The Cosmological Constant Problem in Brane-Worlds and Gravitational Lorentz Violations

Csaba Csáki+, Joshua Erlich+ and Christophe Grojean#,*

+Theory Division T-8, Los Alamos National Laboratory, Los Alamos, NM 87545 USA

#Department of Physics, University of California, Berkeley, CA 94720 USA

*Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA

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The Cosmological Constant Problem in Brane–Worlds and Gravitational Lorentz Violations

Csaba Csáki[a,1], Joshua Erlich[a] and Christophe Grojean[b,c]

[a] *Theory Division T–8, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
[b] *Department of Physics, University of California, Berkeley, CA 94720, USA*
[c] *Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
email addresses: csaki@lanl.gov, erlich@lanl.gov, cmgrojean@lbl.gov

Abstract

Brane worlds are theories with extra spatial dimensions in which ordinary matter is localized on a (3+1) dimensional submanifold. Such theories could have interesting consequences for particle physics and gravitational physics. In this essay we concentrate on the cosmological constant (CC) problem in the context of brane worlds. We show how extra-dimensional scenarios may violate Lorentz invariance in the gravity sector of the effective 4D theory, while particle physics remains unaffected. In such theories the usual no-go theorems for adjustment of the CC do not apply, and we indicate a possible explanation of the smallness of the CC. Lorentz violating effects would manifest themselves in gravitational waves travelling with a speed different from light, which can be searched for in gravitational wave experiments.

[1] J. Robert Oppenheimer fellow.

It is believed that Einstein's General Relativity (GR) is an inadequate description of gravity at high energies because at energies near the Planck scale ($M_{Pl} \sim 10^{19}$ GeV) the Schwarzschild radius of a system ($G_N m/c^2$) becomes of the same order as its Compton length ($\hbar m/c^2$), and the effects of quantum gravity are important. However, the Planck scale may not necessarily be the scale at which modifications to Einstein's theory of gravity first appear. In fact, contrary to many aspects of particle physics, gravity has not been measured at distances smaller than about a millimeter. Therefore, in principle gravity could begin to deviate from ordinary GR at such scales. One way of modifying gravity is by introducing extra dimensions, whose effective size is below the mm scale. However, when modifying physics at these relatively low energy scales one has to make sure that the standard model of particle physics, which has been extremely well tested up to energy scales of the order of 100 GeV ($\sim 10^{-16}$ cm), is not also modified at those scales. One way to modify gravity at low energies without affecting particle physics is to assume that all the fields of the standard model are localized in space to a three-dimensional submanifold ("3-brane") in the higher dimensional world, in which case only gravity or other non-standard model fields probe the presence of extra dimensions. While this "brane world" approach may seem *ad hoc*, these principles are naturally realized in string theory (which is so far the only known consistent theory of quantum gravity). In fact, string theory necessarily requires extra dimensions for the consistency of the theory, and gauge theories localized on branes are a natural part of the theory.

In the context of brane world models several problems in the Standard Model of particle physics and in gravity have been reformulated, with new approaches suggested to their solutions. One of the main motivations for considering brane worlds is that they suggest new resolutions of the hierarchy problem (the question of why the energy scale of electroweak interactions is so much smaller than the scale of gravity). One way of explaining the hierarchy is by assuming that there are flat extra dimensions, which are much larger than their natural value $1/M_{Pl}$ [1]. The presence of such large extra dimensions could lower the *fundamental, higher dimensional* Planck scale all the way to the TeV scale, thereby eliminating the hierarchy in the fundamental scales of physics. In this scenario particle physics experiments with accelerators like the Tevatron or LHC would directly probe the full theory of quantum gravity, which itself would be at the TeV scale. Another explanation for the hierarchy between the weak scale and the Planck scale may be that there is curvature along the extra dimensions, causing the natural scale of the effective 4D theory to depend on the position of the brane along the extra dimension [2]. A typical metric describing such "warped spacetimes" is of the form

$$ds^2 = a(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{1}$$

where the warp factor $a(y)$ can induce an exponential hierarchy between the weak and the Planck scales, and even more strikingly, ensure that the brane world observer sees 4D Einstein gravity at larger distances even without compactification of the extra dimensions.

The main focus of this essay is to explain how extra dimensions may also help to tackle the cosmological constant problem. It was first pointed out by Rubakov and Shaposhnikov [3] that the cosmological constant problem is reformulated in brane world models. In ordinary four dimensional cosmology the contributions to the vacuum energy from gravitational loops ($\mathcal{O}(M_{Pl}^4)$), the electroweak phase transition ($\mathcal{O}(10^{-64} M_{Pl}^4)$) and chiral symmetry breaking ($\mathcal{O}(10^{-76} M_{Pl}^4)$) have to cancel each other to $\mathcal{O}(10^{-120} M_{Pl}^4)$ to be consistent with current bounds on the cosmological constant. However, in the presence of extra dimensions the four dimensional vacuum energy on the brane does not necessarily give rise to an effective four dimensional cosmological constant. Instead the vacuum energy can *warp* the spacetime and introduce a curvature in the bulk while maintaining a static four dimensional brane world: in a sense, the energy-induced curvature flows off the brane. The cosmological constant problem is then reformulated as the question of why the background warps in the appropriate fashion without introducing an effective 4D cosmological constant; that is, why there would be an exact cancellation between the brane and the bulk cosmological constants. *Per se* such extra dimensional cancellation mechanisms of the four dimensional cosmological constant are reminiscent of purely four dimensional cancellation mechanisms. As proposed by Hawking [4] for example, a four form field strength would provide a contribution to the cosmological constant whose magnitude is not fixed by the field equations but appears instead as a constant of integration. However, the anthropic principle must still be invoked in order to explain why that integration constant happens to be chosen so as to cancel the other contributions to the cosmological constant. Or to say it differently, among the three classes of maximally symmetric solutions (flat, de Sitter or anti-de Sitter) why is the flat solution singled out? As argued by Weinberg [5] all known adjustment mechanisms of the cosmological constant in a purely four dimensional scenario suffer from similar problems.

In order for extra dimensional scenarios to provide a resolution of the cosmological constant problem it would then have to give a positive answer to the following questions:

1. Can the brane vacuum energy vary continuously in a "natural" range (around the weak scale, for instance) while still maintaining a vanishing effective 4D cosmological constant?

2. Is there a way to select the flat solution among maximally symmetric solutions?

In order for an extra dimensional theory to provide a satisfactory resolution to the cosmological constant problem it must somehow evade Weinberg's no-go theorem for the adjustment of the cosmological constant. The difficulty is that for an extra dimensional theory to agree with our observed four dimensional universe, the theory has to have an effective 4D description at large distances. However the cosmological constant has to be extremely small, at most of $\mathcal{O}(10^{-3}\ \mathrm{eV})^4$, so the CC problem is in some sense a low-energy physics problem. Then one should also be able to understand the cancellation mechanism directly from the effective 4D theory, and one seems to be stuck with Weinberg's no-go theorem. This is, however, not quite true. The freedom of extra dimensional theories to modify the behavior of gravity in the bulk while the gauge interactions live on the brane allows for the possibility of an effective low-energy description which weakly violates 4D Lorentz invariance, without contradicting any current observations. Thus one might hope to circumvent the no-go theorem for the adjustment of the cosmological constant because these theories are fundamentally different than those considered by Weinberg. To construct such a theory we will consider some higher dimensional geometries in which not only the 4D distance scales vary along the extra dimensions as in usual warped scenarios, but also in which the spatial and time scales vary in a slightly different way ("asymmetric warping"). A prototypical example will be given by 5D metrics of the form

$$ds^2 = -n^2(r)\,dt^2 + a^2(r)\left(\frac{d\sigma^2}{1-k\sigma^2} + \sigma^2 d\Omega_2^2\right) + b^2(r)\,dr^2. \tag{2}$$

The coordinate r corresponds to the extra dimension transverse to the brane and $k = \pm 1, 0$ is the spatial curvature of the 3D sections parallel to the brane. The induced geometry at the 4D sections of constant r may still be flat, implying that (up to tiny quantum gravitational corrections) particle physics on the brane will see a Lorentz invariant spacetime. However, different 4D sections of the metric (2) have a differently defined Lorentz symmetry: the local speed of light depends on the position along the extra dimension as $c(r) = n(r)/a(r)$. Therefore the spacetime (2) globally violates 4D Lorentz invariance, leading to apparent violations of Lorentz invariance from the brane observer's point of view due to bulk gravity effects. The important point is that these effects are restricted to the gravity sector of the effective theory, which has not been very well measured, while the extremely well measured Lorentz invariance of particle physics remains unaffected in these scenarios.

Such asymmetrically warped space-times are actually quite generic: indeed, Birkhoff's theorem ensures that the most general solution in the bulk can be transformed into the black hole metric of the form (2) with

$$a(r) = r \quad \text{and} \quad n(r) = \frac{1}{b(r)} = k + \frac{r^2}{l^2} - \frac{\mu}{r^2} + \frac{Q^2}{r^4}, \tag{3}$$

where l is the radius of curvature of the bulk induced by the 5D vacuum energy. The precise geometry of the black hole depends on the type of sources and fields which propagate in the bulk; in general, it would be charaterized by its mass, μ, and its charges, Q, which appear as constants of integration of the equations of motion and parametrize the asymmetry of the 5D space-time. The 4D Lorentz symmetry is restored only when both μ and Q vanish. These new parameters can now be used to eliminate the fine-tunings that plague the symmetrically warped models.

Indeed, it can be shown [6] by solving the Einstein equations around the brane where

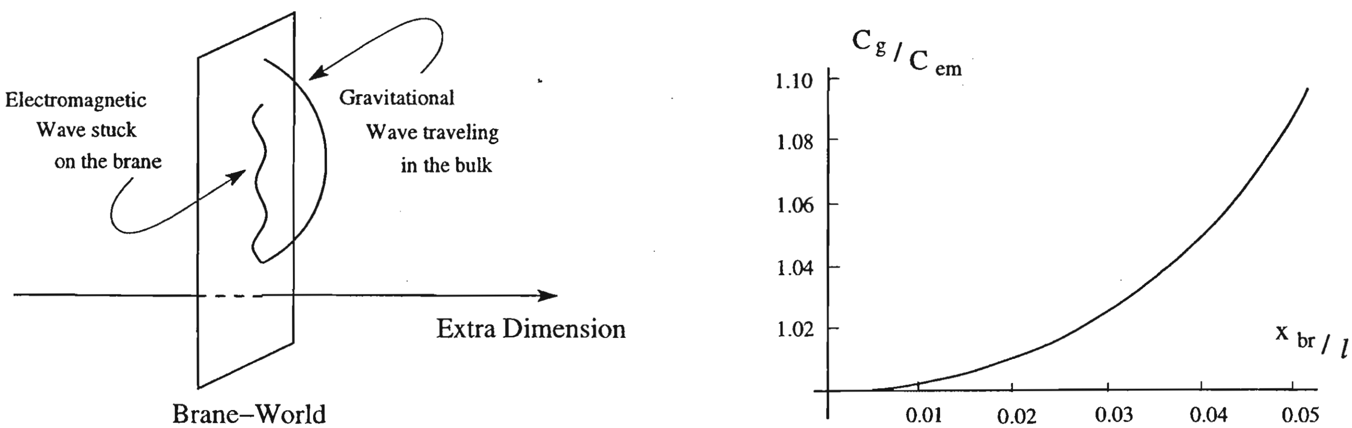

Figure 1: A graviton emitted on the brane will travel along a geodesic in the bulk before returning to the brane. A photon emitted at the same time can propagate only along the brane and may wander a shorter distance along the brane than the graviton in the same time. The 4D effective propagation speed of gravity is distance dependent (x_{br} is the distance travelled along the brane and l characterizes the curvature of the bulk).

standard model fields are localized (that is, solving the "Israel junction conditions") that the induced metric on a brane embedded in such a black-hole background can remain flat whatever the vacuum energy density on the brane. For instance, a phase transition on the brane might not affect the geometry of the brane, but may instead be compensated for by a change of the mass and charge of the BH due to emission or absorption of gravitational/electromagnetic waves. Hence, no parameter of the action would have to be tuned to keep the brane flat. This answers the first requirement on solutions of the cosmological constant problem posed above. However, one still needs to answer the second question, namely what selects this flat solution among the 4D maximally symmetric solutions. The point is that other maximally symmetric induced metrics on the brane require either the mass or the charge of the BH to vanish and so the flat solutions for which $\mu \neq 0$ and $Q \neq 0$ are not continuously connected to the (anti-)de Sitter solutions. Being an isolated point in the moduli space, the flat solution is *ipso facto* a stable vacuum. From an effective four dimensional point of view, Weinberg's no-go theorem is evaded by the Lorentz violating corrections to four dimensional Einstein gravity.

Perhaps the most remarkable property of these asymmetrically warped backgrounds is that, in addition to providing a possible resolution of the CC problem, these backgrounds have consequences that can be verified through gravitational measurements. As we have already stressed, asymmetrically warped spacetimes break 4D Lorentz invariance in the gravitational sector. Particle physics will not feel these effects, but gravitational waves are free to propagate into the bulk and they will necessarily feel the effects of the variation of the speed of light along the extra dimension. The propagation of gravitational waves in asymmetrically warped spaces is analogous to the propagation of electromagnetic waves through a medium with a varying index of refraction. Gravitational wave propagation reflects Fermat's principle, and if the local speed of light is increasing away from the brane then gravitational waves propagating between two points on the brane will take advantage by bending slightly into the bulk, and will arrive earlier than the electromagnetic waves which are stuck to the brane (see Figure). Thus in these theories gravitational waves can travel faster than light! However, these faster than light signals do not violate causality with the usual associated paradoxes. The apparent violation of causality from the brane observer's point of view is due to the fact that the region of causal contact is actually bigger than one would naively expect from the ordinary propagation of light in an expanding Universe, but there are no closed timelike curves in the 5D spacetime that would make the theory inconsistent.

The beauty of these models is that even extremely small Lorentz violating effects may be measured in gravity wave experiments. For example, an astrophysical event such as a distant supernova might generate gravitational waves which would reach future gravity wave detectors before we actually "see" the event. Thus future gravitational wave experiments like LIGO, VIRGO or LISA might discover this unique signature of the existence of extra dimensions. If found, such evidence may strongly influence future developments in elementary particle physics, cosmology and astrophysics.

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