The End of Black Hole Uniqueness

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Abstract

Are higher-dimensional black holes uniquely determined by their mass and spin? Do non-spherical black holes exist in higher dimensions? This essay explains how the answers to these questions have been supplied by the discovery of a new five-dimensional black hole solution. The existence of this solution implies that five-dimensional black holes exhibit much richer dynamics than their four-dimensional counterparts.

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No macroscopic dynamical object can be simpler than a black hole. Such is the remarkable conclusion of a number of theorems [1], obtained in the sixties and seventies, which prove that a stationary, asymptotically flat, vacuum black hole must be described by the Kerr metric and is uniquely specified by its mass and angular momentum. By measuring just these two quantities near asymptotic infinity, an observer knows in every detail the configuration of the gravitational field all the way down to the horizon of the black hole.

Despite the care with which the uniqueness theorems are formulated, there is one assumption that usually remains unstated: the theorems have only been proven for black holes in *four dimensions*. At the time, this probably sounded too evident a condition to need explicit mention. However, the most promising candidate for a theory of quantum gravity and matter, namely string theory, requires the existence of additional spatial dimensions.

String theory has had some spectacular successes in answering some of the deep questions raised by black hole thermodynamics, at least for certain types of extremal and near-extremal charged black holes. However, in spite of this success in understanding particular black hole solutions, very little is known about the general properties of black holes. This is not just a quantum mechanical issue: we know very little about general *classical* features of higher-dimensional black holes. This contrasts with the four-dimensional case, where the uniqueness theorems provide a complete answer.

Research into higher-dimensional black holes has largely been based on understanding solutions which are in many respects very similar to four-dimensional black holes. For example, in four dimensions the uniqueness theorems imply that (a constant time slice through) the event horizon must have spherical topology. Since all known higher-dimensional black hole solutions also have this property, there may have been a tendency to assume that it is true in general. However, if this assumption were incorrect then there could be a large class of higher-dimensional black holes with properties very different from those investigated so far.

This is not of solely theoretical significance. One of the most striking realizations of recent times is that the existence of extra dimensions may be a reality accessible to experiment. Models have been proposed in which the fundamental higher-dimensional Planck scale is near 1 TeV and matter fields are confined to a hypersurface in the higher-dimensional spacetime [2]. In these scenarios, understanding the full spectrum of higher-dimensional black holes may be essential for understanding experimental results from the next generation of particle accelerators [3].

The purpose of this essay is to address the questions raised above, namely[2]

Q1. Do the uniqueness theorems extend to higher dimensions? In other words, is a D-dimensional black hole solution uniquely determined by its mass and angular momenta?

Q2. Do topologically non-spherical black holes exist in higher dimensions?

Black hole solutions were found for all dimensions $D > 4$ by Myers and Perry (MP) [4]. Consider the case of $D = 5$. Asymptotically flat solutions can be characterized by the ADM mass M and two angular momenta J_1 and J_2. MP black hole solutions exist if

$$\eta \equiv \frac{27\pi J^2}{32GM^3} < 1, \tag{1}$$

where $J = |J_1| + |J_2|$ and G is Newton's constant. If the answer to Q1 is "yes" then the MP solutions must be the only black holes satisfying (1).

Let us set $J_2 = 0$ to simplify things further. Unlike the extremal Kerr solution in $D = 4$, the MP hole becomes nakedly singular when the bound (1) is saturated. Solutions which are near to saturating this bound have horizons that are almost flattened into a disc lying in the plane of rotation. The circumference of this disc tends to infinity as the singular solution is approached. This bizarre geometry suggests that these solutions might be unstable. So we have another question to add to our list:

Q3. Are some of the Myers-Perry black holes unstable? If so, what is the end-point of the instability?

In the remainder of this essay, we will provide rigorous answers to Q1 and Q2, and discuss possible answers to Q3, all in the case of five dimensions. The answers are provided by an explicit example.

A recent paper [5] has presented a new 2-parameter black hole solution in five dimensions. What is remarkable about this solution is that the event horizon has topology $S^1 \times S^2$: it describes a "black ring". The solution has a simple physical interpretation as a rotating loop of black string. The centrifugal force balances the tendency of the ring to collapse under its own tension and gravity.

[2]Here, and in the rest of this essay, the phrase "black hole" will be used as an abbreviation for "stationary, asymptotically flat black hole solution of the vacuum Einstein equations".

The black ring is the first example of a non-spherical black hole. Its existence proves that the answer to Q2 is "yes", at least in five dimensions. What about Q1? The black rings of [5] form a 2-parameter family of solutions with $J_2 = 0$ and hence $J = J_1$. The solutions exist if, and only if,

$$\eta \geq \eta_* \approx 0.84. \tag{2}$$

Note that this overlaps with the range for existence of MP black holes (1). Hence there are MP black holes and black rings with the same values of M and J. This proves that the answer to Q1 must be "no", at least in five dimensions. In fact, the situation is even more complicated because it turns out that if (1) and (2) are both strictly satisfied then there are *two* black ring solutions with the same values of M and J, as well as the MP black hole! If the bound (1) is exceeded then there is a unique ring solution. The situation is depicted in figure 1, which plots the areas of the MP black hole and black rings as functions of η.

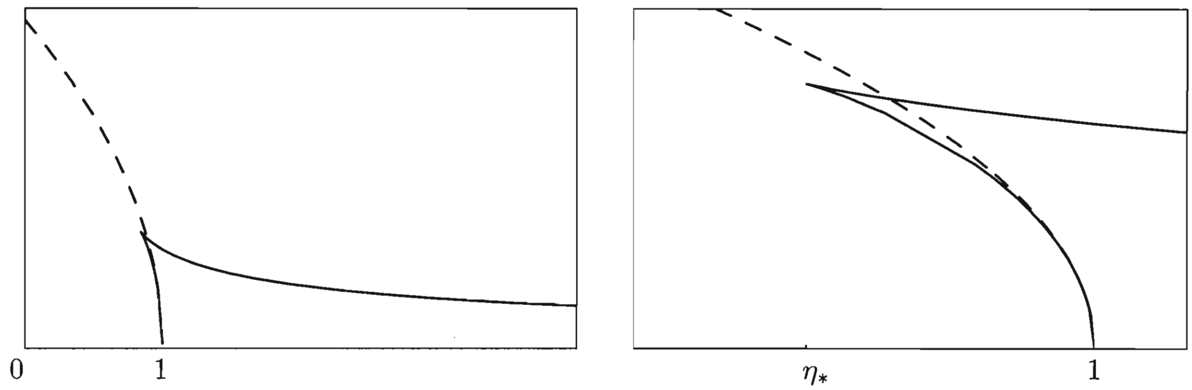

Figure 1: Plot of the area of the MP black hole (dashed) and black rings (solid) as functions of η. The plot on the right is a blow-up of the region where a black hole and two black rings with the same values of M and J exist. To fix the scale for a given mass, the area has been divided by $(GM)^{3/2}$.

It is natural to ask whether these new solutions are stable. Consider first the minimally spinning black ring, which saturates (2). Imagine dropping a test particle with vanishing angular momentum into it. If the ring were stable, one would expect it to settle down to a new ring solution with the same value of J and larger M, *i.e.*, smaller η. But this would violate (2)! This is a strong indication that the black ring with $\eta = \eta_*$ has to be *classically* unstable. However, the geometry of this ring is not qualitatively different from that of other rings (in particular, the horizon is non-degenerate). So we expect that rings close to saturating (2) will also be unstable. The second law of black hole mechanics implies that the area of the event horizon must increase during the evolution of the instability. We see from figure 1 that these rings can collapse into MP holes (possibly with emission of gravitational waves) consistently with the second law.

Now consider a ring with $\eta \gg 1$. Such a ring is large and skinny. To a good approximation, it looks locally like a boosted straight black string and will therefore suffer from the Gregory-Laflamme instability [6]. It has recently been proposed that the horizon of a black string becomes "lumpy" (translationally inhomogeneous) under the evolution of this instability [7]. This suggests that a black ring with large η will develop lumps around the ring. This cannot be the endpoint of the instability because the ring is rotating. The rotating lumps will give rise to a varying quadrupole moment and hence the ring will emit gravitational waves, losing energy and angular momentum. From fig 1, we see that the second law implies that η must decrease. Simple estimates suggest that this instability will persist down to $\eta = \eta_*$, at least for the larger black ring. The endpoint of the instability is presumably the collapse into a MP black hole.

The region of overlap between (1) and (2) is likely to contain rich dynamics that has no four-dimensional analogue. For example, consider black hole thermodynamics. According to the generalized second law, the solution with the event horizon of largest area (entropy) should be the one that is thermodynamically preferred for given M and J.[3] Figure 1 exhibits clearly the situation. For η near to η_*, both rings have lower entropy than the MP hole and must therefore be thermodynamically unstable.

As η is increased further, a point is reached beyond which the larger black ring has greater entropy than the MP hole. Hence the MP hole becomes thermodynamically unstable. It is satisfying that, thanks to the existence of a regular ring solution, the singular solution with $\eta = 1$ and zero area is hidden from the thermodynamic point of view. We argued above that the black ring is itself classically unstable so the unstable MP hole won't turn into a black ring. Instead it will emit radiation, losing mass and angular momentum and eventually settle down to a stable MP hole with a smaller value of η. Thus, we have a thermodynamic answer to Q3: the MP holes that are close to saturating (1) *are* unstable, and will settle down to black holes that rotate more slowly. It is natural to conjecture that the classical answer to Q3 will be very

[3]Thermodynamical ensembles are not well-defined for gravity in asymptotically flat space, so we are using the microcanonical ensemble, corresponding to putting the system into a box. This allows the possibility of a black hole coming into stable thermal equilibrium with its own radiation.

similar.

To summarize, the recent discovery of black ring solutions supplies answers to two long-standing questions. First, it shows that black holes with horizons of non-spherical topology do exist. Second, it proves that the four-dimensional black hole uniqueness theorems cannot be extended in a straightforward manner to five dimensions. We have also shown that the existence of black rings implies that some spherical black holes must be thermodynamically unstable, and suggests that they will also be classically unstable.

The discovery of black rings raises many new questions. For example, is there a generalization of the black ring that carries non-zero J_2? Charged black rings are likely to exist, but what about supersymmetric rings with regular horizons? Such rings would certainly be stable.

What is the answer to Q1 and Q2 in dimensions higher than five? One might view the ring as the result of bending a straight black string into a circular shape, and adding enough rotation to achieve a balance. Can one similarly "bend and balance" the black strings and black branes of higher dimensions into asymptotically flat solutions with finite horizons of non-spherical topology? More generally, are there any restrictions on the topology of higher-dimensional black holes? In five dimensions this question might be tractable because (a constant time slice through) the horizon is then a three-manifold, and a lot is known about the topology of three-manifolds.[4] Higher dimensions pose a much harder problem.

Even if the strongest uniqueness theorem does not extend to five (or more) dimensions, is a weaker version, such as Israel's uniqueness theorem for *static* black holes, still valid? In other words, do there exist static black hole solutions other than the Schwarzschild solution? Finally: is it possible to formulate a concept of uniqueness in higher dimensions that is still general enough to be useful?

Note added: After completion of this essay, the uniqueness of higher-dimensional static black holes was proved in [9].

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[4]This was recently exploited [8] to show that the event horizon of any *static* five-dimensional black hole must have topology given by a connected sum of S^3 and $S^1 \times S^2$ terms.

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