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BLACK HOLES:

The Strongest Energy Storehouse in the Universe*

Demetrios Christodoulou

Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08540

and

Remo Ruffini

School of Natural Sciences
Institute for Advanced Study
Princeton, New Jersey 08540
and
Joseph Henry Laboratories
Princeton University
Princeton, New Jersey 08540

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SUMMARY

Recent theoretical advances have clearly put in evidence that black holes are not only the only astrophysical objects that are fully describable by general relativity alone, but also to be the strongest storage of energy in the Universe. In this paper we give a new formula establishing for the first time the energetics limit of a black hole (50% of the total mass energy can be extracted from a black hole!); we also give two new mechanisms of extraction of energy of unprecedented efficiency. We finally shortly discuss the singularity problem in a realistic Black Hole.

Nothing has given more impetus to the research on gravitation in recent years than the clear experimental evidence for the existence of collapsed objects. The evidence still hypothetical from the case of the Quasars and of the powerful Jets in radio sources has become imperative and irrefutable after the discovery of pulsars and of pulsating radio sources. The experimental evidence cannot yet point out in favor of either one of the two more typical collapsed objects: neutron star or black holes.

Neutron stars, mainly characterized by matter at nuclear and supranuclear density, have radii varying from 10-100 km and mass in the range $0.1 \lesssim M/M_{\odot} \lesssim 1$.[1] In their description different fields of physics have to come in, equally powerful: (1) elementary particle physics in the innermost core, (2) nuclear physics in the main body of the star, (3) solid state physics in the crust and (4) general relativity.

Different in many respects and extremely similar at the same time Black Holes! Their mass starts near the limiting mass of the neutron star equilibrium configurations. The radius for a black hole of 2 $M_{\odot}$ is ~ 6 km. But the theoretical framework to describe these objects is completely different: no equation of state here, no nuclear physics, no solid state physics - only generaly relativity with its full power comes in: Black Holes are tailored in the vacuum pure geometry of space and time!

The name Black Hole [2] was introduced only a few years ago in connection with the Schwarzschild solution:

a) Black: light can be emitted only radially out from such an object and moreover this extremely narrow beam of light will reach a faraway observer with zero frequency (infinite redshift!).

b) Hole: once that the border of these objects has been crossed nothing can come out, the light cone is tilted inward and everything is forced to collapse toward a central singularity.

In the meantime very much more has been known: we have an entire new family of black holes (see Fig. 1). More important, all the newly theoretically discovered black holes differ drastically from the Schwarzschild one: They are surrounded by an "ergosphere" (3) in which energy can be extracted from the black hole.

Let us now give an evaluation of the total energy which can be extracted from a black hole.

The most general black hole with a stationary space time at infinity is believed to be characterized only by the mass m, charge e, angular momentum L and linear momentum p. As a consequence of these parameters the black hole will also have a magnetic moment equal to Le/m (geometrical units; $L = G\,L_{conv.}/c^3$, $e = G\,e_{conv.}/c^{1/2}$, $m = G\,m_{conv.}/c^2$). The range of the parameters m, L and e are such that the following relation has always to be fulfilled

$$m^2 \geq a^2 + e^2 \qquad (a \equiv L/m)\,. \tag{1}$$

The equal sign denoting an "extreme" black hole configuration. We know, of course, that there exist in nature astrophysical objects with angular momentum which violates this condition. We believe that during the gravitational collapse of such objects the extra angular momentum will be radiated away. We have been able to find a closed form formula for the total energy of a black hole,(4)

$$E^2 = \left(m_{ir} + \frac{e^2}{4m_{ir}}\right)^2 + \frac{L^2}{4m_{ir}^{\;2}} + p^2\,, \tag{2}$$

where m_{ir} is the "irreducible" mass of the black hole.(5) The rotational and electromagnetic (Coulomb) contributions to the squared mass can be augmented or depleted by (rest particle) reversible transformations.

Reversible transformations keep constant the "irreducible mass" which is thus defined, for a given black hole, to be the mass of the Schwarzschild black hole that results when all the rotational and electromagnetic energy has been taken out by reversible transformations. Thus it is possible to extract 29% of the mass of an extreme Kerr black hole ($L = m^2$, $e = 0$) and 50% of the mass of an extreme Reissner-Nordström black hole ($e = m$, $L = 0$). This is the maximum energy conversion efficiency that has hitherto been found except that in the case of matter-antimatter annihilation!

All other test particle transformations are irreversible and always increase the "irreducible" mass, no process existing capable of decreasing it. The area of the Killing horizon is given by

$$S = 4\pi\,(r_{horizon}^2 + a^2) = 16\pi\, m_{ir}^2 \,.$$

Thus one area of the horizon can only increase or stay constant its reduction being impossible.

We examine now two processes of emission from a black hole:

1) A particle of mass μ and charge ε orbiting a magnetic black hole with charge e has the most bound orbit lying on the innermost surface of the ergosphere $r = m$. For an extreme black hole the particle will have energy

$$\frac{E}{\mu} = [(2a^2-m^2)\lambda + a\sqrt{\lambda^2 m^2 + 4a^2 - m^2}\,]/(4a^2-m^2) \text{ with } \lambda = \varepsilon e/\mu m$$

(such orbits will be stable and will be the orbit of lowest energy if and only if $E \le \varepsilon\mu/m$ and $\varepsilon e < 0$).

The orbit with absolute largest binding will have $\varepsilon/\mu = -(2a^2-m^2)/e^2$ and $E = e\mu/m$. From Eq. (3) it clearly follows that in the limit $\varepsilon/\mu \to -\infty$ $e \to 0$ $(a \to m)$, but with $\lambda = \varepsilon e/m\mu \to -\infty$, the binding energy of the orbit tends to 100% of the rest mass of the particle! If, therefore, a particle of large charge to mass ratio is initially orbiting a black hole of small charge to mass ratio and maximal angular momentum, the particle will loose all of its mass, mainly the electromagnetic radiation (classical analog of particle-antiparticle annihilation!). One hundred percent efficiency!

b) Here we exemplify a method of extracting energy from a black hole which could be of interest in astrophysical applications.

Let us have a magnetic black hole of solar mass. Send to it a blob of neutral matter of dimension smaller, bur comparable to that of the black hole. The gravitational tidal force will squeeze the star, while the electric field of the black hole will polarize the blob of matter. The two forces are in the same direction and reinforce each other. Choose suitable dimension for the object such that the sum of the tidal and polarization forces overcomes the binding forces of the blob of matter in the ergosphere of the black hole. There the blob will be broken apart, the negative charges (black hole assumed positively charged) falling toward the black hole reducing not only its charge and angu'ar momentum but also its mass, the whole positive charges escape to inifnity with rest plus kinetic energy larger than the total energy of the initial blob. The energy gained in the process could be up to 10^{20} times the initial energy of the blob if total separation of the charges is achieved in the neighborhood and if only the negative charges fall into the black hole.

Finally, one of the most important features of black holes is that it takes an infinite time for a test particle to fall inside and an infinite time is also required for the formation of a black hole itself! In the Schwarzschild case as the collapsing star approaches the Schwarzschild radius, the time of a far away observer, in approximately flat space increases exponentially with respect to the time of an observer comoving with the surface of the star. Therefore since the lifetime of the Universe is infinite, an observer on the surface of the collapsing star will be able to calculate, tracing back the light signal received, that the Universe is rapidly passing the moment of maximum expansion and recontracting. However, when the radius of the Universe becomes of dimensions comparable to the radius of the collapsing star, no far away observer exists and the collapsing star as well as the rest of the Universe will form a three sphere and the system as a whole will collapse to one and only one singularity! The same reasoning applies, of course, if we have a system of collapsing stars.

See Fig 2

It is indeed a pleasure to thank Prof. John A. Wheeler for exciting discussions.

Bibliography

(1) C. E. Rhoades, Jr. and R. Ruffini, Ap. J. Letts. 163, L83 (1971) and references quoted there.

(2) See, e.g. R. Ruffini and J. A. Wheeler, Physics Today, Jan. 1971. For recent interesting experimental reports, see Giacconi et al., M. I. T. preprint ASE-2643 to appear in Ap. J. Letters; also, W. Sullivan, New York Times, April 4 and April 1, 1971.

(3) For the definition of ergosphere, see R. Ruffini and J. A. Wheeler, The Significance of Space Research for Fundamental Physics, E.S.R.O. - Paris.

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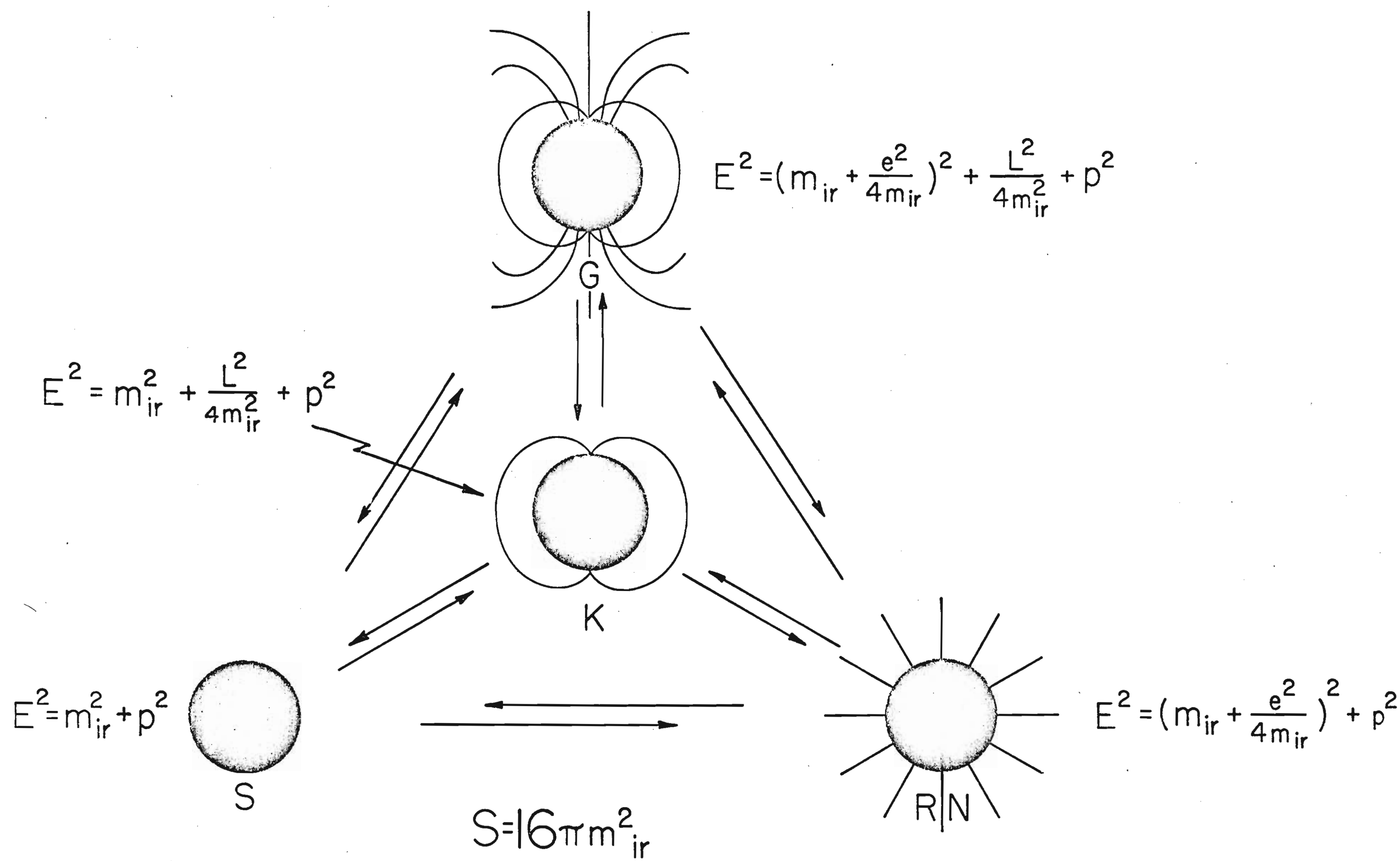
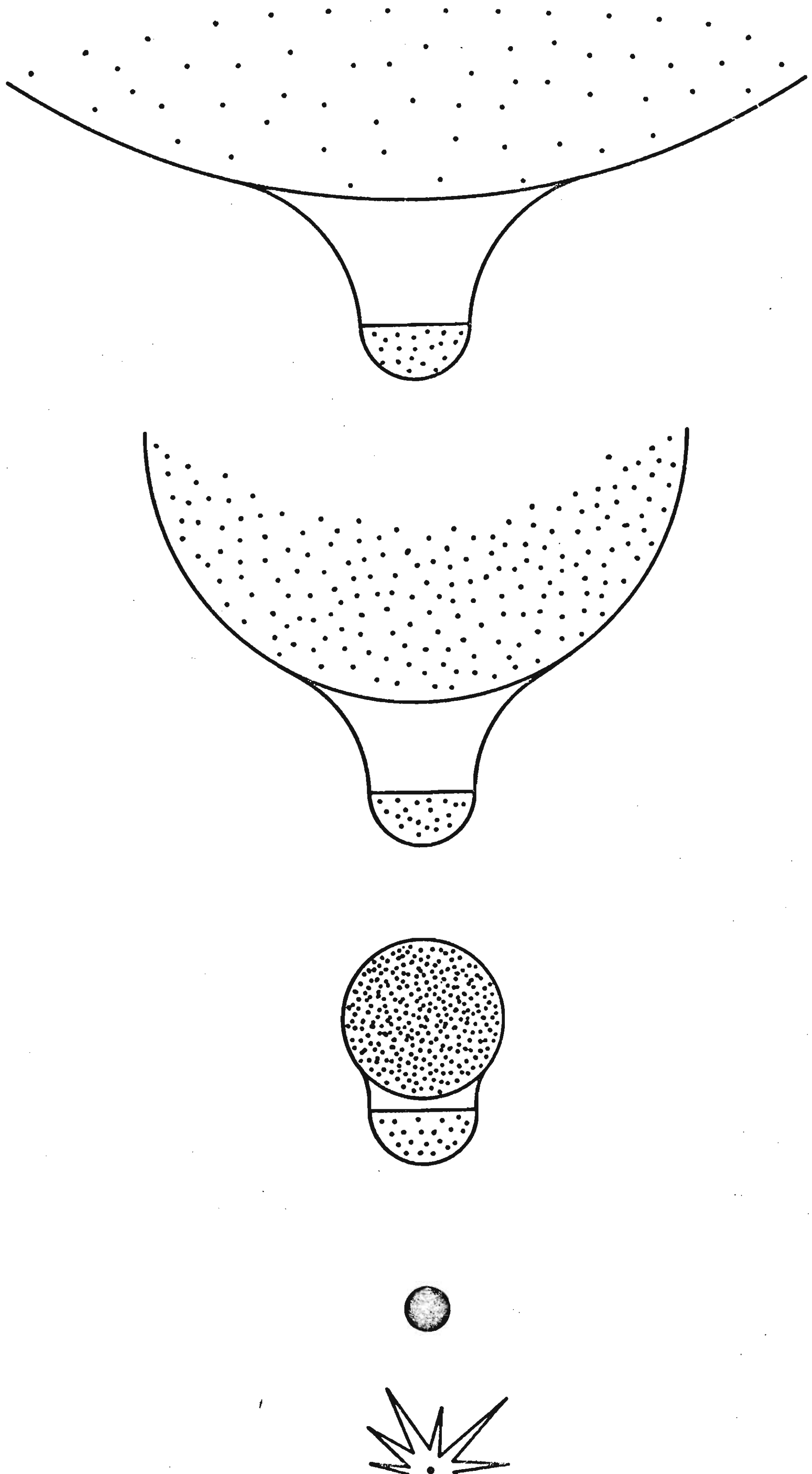
(5) D. Christodoulou, Phys. Rev. Letts. 25, 1596 (1970).

Figure 1. The entire family of black holes is here presented with the formula determining their energy as a function of these characterizing parameters.

Figure 2. Final state of collapse of a black hole matched to a closed Universe.



$E^2 = (m_{ir} + \frac{e^2}{4m_{ir}})^2 + \frac{L^2}{4m_{ir}^2} + p^2$
G
$E^2 = m_{ir}^2 + \frac{L^2}{4m_{ir}^2} + p^2$
K
$E^2 = m_{ir}^2 + p^2$
S
$S = 16\pi m_{ir}^2$
RN
$E^2 = (m_{ir} + \frac{e^2}{4m_{ir}})^2 + p^2$



Demetrios CHRISTODOULOU

Born 19 October 1951 in Athens, Greece. Came to Princeton at the age of 15, February 1968 and directly admitted to the graduate school as qualifying student. Regular graduate student from September '68. Expected to receive Ph.D. degree 5 May 1971 at the age of 19! Major contributions in general relativity: some fundamental works in gravitational collapse and black hole physics. Mr. Christodoulou has introduced for the first time the concept of irreducible mass.

Remo RUFFINI

Born 17 May 1942 in La Brigue, A.M., France. Obtained the doctorate degree from the University of Rome in 1967. Assistant professor at Rome University in the academic year 1967-68. Visiting fellow- Princeton University 1968-69. Since 1969 Member of the Institute for Advanced Study, Princeton, N. J. Major works in general relativity, gravitation physics and astrophysics. A selected list of publications is attached. (sent only to Dr. Witten – only 1 copy)

Honors:

"Adriano Olivetti" Mathematical Physics Award for graduate student (First Prize)

"Confindustria Prize for Theses in Sciences" (First Prize)

"Award for Essay on Gravitation" (Third Prize), 1970 - with R. B. Partridge;

Member of European Physical Society, Member of American Physical Society, Member of Sigma Xi.