A Search for Anisotropy of Inertia

Giuseppe Cocconi and Edwin Salpeter
Cornell University
Lab. of Nuclear Studies
Ithaca, N.Y.

Summary

A preliminary condition for the control of gravity seems to be that at least some aspects of presently accepted theories of Gravitation and Inertia be proved wrong. In this essay it is suggested that, contrary to present theories, the presence of our galaxy might introduce anisotropies in the inertia of terrestrial bodies. An experimental test is suggested which utilizes microwave spectroscopy of anisotropic quantum states of atoms. This method promises to be sensitive enough to detect the expected anisotropies.

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One of the fundamental assumptions of present-day mechanics and gravitational theory is that the equality of the inertial and gravitational masses of a body is exact, and independent of the position and motion of the body. This assumption is an extrapolation to infinite accuracy of observations which obviously have a finite precision. It is also an observational fact (of finite precision) that the so-called "fixed stars" do in fact represent an inertial reference frame.

This last observation has led to the formulation of Mach's principle, which asserts that the inertia of a body at any point in the Universe is determined by "the fixed stars", which presumably must be understood as the total distribution of matter in the Universe.

This distribution, however, is uniform and isotropic only to a certain degree of approximation. If Mach's principle holds, we might then expect that the slight asymmetries in the distribution of matter at large would result in slight deviations from at least some of the laws of mechanics and gravitation which are commonly assumed to be exact. It is thus desirable to find tests that can demonstrate the validity of Mach's principle and of its consequences. In this essay we shall concentrate on the

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question of the isotropy of inertia and describe a relevant experiment more sensitive than those performed thus far.

To good accuracy, the acceleration produced on any body by a force points in the same direction as the force, and the ratio of force to acceleration, the inertial mass, is independent of the direction of the force; i.e. inertia is isotropic, and inertial mass is a scalar quantity. However, if Mach's principle holds, these statements should be true only in first approximation and asymmetries in the matter distribution at large, say concentration of matter near the center of our galaxy, should produce asymmetries in inertia. It is an anisotropy of this kind that one should try to detect.

Mach's principle alone does not specify the nature of the effect that matter at large has on the inertia of a body. Rather plausible possibilities are the following:

(a) that the contribution to the inertia of a body from the masses M at a distance r away from it is proportional to M and to a negative power, $-\nu$, of the distance r;

(b) that the contribution is larger for motions of the body in directions pointing toward or away from M than for motions perpendicular to these directions(1).

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(1) The inertia might also be larger when the body is, e.g., accelerated toward M and smaller when accelerated in the opposite direction. This possibility will be discussed further at the end of this essay.

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The order of magnitude of the expected anisotropy can be evaluated as follows: If ϱ is the average density of matter in the Universe, the isotropic part $\mathcal{M}$ of the inertia of a body on the Earth is proportional to

$$\text{Inertial Mass} = \mathcal{M} \propto \int_0^R \frac{4\pi r^2 \varrho}{r^\nu}\, dr = \frac{4\pi\varrho}{3-\nu}\, R^{(3-\nu)},$$

where R is the maximum distance from which matter still influences the inertia on the Earth. The anisotropic contribution $\Delta\mathcal{M}$ to the local inertia from a body of mass M at a distance r away is thus

$$\frac{\Delta\mathcal{M}}{\mathcal{M}} = \frac{M}{r^\nu} \; \frac{3-\nu}{4\pi\varrho\, R^{(3-\nu)}} \,. \qquad (1)$$

To give a numerical example, assume that $\varrho = 10^{-28}$ g cm^{-3} and that R = c T = 5 10^{27} cm, the so-called radius of the Universe as deduced from the Hubble constant. Then for $\nu = 1$(2) the ~~extent~~ $, \Delta\mathcal{M}/\mathcal{M},$ ~~of the~~ anisotropy due to the matter near the center of the Galaxy would be of the order of 1 part in 10^7 and for $\nu = 0.25$ 1 part in 10^{11}.

It must be emphasized here that these numbers depend strongly on the values assumed for the density and the radius of the Universe, quantities which are not yet known with any

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(2) Values of n larger than 1 can be excluded a priori since they would make the masses of the Sum end of the Earth prominent.

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accuracy. (The corresponding quantities for the Galaxy are instead rather sound.) The value of R is particularly uncertain since the value of the Hubble constant is derived only by extrapolation of the red shift from relatively small distances and the detailed relation between optical red shift and decrease of inertial influence of matter is not known. In fact, it is likely that our assumed value of R is an underestimate.

In principle, this anisotropy would be observable in classical macroscopic experiments; e.g. there should be a diurnal variation in the period of a quartz crystal (or a pendulum) clock if one assumes that the anisotropy of electric forces (or gravitational mass) is either absent or at least different from that of the inertial mass. However, present-day observations place an upper limit of about 1 part in 10^8 on a possible anisotropy observable with a quartz clock.

We want to propose a more sensitive test that utilizes microwave measurements of the splitting of atomic energy levels. In principle it consists in the comparison of the periodic motion of two atomic electrons, one moving in a line pointing towards our galactic center, the other moving in a plane perpendicular to this direction. Since electronic motion is governed by quantum mechanics, orbits of precisely this nature cannot be achieved; however, one can select quantum states of atomic electrons in

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which there is a preponderance of motion in a specific direction (or in a plane perpendicular to it). For an atom placed in a homogeneous magnetic field, this can be achieved by selecting atomic energy levels with specific values of the principal quantum number n, orbital quantum number ℓ, total angular momentum quantum number j, and of the component m of this angular momentum in the direction of the field. ~~Extremely~~ Very high precisions can be achieved by measuring not the absolute value of the energy in a single atomic level but the energy difference between two extremely close-lying levels with different spatial orientations of the electron motion as described above.

More specifically, what must be measured is either the fine structure splitting between two single-electron levels with the same value of m and differing values of j, or the Zeeman splitting between two levels with the same value of j and different values of m. What one needs is only the <u>variation</u> of such a fine structure or Zeeman splitting as the orientation of the magnetic field (which determines the m-axis) relative to the galactic center is changed. This could be observed either by rotating the experimental apparatus or merely by looking for sidereal variations (as the orientation of the laboratory coordinates relative to the galactic center is modified by the Earth's rotation).

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For a given value of $\Delta\mathcal{M}/\mathcal{M}$ (Eq. (1)), one can explicitly calculate ΔE, the maximum variation with orientation of the energy splitting discussed above:

$$\Delta E = K \frac{\Delta\mathcal{M}}{\mathcal{M}} E_o \qquad (2)$$

where E_o is the unperturbed total binding energy of either level, and K is a numerical factor of the order of unity (obtained in terms of Clebsch-Gordan coefficients), depending on the chosen values of ℓ, j, and m. The calculation is somewhat similar to that used for evaluating the effect on atomic energy levels of a weak quadrupole field. For example, in the case of fine structure splitting between two levels with $j = 1/2$ and $3/2$ respectively (both with $\ell = 1$ and $m = 1/2$), $K = 1/5$.

Precision experiments of the kind described above have in fact been performed, although they were not designed for investigating anisotropy of inertia. For example, one is the famous microwave experiments of Lamb and collaborators on fine structure splitting in hydrogen and deuterium, which measures the energy difference between a $2P_{1/2}$ and a $2P_{3/2}$ state, each with quantum number $m = 1/2$. This splitting is about 10,971 Mc/sec with an error for the mean of many individual measurements of about ± 0.2 Mc/sec. To within experimental errors no obvious variation of this splitting with sidereal time was observed(3).

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(3) Private communication from W. Lamb.

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However, since the series of measurements were not intended for observing sidereal variations, the various hours in the sidereal day were not covered in a systematic manner, and the orientation of the apparatus (relative to the polar axis) was not chosen to give the maximum sidereal variation(4).

Nevertheless, the negative result in Lamb's experiments places an upper limit of about 1 Mc/sec for the sidereal energy variation ΔE, defined in Eq. (2), which corresponds to an upper limit of $6\ 10^{-9}$ for $\Delta\mathcal{M}/\mathcal{M}$. It is likely that the same experiment repeated with the purpose of studying sidereal variation could detect values of $\Delta\mathcal{M}/\mathcal{M}$ about 10 times smaller than the present upper limit. Further improvement might be achieved by observing fine structure or Zeeman effect splittings in electronic states in heavier atoms with larger values of the principal quantum number n (the ratio of the energy variation ΔE to the natural line-width of an energy level is proportional to n). With these methods, one may hope to reach accuracies for $\Delta\mathcal{M}/\mathcal{M}$ of the order of one part in 10^{10}. With such an accuracy any anisotropy due to the presence of our galaxy would be detectable if the parameter ν in Eq. (1) is around unity even if the value of R is substantially larger

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(4) At latitudes near 45° N the most favorable direction is the NS direction in a horizontal plane. This line points towards the center of our galaxy (due S) at sidereal time about $5.^{30}$ h; twelve hours later the galactic center is at the nadir.

than that used in the example.

The possible effects discussed in this essay, even if found, would be extremely small and not directly related to the control of gravity. Nevertheless, they would have far-reaching consequences: They would show that both Newtonian Mechanics and General Relativity break down. According to present-day theories of mechanics and gravitation, the effect of gravitational forces cannot be modified. A demonstration of the breakdown of present-day mechanical laws would open up possibilities for such modifications.

As an example of the far-reaching consequences of even a minute alteration of present-day laws, consider the possibility mentioned in footnote (1): In such a situation all forces, including gravitation, would no longer be conservative. As a net result, with the signs as in footnote (1), all compound systems with internal forces would be repelled by matter proportionally to the strength of the internal forces. An effect of this kind might be related to the observed expansion of the Universe. One way to look for this kind of asymmetry of inertia would be to perform atomic microwave experiments similar to those described above but with Stark effect splitting substituted for Zeeman or fine structure splitting.

Biographical Sketches

Giuseppe Cocconi

Born in Como, Italy, in 1914.

High school in Como, B.S. and "Libera Docenza" in Milano, Italy. Professor of Physics in Milano (1940), Rome (1941-1944), and Catania (1945-1947). Now Professor of Physics and Nuclear Studies at Cornell University, Ithaca, New York.

Main fields of interest: Cosmic Rays and Physics and High Energy Particles.

Edwin E. Salpeter

Born in Vienna, Austria, in 1924.

Finished high school and B.S. in Sydney, Australia; obtained Ph.D. degree in Theoretical Physics in Birmingham, England, in 1948. Now Professor of Physics and Nuclear Studies at Cornell University, Ithaca, New York.

Main fields of interest: Theoretical nuclear physics and theoretical astrophysics.