AN ESSAY ON

THE APPLICATION AND PRINCIPLE OF GRAVITY-ASSIST TRAJECTORIES FOR SPACE FLIGHT

By

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SUMMARY

In January, 1970 NASA authorized the Mariner Venus/Mercury 1973 Project. This action was historic because, for the first time, a highly beneficial use of a planet's (Venus) gravity field will be made in spaceflight. This will be accomplished by employing a "gravity assist" trajectory to fly from Earth to Mercury. This essay describes the application and principle of gravity assist trajectories, i.e., the use of a planet's gravity field to sharply alter a spacecraft's course. Discovery of this principle has opened a new frontier of solar system exploration at considerable savings in launch vehicle size and cost.

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

I. APPLICATIONS OF GRAVITY ASSIST TRAJECTORIES

In January, 1970 the National Aeronautics and Space Administration officially authorized the Mariner Venus/Mercury 1973 Project. This action was historic in that, for the first time, a highly beneficial use of the gravitational field of a planet (Venus) will be made in spaceflight. This will be accomplished by employing a "gravity-assist" trajectory to fly from Earth to Mercury. A Mariner spacecraft launched from Earth in October 1973 will be guided to fly within 5500 kilometers of Venus in February, 1974. The closest approach point to Venus will be carefully chosen so that Venus' gravity field will sharply bend the path of Mariner such that it will be diverted toward the planet Mercury. It will arrive there in late March, 1974.

The effect of Venus' gravity in diverting the spacecraft's course is very large. It causes a velocity change of 4.5 kilometers/second. Such a change is equivalent to that produced by a rocket stage with a specific impulse of 300 seconds and expending a fuel load equal to 78% of the combined stage and Mariner weights. However, use of Venus' gravity renders such a rocket unnecessary. Alternately, the spacecraft could be launched directly from Earth to Mercury. To do so, however, would require a launch vehicle which would have to provide more than twice as much injection energy[1] (> 45 km^2/sec^2)

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[1] The injection energy is twice the total energy per unit mass required to place a spacecraft in a prespecified orbit.

to Mariner than the energy required ($\sim 21\ \text{km}^2/\text{sec}^2$) to fly to Mercury via Venus.

The lower energy of 21 km^2/sec^2 is within the capability of the Atlas/Centaur launch vehicle for the 900-lb. Mariner. But a much larger, and far more costly, Titan/Centaur would have to be used for the higher-energy direct flight to Mercury. Alternately, a third stage, again far more costly, would have to be added to the Atlas/Centaur for the direct flight.

In addition to the Mariner Venus/Mercury 1973 Mission, President Nixon has included the Grand Tour missions, e.g., Earth-Jupiter-Saturn-Uranus-Neptune, Earth-Jupiter-Saturn-Pluto, in the United States space program plan for the 1970 decade. These missions also will use "gravity-assist" trajectories and make possible a unique opportunity to explore the outer planets by employing a launch vehicle of the Titan III C class, which is currently being used in Air Force programs.

In essence, then, very significant cost savings and unusually good opportunities for exploration of the solar system are made available by use of the "gravity-assist" trajectory. In particular, two or more planets can be explored on one flight using smaller, less costly, launch vehicles.

The "gravity-assist" trajectory was originally conceived and demonstrated through analysis and calculation by the author of this essay in early 1961. Extensive development and calculation of a variety of gravity-assist trajectories, e.g., Earth-Venus-Mercury, Earth-Venus-Mars, Earth-Venus-Mars-Earth return, was conducted by M. Minovitch (reference 1) under the supervision of the author at the Jet Propulsion Laboratory in the 1961-63 time period.

It is thus for the original concept, demonstration, and development of this highly practical and beneficial application of the phenomenon of gravitation that an award is sought. The principle of the gravity-assist trajectory is explained in the following section.

II. PRINCIPLE OF GRAVITY ASSIST TRAJECTORIES

A. Simple Planet-to-Planet Transfer

In 1960-61 the author was engaged in extensive computations of ballistic interplanetary trajectories from Earth to Venus, Mars, Mercury, and Jupiter. The results of these computations are given in references 2-4. The simple, but accurate, analytical model used in the calculation of planet-to-planet trajectory parameters consisted of three distinct phases of two-body motion: (1) an exit hyperbola near the launch planet, (2) elliptical motion in the gravity field of the Sun only, and (3) entry hyperbolic motion near the target planet. Solution of the heliocentric elliptic motion is obtained under the following assumptions:

(1) The launch and target planets move in orbits about the Sun as given in the national ephemerides.

(2) The launch and target planets are massless. Only the Sun's gravitational force acts on the spacecraft.

(3) The position of the spacecraft at launch into the heliocentric orbit is the center of the massless launch planet. Its position at arrival is the center of the massless target planet.

Thus, for solution of the heliocentric phase of motion, the attractions of the launch and target planets are <u>temporarily</u> disregarded. Key parameters obtained from the solution of this problem is determination of the hyperbolic-excess velocity vectors relative to the launch and target planets. Typical results (for Earth to Venus flight in 1962) are illustrated in figures 1-8. In figure 1, the time of flight from Earth to Venus is plotted as a function of launch date. A set of characteristic closed-contour curves evolve, parametric with launch energy C_3, which is the square of the hyperbolic-excess speed, V_{hL}, at launch. Note that for a launch date of August 24, 1962, four distinct flight times (97, 126, 168, 184 days) from Earth to Venus are possible for an energy of $.13 \times 10^8$ m^2/sec^2. Similarly, the other five key parameters (declinations[2] and right ascensions[2] of the launch and target planet asymptotes and hyerbolic speed relative to Venus) also are typified by closed contours, parametric with launch energy and each having four values for each launch date. The key point to be made here is that characteristic curves of this type are calculable for transfer between any two planets.

B. Principle of Gravity Assist

Suppose that now we have calculated planet-to-planet trajectory characteristics from planet P_1 to planet P_2 for a launch date interval, T_{L1} to T'_{L1}. These trajectories arrive at P_2 in the period T_{A2} to T'_{A2}. Next we calculate characteristics for trajectories from planet P_2 to

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[2] Declination and right ascension of the launch and target planet asymptotes define the direction angles of their respective hyperbolic excess velocity vectors.

planet P_3 for a launch date interval at P_2 which is identical to the previous arrival dates at P_2 of T_{A2} to T'_{A2}. These trajectories arrive at planet P_3 at a still later arrival date interval, T_{A3} to T'_{A3}. The geometry is illustrated in figure 9. In early 1961, the author conceived that a value of arrival energy[3] at planet P_2 could possibly be found which exactly equaled a launch (or departure) energy from planet P_2 to P_3 for a particular date at P_2. This possibility occurred especially since there are four values of arrival energy available on any given date at P_2. If such an energy match could be found, it would then only be necessary to match the directional parameters of the arrival and departure hyperbolic-excess velocity vectors at planet P_2. This match is made possible by using the gravity field[4] of P_2 to cause a velocity direction change, as illustrated in figure 10. In this figure, the spacecraft arrives at planet P_2 along an asymptote with asymptotic velocity $\overline{V}_{ha}$. The heliocentric arrival velocity, $\overline{V}_{a2}$ is computed from

$$(1) \qquad \overline{V}_{a2} = \overline{V}_{p2} + \overline{V}_{ha}$$

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[3] The arrival energy is the square of the asymptotic speed relative to P_2.

[4] Here now, the gravity field of P_2 is re-introduced and has a sphere of influence whose radius is found by multiplying the planet-Sun distance by the planet/Sun mass ratio raised to the 2/5 power.

The spacecraft departs from P_2 in a direction determined by the bending due to the gravitational field of P_2. The asymptotic departure speed, V_{hd}, is equal to the arrival speed, i.e.,

$$(2) \qquad \left|\bar{V}_{hd}\right| = \left|\bar{V}_{ha}\right|$$

The heliocentric departure velocity is then

$$(3) \qquad \bar{V}_{d2} = \bar{V}_{p2} + \bar{V}_{hd}$$

Subtracting equation (1) from equation (3)

$$(4) \qquad \bar{V}_{d2} - \bar{V}_{a2} = \bar{V}_{hd} - \bar{V}_{ha}$$

<u>or the change in heliocentric velocity is exactly equal to the change between the arrival and departure hyperbolic-excess velocity vectors</u>. Thus, a heliocentric trajectory can be drastically altered by passing through the gravitational field of a planet. <u>In particular, if the point of closest approach is properly selected, the velocity change</u> $\bar{V}_{d2} - \bar{V}_{a2}$ <u>can cause the spacecraft to be diverted onto a trajectory to a third planet</u> P_3. By induction then, it is obvious that the process of diverting a spacecraft from one planet to another might be continued indefinitely, if the planets were in favorable positions. In fact, Minovitch (reference 1) has calculated an eight-planet gravity assist trajectory, Earth-Venus-Mars-Earth-Mars-Earth-Venus-Earth.

The case used by the author in early 1961 to demonstrate this principle was a simple Earth-Venus-Earth return. A set of Earth-Venus trajectories was calculated in the launch interval July to November, 1962. These trajectories arrived at Venus in the interval November 24, 1962 to April 4, 1963. Next, a second set of trajectories were calculated from Venus to Earth for a November 24, 1962 to April 11, 1963 Venus launch interval. By comparison of arrival energies of the first set with launch energies from the second set, an energy match was found. The distance of closest approach, aim point, and amount of bending at Venus was next calculated. The resulting values were found to be entirely within reason, i.e., the closest approach distance was greater than Venus' radius plus atmospheric height.

III. CONCLUSION

In summary, the discovery of the gravity assist trajectory has opened a new frontier for exploration of the solar system at considerable savings in launch vehicle size and cost. This is perhaps the most practical application of gravity since hydroelectric power was first developed. Much credit is due to M. Minovitch for his subsequent rigorous analysis and extensive calculations of this concept.

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2. Clarke, Jr., V. C., A Summary of the Characteristics of Ballistic Interplanetary Trajectories, 1962-77, Technical Report 32-209, Jet Propulsion Laboratory, January 15, 1962.
3. Clarke, Jr., V. C., Bollman, W. E., Roth, R. Y., and Scholey, W. J., Design Parameters for Ballistic Interplanetary Trajectories, Part I. One-Way Transfers to Mars and Venus, Technical Report 32-77, Jet Propulsion Laboratory, January 16, 1963.
4. Clarke, Jr., V. C., Bollman, W. E., Feitis, P. H., and Roth, R. Y., Design Parameters for Ballistic Interplanetary Trajectories, Part II. One-Way Transfers to Mercury and Jupiter, Technical Report 32-77, Jet Propulsion Laboratory, January 15, 1966.

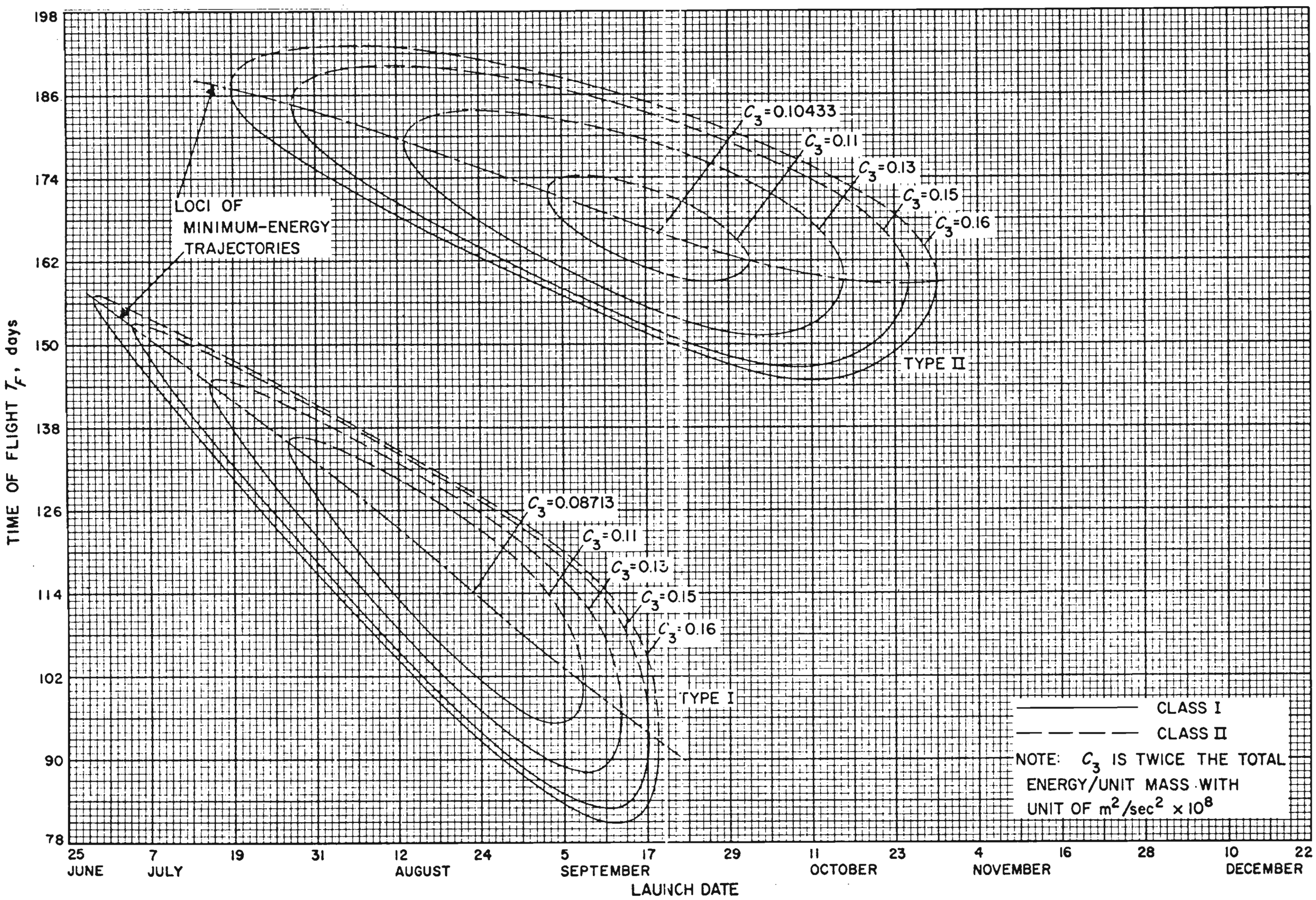

Figure 1. Venus 1962: Time of flight vs launch date

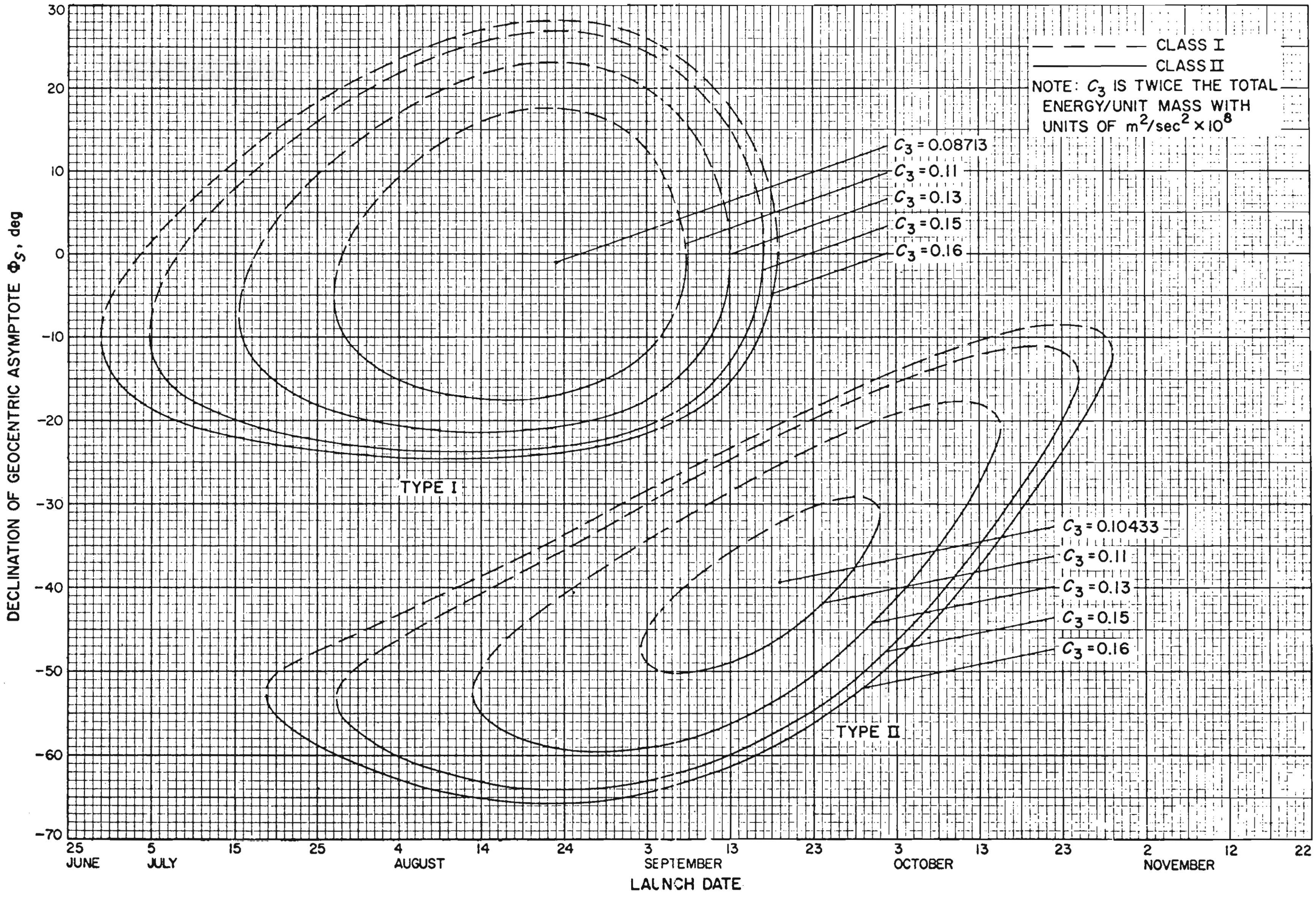

Figure 2. Venus 1962: Declination of geocentric asymptote vs launch date

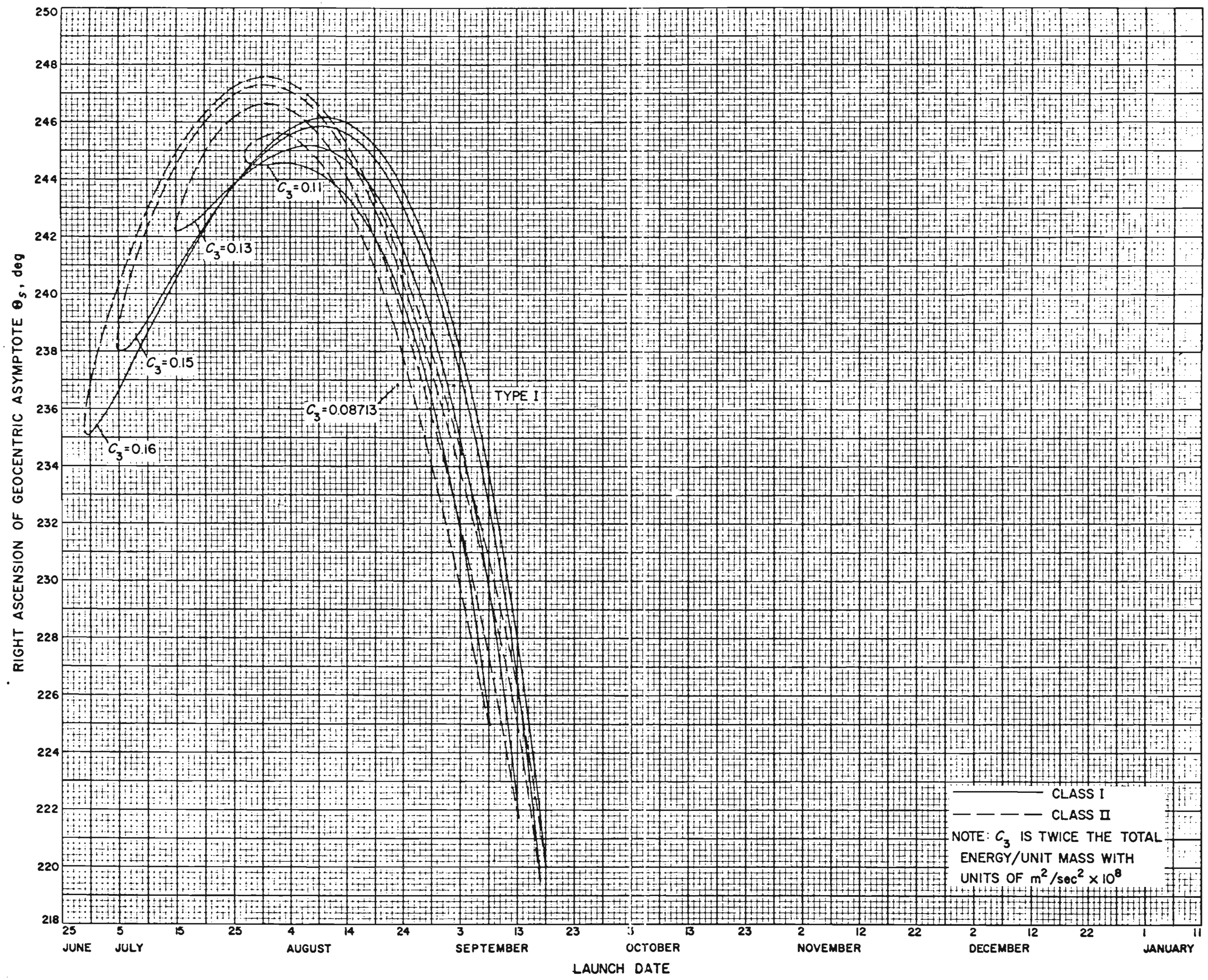

Figure 3. Venus 1962: Right ascension of geocentric asymptote vs launch date, Type I

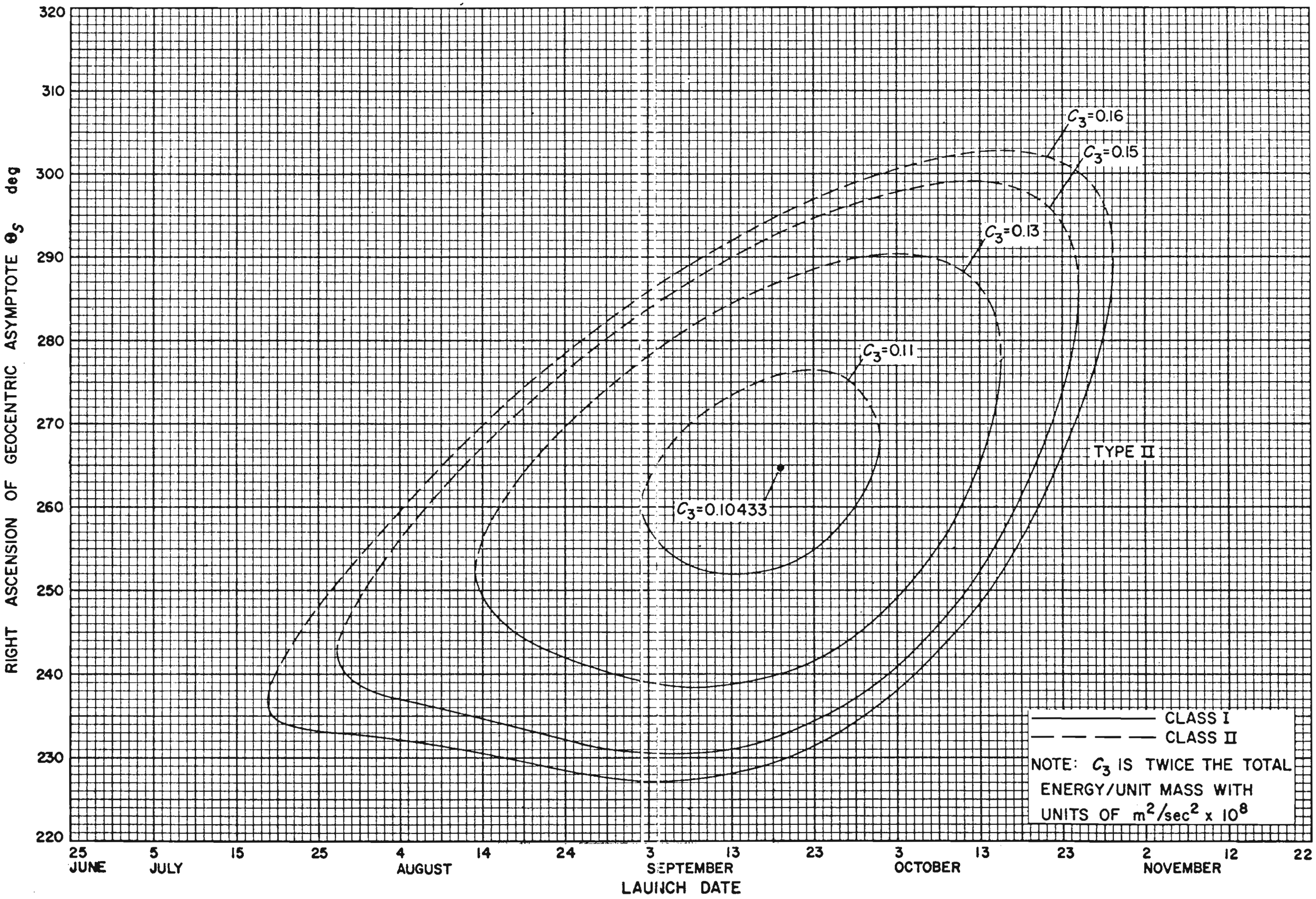

Figure 4. Venus 1962: Right ascension of geocentric asymptote vs launch date, Type II

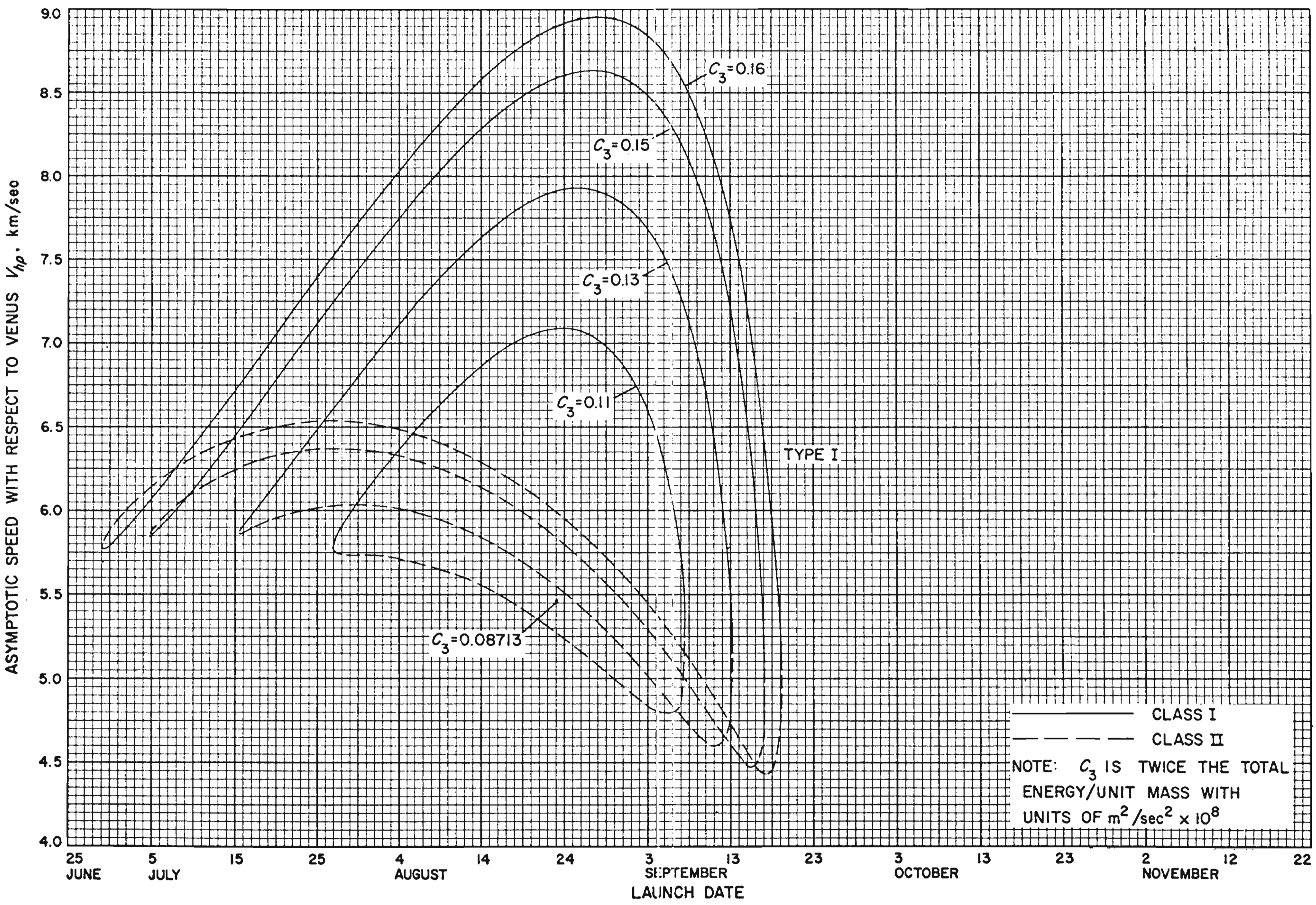

Figure 5. Venus 1962: Asymptotic speed with respect to Venus vs launch date, Type I
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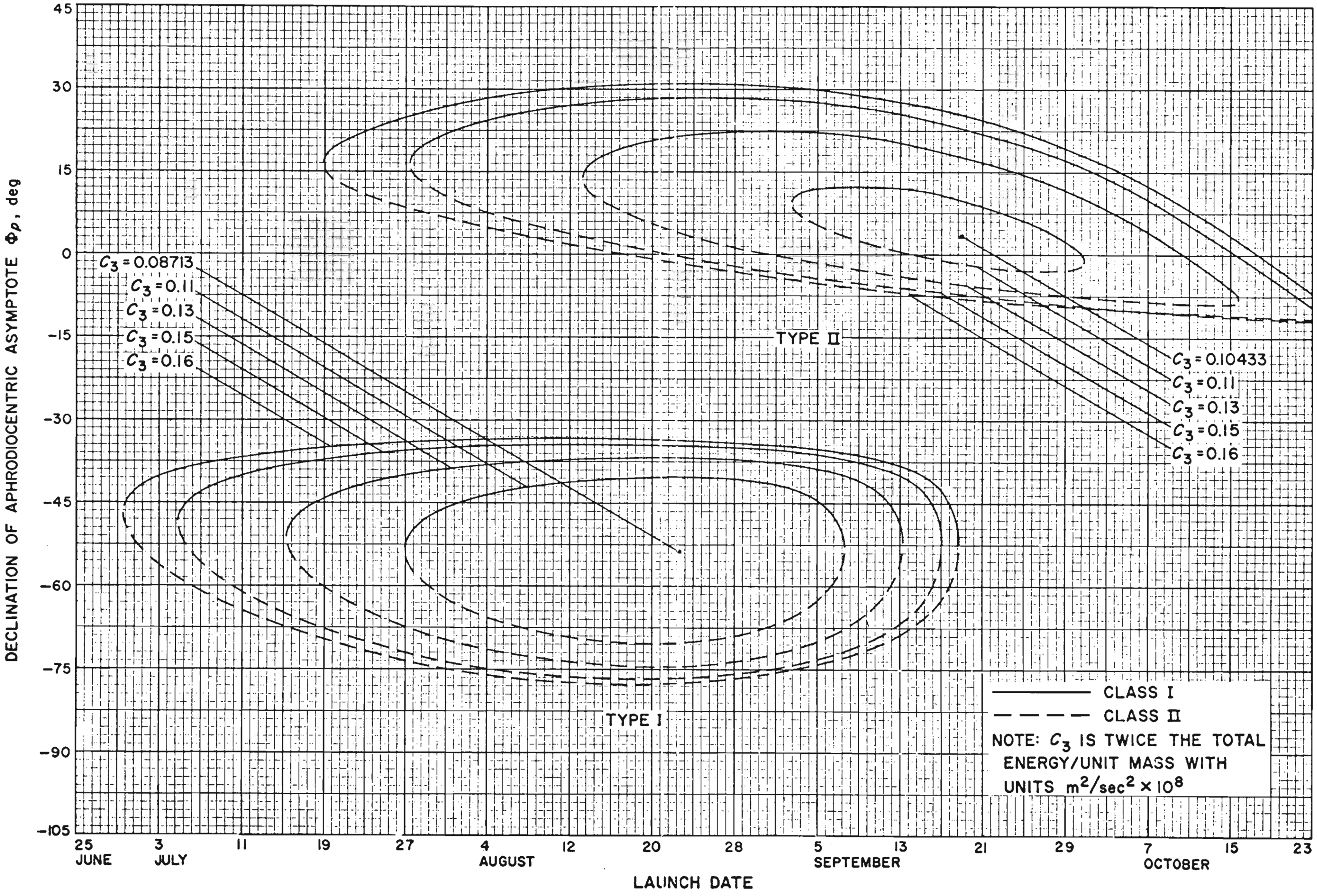


Figure 6. Venus 1962: Declination of aphrodiocentric asymptote vs launch date

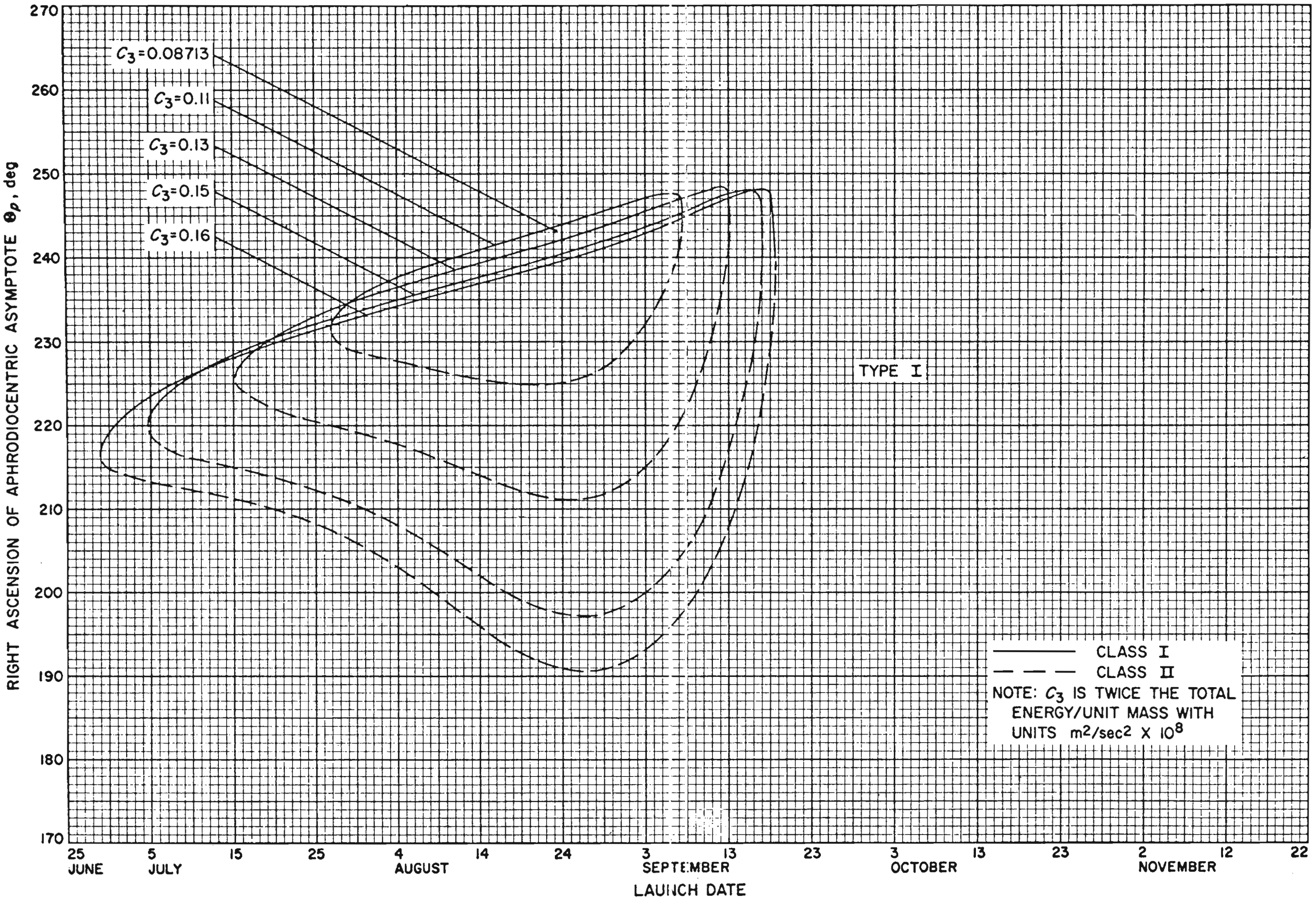


Figure 7. Venus 1962: Right ascension of aphrodiocentric asymptote vs launch date, Type I

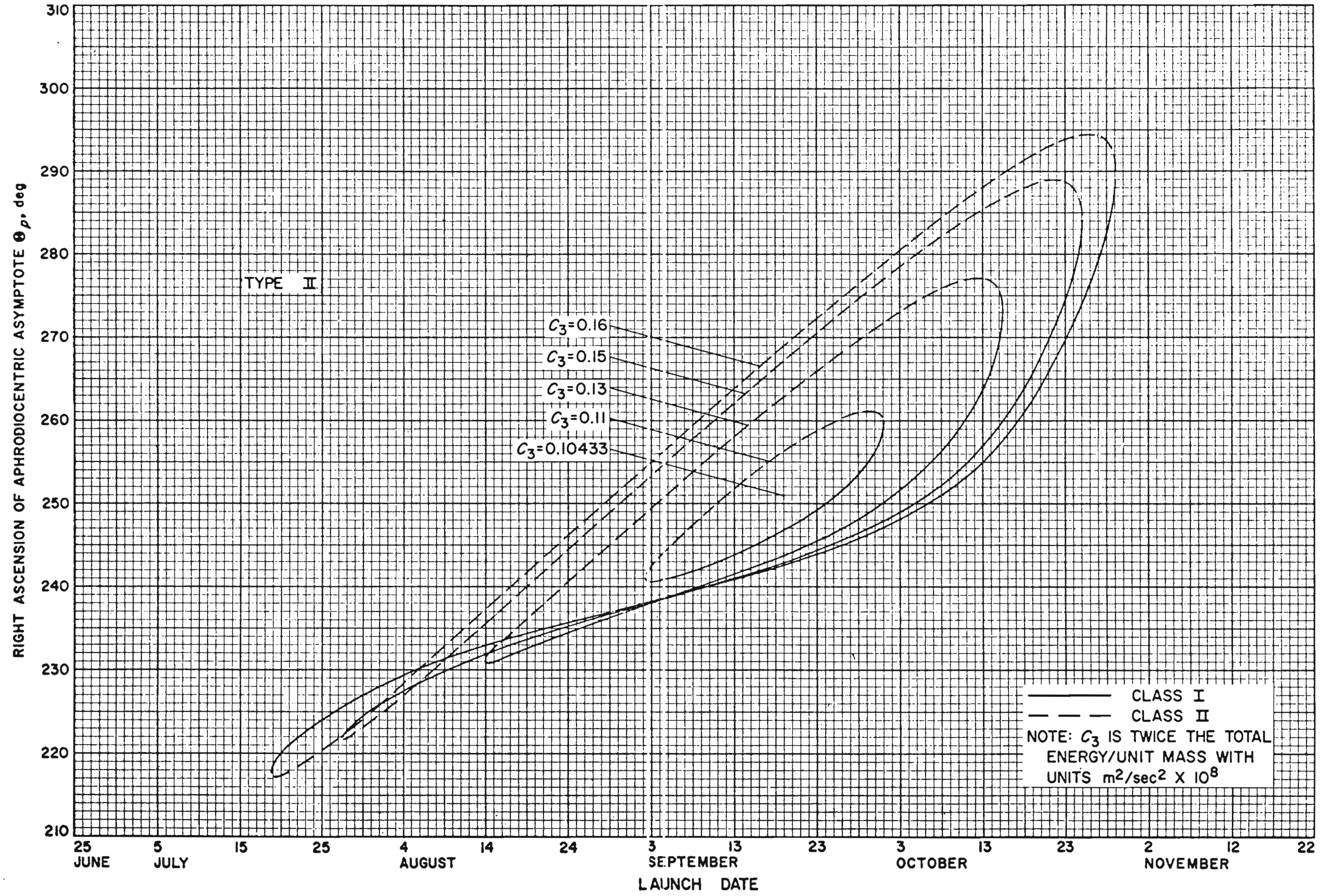


Figure 8. Venus 1962: Right ascension of aphrodiocentric asymptote vs launch date, Type II

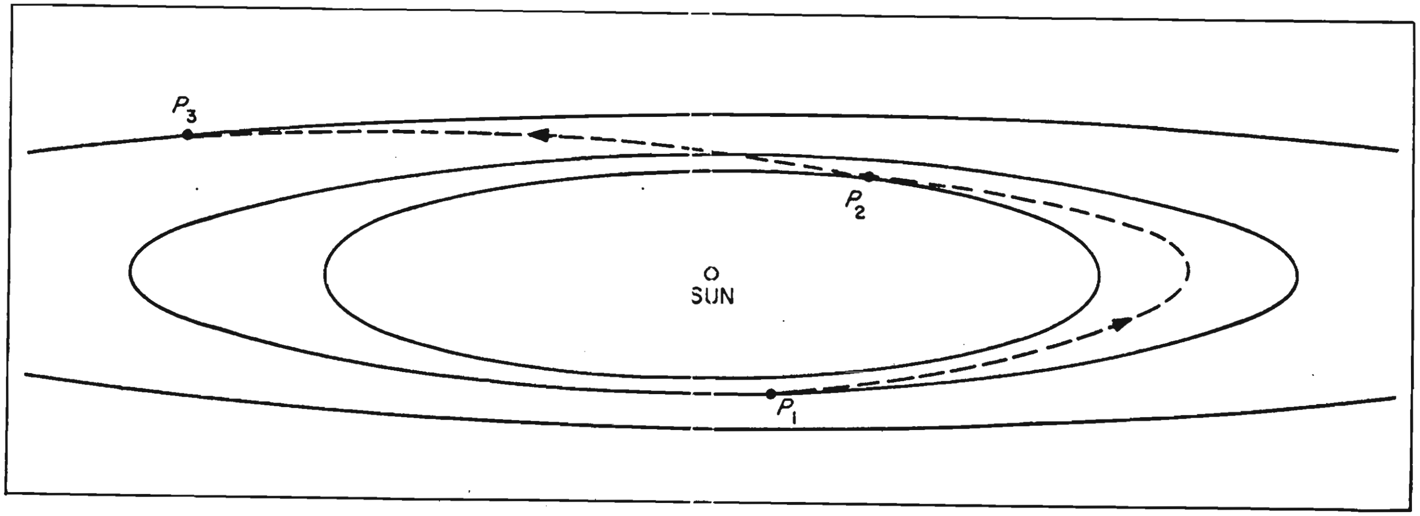
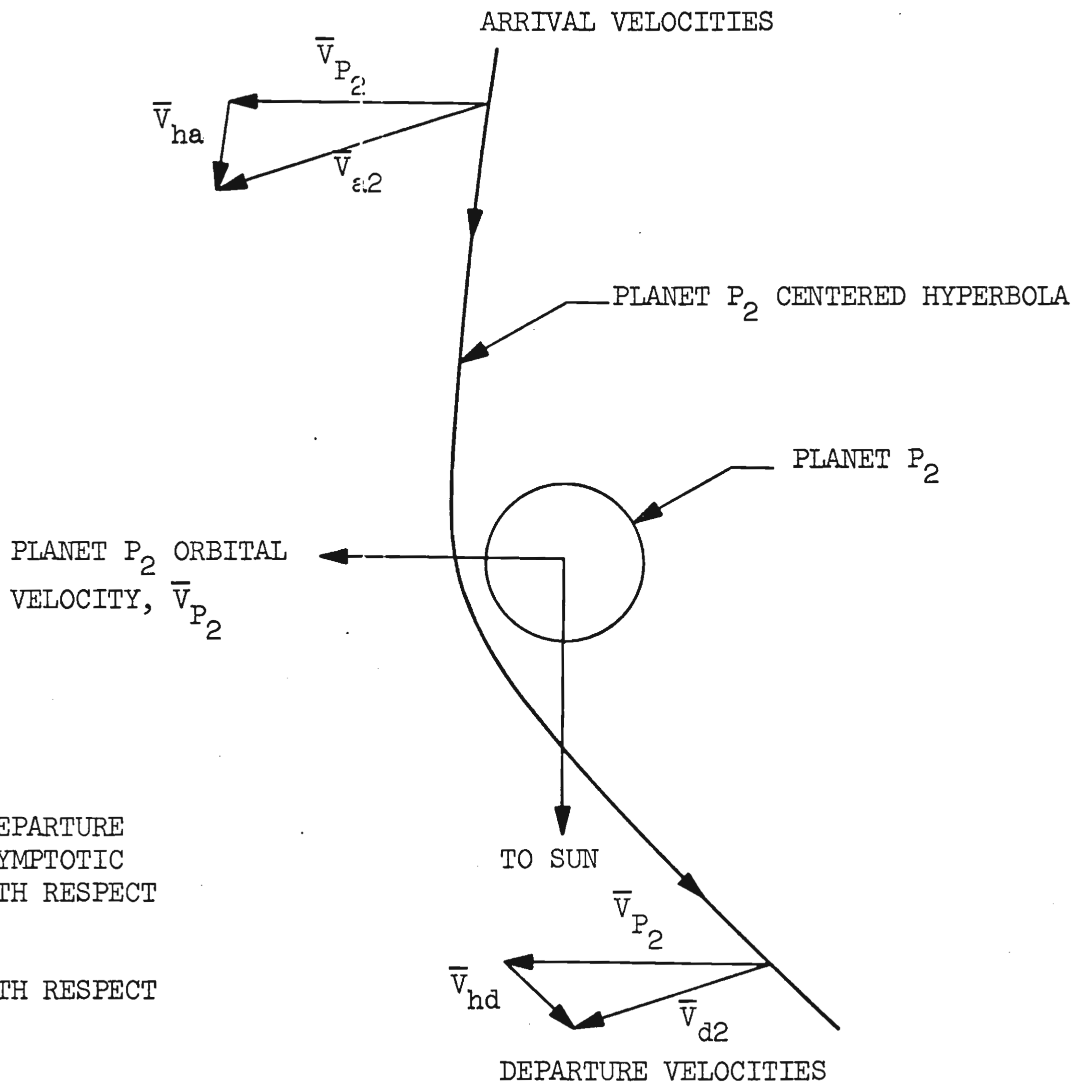


Figure 9. Typical gravity assist trajectory



$\overline{V}_{ha}$, $\overline{V}_{da}$ = ARRIVAL AND DEPARTURE HYPERBOLIC ASYMPTOTIC VELOCITIES WITH RESPECT TO PLANET P_2

$\overline{V}_{a2}$, $\overline{V}_{d2}$ = VELOCITIES WITH RESPECT TO SUN

Figure 10. Velocity charges during planet P_2 flyby

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Biography of Victor C. Clarke, Jr.

Victor C. Clarke, Jr., Mission Analysis and Engineering Manager for the Mariner Venus-Mercury 1973 Project and the extended mission of Mariner Mars 1969 at the Jet Propulsion Laboratory, has been associated with lunar and interplanetary trajectory design and mission analysis for more than ten years.

He was born in Cleveland, Ohio, on October 29, 1927, and received a Bachelor's Degree in Physics from Miami University, Oxford, Ohio, in 1949, and a Master's Degree in Electrical Engineering from the University of Arizona in 1958.

Mr. Clarke joined JPL in 1958 and became Supervisor of the Trajectories and Performance Group of the Systems Analysis Section. There he was responsible for the design of lunar and interplanetary trajectories, and did pioneering work in the study of gravity-assisted trajectories like that planned for the 1973 Mariner mission. He also headed the Trajectory and Performance Working Groups formed to coordinate information and requirements for the Centaur and Agena space launch vehicles.

In 1964 he became Mission Analysis and Engineering Manager for the Surveyor Project, where he was responsible for mission design and planning and flight path design and analysis.

In 1967 Mr. Clarke joined the Mariner Mars 1969 Project as Mission Analysis and Engineering Manager. He was responsible for the analysis of launch, cruise, and encounter analysis, trajectory design, and mission planning to maximize the scientific return. In October 1969 he was awarded the NASA Exceptional Service Medal for his contributions to the successful Mars mission of 1969. He lives in Flintridge with his wife and six children.