological constant and the possibility of mass originating in the gravitational field.

We briefly review gauge invariance of the second kind in quantum electrodynamics. A charged particle may be represented by a complex field φ. The local charge density $\varphi^* \varphi$ is invariant under a transformation of phase $\varphi \to \varphi e^{i\chi}$. If χ is a constant this is called gauge invariance of the first kind which already has a consequence conservation of local charge. This conservation, however, is destroyed if χ is allowed to depend on space time x. It may be restored by introduction of a new field coupled to the charged particles through the minimal electromagnetic coupling (i.e. $p_\mu \to p_\mu - \frac{e}{c} A_\mu$). No experiment has yet been shown to be in disagrement with this theory and its quantitative success is one of the great achievement of modern theoretical physics.

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Turning now to the gravitational field, let us for the sake of simplicity consider a real, spinless boson field [1]. This has an unperturbed Lagrangian density (we use standard sommation convention)

$$\mathcal{L} = \frac{\partial\varphi}{\partial x^{\mu}}\frac{\partial\varphi}{\partial x_{\mu}} + m^{2}\varphi\varphi \qquad (1)$$

Such an expression is invariant under Lorentz transformations and this implies conservation of energy and momentum. However just as electrodynamics was founded on the principle of extending the gauge group from global to local, we wish to extend global Lorentz invariance to local Lorentz invariance. Before speculating on the philosophical interpretation of this assumption, we first present some of its elementary consequences.

Energy is no longer locally conserved but will be restored by the invention of a new field. Consider an inhomogeneous non uniform transformation $x^{\mu} \to x^{\mu} + a^{\mu}(x)$

This induces a transformation in the derivatives containing both an antisymmetric and a symmetric part corresponding respectively to a rotation of space-time and to a strain

$$\frac{\partial}{\partial x^{\mu}} \to \frac{\partial}{\partial x^{\mu}} + \omega_{\mu}{}^{\nu}\frac{\partial}{\partial x^{\nu}} + \epsilon_{\mu}{}^{\nu}\frac{\partial}{\partial x^{\nu}}$$

$$\omega_{\mu\nu} = \frac{1}{2}\left[\frac{\partial a_{\mu}}{\partial x^{\nu}} - \frac{\partial a_{\nu}}{\partial x^{\mu}}\right] \qquad (2)$$

$\mathcal{L}$ as defined by (1) is not invariant under the strain. This necessitates introducing a symmetric tensor field $g^{\mu\nu}$ coupled to equation (1) in the following way

$$\mathcal{L}' = g^{\mu\nu}\left[\frac{\partial\varphi}{\partial x^{\mu}}\frac{\partial\varphi}{\partial x^{\nu}}\right]\sqrt{g} + m^2\varphi\varphi\sqrt{g} \qquad (1')$$

where the indices of $g^{\mu\nu}$ transform contravariantly to the gradient

Having established in this way the existence of the tensor field $g^{\mu\nu}$ and its coupling to matter, we now must look for its equations of motion. In addition to $g^{\mu\nu}$ itself it is then necessary to construct other tensors which may be contracted to scalars in order to form a Lagrangian density. The technique for doing this is found in a classical article by Yang and Mills[2]. Their method is readily applied to our field $g^{\mu\nu}$ and one proves that the quantity $R^{\sigma}{}_{\mu\nu\lambda}$ is a tensor where

$$R^{\sigma}{}_{\mu\nu\lambda} = \frac{\partial\Gamma^{\sigma}{}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\partial\Gamma^{\sigma}{}_{\mu\lambda}}{\partial x^{\nu}} + \Gamma^{\sigma}{}_{\rho\nu}\Gamma^{\rho}{}_{\mu\lambda} - \Gamma^{\sigma}{}_{\rho\lambda}\Gamma^{\rho}{}_{\mu\nu} \qquad (3)$$

$\Gamma^{\sigma}{}_{\mu\nu}$ is a non tensor field which is necessarily introduced in the formation of a tensor involving derivatives. A prescription will be stated later to express $\Gamma^{\sigma}{}_{\mu\nu}$ in terms of $g^{\mu\nu}$. In the conventional Einstein theory these turn out to be the Christoffel symbols.

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Before discussing the free field Lagrangian, we wish at this point to speculate on the meaning of extended Lorentz invariance. It is customasily asserted that a free particle travels in a straight line, however upon further examination of this proposition one sees that it is a tautology for it defines either what is a free particle (if one assumes prior knowledge of what is a straight line) or a straight line (if one knows all about free particles). Thus to describe free particles it is required to have an a priori geometry which may be chosen arbitrarily to be a flat (Minkowskian) space. This choice has the immediate implication through translational invariance of space and time that energy and momentum are conserved. This conservation law at this stage of the argument follows by definition. In order to escape the tautology while remaining consistent with the arbitrariness of this description, we must

1°) allow any departure of energy conservation by allowing any departure from Lorentz invariance.

2°) restore energy conservation by interpreting this departure as due to a new agent ("field") acting in flat space such that with this new agent energy is again restored.

Any scalar density formed from the $g^{\mu\nu}$ and $R^{\sigma}{}_{\mu\nu\lambda}$ may be used as a free Lagrangian density. If "simplicity" is the only guide one may take advantage of the existence a scalar R obtained by contracting $R^{\sigma}{}_{\mu\nu\lambda}$ and $g^{\mu\nu}$. This constitutes an essen-

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tial difference between the theory of gravitation and that of electromagnetism where the field tensor $F_{\mu\nu}$ cannot be contracted. The resulting Lagrangian

$$L = \int [\sqrt{|g|}\, R + \mathcal{L}'] \, d^4x \tag{4}$$

Yields, after independant variation of $g^{\mu\nu}$ and $\Gamma^{\sigma}_{\mu\nu}$ Einstein's field equation coupled to matter.

This Lagrangian is subject to criticism from our point of view of gauge invariance. Indeed R has dimensions (length)$^{-2}$ and therefore the coupling with matter will involve a coupling constantwith dimension (length)^2 (the so called gravitational coupling constant). But as our reference geometry is entirely arbitrary, its scale is also arbitrary, thus the coupling constant with dimensions may be varied at will. It is only possible to escape such a contradiction if the coupling constant, as in electrodynamics, is dimensionless. This may be done by adimensionless coupling of $g^{\mu\nu}$ to some kind of new field much as an intermediate boson can be introduced to eliminate the dimensionality of coupling constants in weak interactions. However here the possibility also arises that this new field be the gravitational field itself; in the remaining we will discuss this latter possibility. This requires that the free Lagrangian L_0 be constituted from the three terms

$$\int \sqrt{|g|}\, R^2 \, d^4x \; ; \quad \int \sqrt{|g|}\, R^{\mu\nu} R_{\mu\nu} \, d^4x \; ; \quad \int \sqrt{|g|}\, R^{\sigma}_{\ \mu\nu\lambda} R_{\sigma}^{\ \mu\nu\lambda} \, d^4x.$$

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These terms were first studied by Pauli[3] and Weyl[4] and recently revived by Lanczos[5]. Generalizing the gravitational theory to include electromagnetic effects involves the two latter terms[3] . Restricting ourselves to a pure gravitation theory we shall then only consider the first term. Note that by scale invariance no constant term may arise in the Lagrangian, which means that a cosmological constant λ cannot exist. Further such a term would be the manifestation of a constant matter density because it would appears in L on the same footing as $m^2 \varphi\varphi$; for this there is no physical justification.

We thus are led to the Lagrangian

$$L = \int \left[\sqrt{|g|}\, R^2 + \mathcal{L}' \right] d^4x \tag{5}$$

The free field equations obtained from (5) differ ~~s~~ from those obtained by Pauli and others because from the gauge point of view the $g^{\mu\nu}$ must be varied independantly of the $\Gamma^{\sigma}{}_{\mu\nu}$. The resulting free equations are identical to one of those derived by Stephenson [6]. The coupling term, however, suggests a natural new interpretation. To see this we first write the equation of motion in the following suggestive form

$$(g^{\mu\nu}/R)_{;\sigma} = 0 \tag{6}$$

$$R_{\mu\nu}\,(g^{\mu\nu}/R) - \tfrac{1}{4}\, g^{\mu\nu} R = \frac{1}{R}\, T_{\mu\nu} \tag{7}$$

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where $T_{\mu\nu}$ is the matter energy momentum tensor.

The essential feature of equation (7) is that it has zero trace so that coupling with mass terms is impossible. A consistent interpretation is however that these equations describe a unified theory of gravitation and mass with $\mathcal{R}$ proportionnal to the mass density. The coupling is then only with particles with zero bare mass. In general one may interprete (6) as defining a Riemann space renormalized by its own curvature which is the mass density (note that the free field Lagrangian density is the renormalized volume element $\sqrt{|g|}\,\mathcal{R}^2 = \sqrt{|g_{\mu\nu}\mathcal{R}|}$

When $T_{\mu\nu}$ vanishes equation (6) (7) yields all results of Einsteinian general relativity on a planetary scale. However on an averaged galactic scale one has a variation of the effective gravitational constant because of the variation of $\mathcal{R}$.

On the elementary particle level, a possible interpretation is that the gravitational equation coupled to bare fields contains solutions which give rise to the masses of the elementary particles.

We close by summarizing our point of view. Through our analysis of the logic of gauge invariance of the second kind as applied to gravity, we are led to chose an equation for the gravitational field that has the following properties

a) no cosmological constant appears

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b) it reduces to Einstein's equation on the macroscopic scale but with a self consistant gravitational coupling constant.

c) it provides a possible theory of the origin of mass through the gravitational field.

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