New Directions for Research in the Theory of Gravitation

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Before anyone can have the audacity to formulate even the most rudimentary plan of attack on the problem of harnessing the force of gravitation, he must understand the nature of his adversary. I take it as almost axiomatic that the phenomenon of gravitation is poorly understood even by the best of minds, and that the last word on it is very far indeed from having been spoken.

Nevertheless, the theoretical investigation of gravitation has received relatively little attention during the last three decades. There are several reasons for this. First, the subject is peculiarly difficult; the existing body of theory on it involves rather recondite mathematics, and the fundamental equations are almost hopeless of solution in all but a very few special cases. Although the accepted theory is motivated by two or three beautifully simple yet profound principles, these guiding principles have so far been of little help in predicting the general features of the solutions of the equations to which they give rise. And, as any researcher in the field knows, one can develop a serious case of "writer's cramp" in that manipulation of tensor indices which is usually necessary in order to prove only a single tediously trivial point.

Secondly, modern gravitational theory has few consequences which are even remotely susceptible of experimental verification. The old Newtonian theory, involving action-at-a-distance, has, for practical purposes, been far too adequate. Consequently, stimuli for the theoretical investigation of gravitation are virtually non-existent, and gravitational research is almost totally unrewarding. It is a field which had its brief brilliant hour, but which has since fallen into a state of near disrepute.

In spite of all this, it is very probable that the phenomenon of gravitation will eventually have to be reckoned with again in respectable circles, and it may well happen that this reckoning will present itself in a rather acute form. It is one of the purposes of this note to suggest that we may be already in the first phases of such a new development, and to point out some new directions into which we are likely to be led as a result.

I shall assume, virtually without question, the validity, in its appropriate domain, of the Einstein theory of gravitation - that is to say, of the original general theory of relativity, as distinct from later embellishments by many workers including Einstein himself. Einstein's theory is, to my mind, far too beautiful and satisfying to be cast aside. And it is so intimately connected with and firmly entrenched in those concepts of invariance and conservation which have come to be regarded as fundamental in physics, that in casting it aside, we should be casting aside much that has been enormously fruitful in the past as well as the present, to the experimenter no less than to the theorist. However, it should be borne in mind that the Einstein theory is a "classical" (that is, *non-quantum*) theory. It forms by itself a logical and self-contained system. Only the fact that the real world around us has taught us that the system may not be quite so self-contained after all, makes the following remarks of some interest.

For the sake of orientation let us reverse the usual order of things and first fix our sights on those grossly practical things, such as "gravity reflectors" or "insulators", or magic "alloys" which can change "gravity" into heat, which one might hope to find as the useful by-products of new discoveries in the theory of gravitation. The use of terms such as "reflector" or "insulator" clearly is based upon analogy with electromagnetism. Now, it is quite true that gravitation is similar to electromagnetism in many ways. Just as the latter can be split into an electric and a magnetic part, so can the former be split into two parts, one being that produced by static matter and the other that produced by moving matter. The *gauge group* of electrodynamics has its counterpart in the coordinate transformation group of gravidynamics. The electromagnetic and gravitational fields both propogate with the speed of light.

In other respects, however, the gravitational and electromagnetic fields differ profoundly. Of prime importance is the extreme weakness of gravitational coupling between material bodies, as compared with electromagnetic coupling (advice of professional weight-lifters notwithstanding!) The weakness of this coupling has the consequence that schemes for achieving gravitational insulation, via methods involving fanciful devices such as oscillation or conduction, would require masses of planetary magnitude. And even if the necessary masses could be manipulated, these schemes would be doomed to failure, for, since quantum forces would not be available for such macroscopic manipulation, non-gravitational force fields would have to be employed. But the existence of such external fields would defeat its own purpose, because every stress, every force-potential, and, indeed, every form of energy produces its own gravitational field. The gravitational field is all-pervading.

These features are built into the Einstein theory as consequences of the fundamental requirements of energy-momentum conservation. One result is that the gravitational field partially produces itself! Mathematically this is reflected in the strong non-linearity of the gravitational field equations, which stands in sharp contrast to the linearity of the electromagnetic field equations.

These considerations are quite sufficient to enable one to state flatly that any frontal attack on the problem of harnessing the power of gravity along the above lines is a waste of time. Indeed, unless the term "gravity" is broadened to include a much wider range of phenomena than hitherto, one may safely pronounce all gravity-power schemes impossible. Such a broadening of terminology may, however, be logically possible, or even necessary. That is the point I wish to make.

In one very important respect the Einstein theory has recently undergone some fundamental broadening. It has been "quantized" - or rather, as matters stand at present, -"nearly quantized". For a generation, two tree-like giants, the quantum theory and the general theory of relativity, have existed side by side, one incredibly fruitful and the other almost totally barren save for one or two golden fruits. Except for the most indirect contacts through cosmological problems they have remained completely independent, although the special theory of relativity has long since been combined with the quantum theory, with results which, while profound, have not been as successful as one might have hoped. However, in 1950, Pirani and Schild[1] and, independently, Bergmann and his co-workers accomplished the *Hamiltonian formulation* of the Einstein field equations. By "Hamiltonian formulation" is meant a certain *canonical* way of writing the equations, which forms the point of departure of a quantum theory of them. With this important accomplishment it became possible for the first time to consider seriously a rigorous quantization of the gravitational field.

When one attempts, however, to pass from the classical Hamiltonian formulation of Einstein's equations to a quantum version of his theory, one runs immediately into several problems. Chief among these is the fact that the *Hamiltonian density* for the theory contains products involving *non-commuting* factors. One does not know *a priori* how these factors should be ordered.

The quantum Hamiltonian density must be an *Hermitian operator.* This implies a symmetrical ordering of the aforementioned factors. One could attempt to use the simplest possible symmetrical ordering scheme, but then one would not know whether an equivalent quantum theory would have been obtained with a similar symmetrization procedure, if another set of variables had been used with which to represent the gravitational field, i.e. if a *point transformation* had been carried out on the field of variables. The representation which is most frequently employed is that in which the components $g_{\mu\nu}$ of the metric tensor of space-time are chosen as the gravitational field variables. The present writer has shown how a geometry can be introduced in a natural way into the 10 - dimensional "space" of the $g_{\mu\nu}$ (to be distinguished from the 4 - dimensional space-time manifold) and has used this geometry to construct an invariant quantization rule. The method is applicable to all systems having Hamiltonians which are quadratic in the *momenta*.

$g_{\mu\nu}$ - space, according to its natural geometry, is found to be non-flat. This is a strict characterization of the fact that Einstein's theory is intrinsically non-linear. There exists no Cartesian representation in which its quantization can be carried out in simple fashion. It is therefore quite fortunate that an invariant quantization prescription nevertheless exists.

In addition to invariance under point transformations of the gravitational field variables, the quantized theory must also be investigated with respect to the more important question of general *covariance* . The *forms* of all quantum equations must remain invariant under general coordinate transformations. The classical Hamiltonian formulation is covariant because it proceeds from a set of covariant equations. After passage is made to the quantum theory, however, the covariance must be proved all over again because (1) the non-commutativity of factors may introduce new difficulties, and (2) the formal appearance of the quantized theory, in the so-called *Schrodinger representation* which one arrives at, is quite different from that of the classical theory. The present writer, in unpublished work, has examined the anatomy of general coordinate transformations, as seen from the viewpoint of the quantum theory, in considerable detail. The unfortunate result of these researches is the discovery that the quantized theory is no longer covariant.

At this point one may well ask to know the reasons for attempting quantization of the gravitational field in the first place. As a matter of fact, the overwhelming weight of opinion of physicists is opposed to the attempt. The prime reason for this is the experimental fact that gravitation has never been observed to take part in physical events on a quantum level, and where there is no evidence it is bad form to speculate. Even if the covariance failure mentioned above could be regarded as definite negative evidence, it would cause no upheaval in physics. It may actually be that the gravitational field is the one and only field which is *not* quantized in Nature. The gravitational field, with its attendant phenomena, could, under these circumstances, constitute the ultimate classical level which must be postulated, even in the quantum theory, in order to have a consistent "quantum theory of measurement".[4] The gravitational field could be produced, not by the quantum *stress tensor* of all the other fields in nature, but rather by the quantum *mean value* of this tenor. If Ψ denotes the quantum *state vector* of the quantized fields, it would satisfy an invariant Schrodinger equation of the form

$$i\hbar c\frac{\partial\Psi}{\partial x^{\mu}} = H\mu\Psi, \quad \text{(the symbol } \hbar = \frac{h}{2\pi} \text{),}$$

where ∂x^{μ} is a "time-like" displacement. The Hamiltonian density Hu would, of course, depend on the metric tensor $g_{\mu\nu}$ as well as on the quantized field variables. But $g_{\mu\nu}$, depending on the mean stress, would depend on Ψ and its adjoint Ψ^*. Hence equation (1) would be non-linear in Ψ, and one of the most fundamental principles of the quantum theory, namely the *principle of superposition of states*, would be invalidated. However, this principle would be invalidated only in the large; it would still be true at the quantum level. The dependence of $H\mu$ on the $g_{\mu\nu}$ would be important only on a cosmic level. And here, the superposition principle is of no consequence. The universe is in one and only one state Ψ^*. There exists, so far as we know, no coupling with forces *outside* the universe which could cause a transition of the universe to a different state. The state vector Ψ will satisfy equation (1) for all time.

However, even if the gravitational field is left unquantized, there remain difficulties. To mention only one, if an "impressed" unquantized gravitational field is allowed to interact with the vacuum fluctuations of those fields which are quantized, a polarization effect will result which is non-calculable owing to *divergence* difficulties, and which cannot even be handled by modern "renormalization" techniques.[5] This is a situation which is not aggravated by quantization of the gravitational field. In point of fact, at the present stage of the game, there is little to choose between the two possibilities. Improved methods of computation and/or interpretation may overcome the difficulty mentioned here, as well as the covariance-failure of the quantized theory mentioned previously.

There still remains a powerful æsthetic argument on the side of quantization. The dream of a "unified field theory" is as tantalizing as ever. But in a consistent unified field theory one could hardly exempt one part and not the other of a "super-field" form quantization. Furthermore, a unified field theory may well be the solution to some of the outstanding fundamental problems of the present day. A unifying principle of some sort is clearly needed to solve the problem of the "mass spectrum" and to bring a semblance of order into the baffling array of odd varieties of elementary particles.

Pais*[6] has recently put forward a theory of heavy particles based on an extension of the manifold of transformations from four dimensions to six. While the idea of adding extra dimensions is by no means new, Pais' particular method is original and interesting. By means of it he is able to predict the existence of particles which have properties similar to, and which he identifies with, nucleous, V particles, and π and τ mesons. Furthermore, he is able to derive the charge independent of nuclear forces and the law of conservation of heavy particles from fundamental invariance conditions.

Although Pais specifically disclaims any metric properties for his extra two dimensions, there is no reason why his theory cannot be made completely geometrical. I should therefore like to suggest the following model for Nature: The universe is six-dimensional, with five space-like dimensions and one time-like dimension. Of the five space dimensions, two are closed on themselves with the topology of a spherical surface. The other three are the familiar dimensions of space. The reason we are not immediately aware of the two closed dimensions is that the spherical radius involved is extremely small. The metric of the six-dimensional manifold varies according to a set of equations derived from a *variational principle* based on the total curvature tensor. The metric tensor describes all *boson* fields, including the gravitational, electromagnetic, π and τ meson fields. The *fermion* fields, including electrons, neutrinos, muons, nucleons, and V particles, are described by a superimposed spinor field[7] together with a corresponding *Lagrangian* function.

Preliminary investigation indicates that this model will possess all the qualitative features of Pais' model, including mass spectrum and stability properties. In addition it will yield necessary cross-couplings that Pais was unable to account for.

The final point of this note is now evident. If the gravitational field is welded into a single entity along with electromagnetic and meson fields, circumstances can arise (at least in the subnuclear

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* In my reference to the work of Pais, I state that my proposal of a six-dimensional space-time is not only equivalent to, but is also a generalization of his ideas. This is not true, as I have discovered by further investigations carried out since I submitted this essay. My proposal is similar to Pais' at a number of points, but proves to be inadequate to describe the experimentally observed physical situation at a number of other points.

domain) in which one field cannot be distinguished from another, and a broadening of the term "gravity" becomes inevitable. Under these circumstances one may well anticipate being able to "harness gravity". The vast riches of Nature in this domain are as yet virtually untouched.

If, however, one is ever to be able to do more than merely sit in contemplation of the delicate interplay of forces, both vast and small, between the elementary particles, one must understand in the clearest possible terms precisely what goes on behind the scenes. At the moment, our understanding of these matters is extremely poor. To the extent that this lack of understanding falls in the domain of gravitational theory (in the largest sense), the unrewarding nature of research in this field is to be blamed. External stimuli will be urgently needed in the near future to encourage young physicists to embark upon gravitational research in spite of the odds.

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4. See, for example, D. Bohm, *Quantum Theory,* New York, Prentice-Hall, Inc. (1951) Chapter 22.
5. B. S. DeWitt, *Physical Review,* 90, 357 (1953). For other cases, in which quantization of the gravitational field leads to no difficulties, reference may be made to my PhD. thesis (Harvard, 1950).
6. A. Pais, *Proceedings of the Lorentz-Kamerlingh Onnes Conference,* Leyden, June 1953.
7. For the treatment of spinors in a unified field theory see W. Pauli, *Annalen der Physik,* 18, 337 (1933). See also B. S. DeWitt and C. M. DeWitt, *Physical Review,* 87, 116 (1952).