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Does String Theory Solve the
Puzzles of Black Hole Evaporation?

M.J.Bowick, L.Smolin and L.C.R.Wijewardhana

Yale University Physics Department

New Haven, CT. 06511.

ABSTRACT

We point out that the massive modes of closed superstring theories may play a crucial role in the last stages of black hole evaporation. If the Bekenstein-Hawking entropy describes the true degeneracy of a black hole - implying loss of quantum coherence and the unitary evolution of quantum states - it becomes entropically favorable for an evaporating black hole to make a transition to a state of massive string modes. This in turn may decay into massless modes of the string (radiation) avoiding the naked singularity exposed by black hole evaporation in the semiclassical picture. Quantum coherence may be maintained if the entropy of an evaporating black hole is much larger than that given by the Bekenstein-Hawking formula. In that case the transition to massive string modes is unlikely. String theories might thus resolve the difficulty of the naked singularity, but it appears likely that they will still imply loss of quantum coherence.

The development of a unified quantum theory encompassing gravity has been a long-standing problem in physics. Such a theory should reduce to known quantum field theories of matter in the limit when gravity is switched off and give classical general relativity in the limit when quantum fluctuations are insignificant. Calculability (finiteness or renormalizability) and unitarity (or at least conservation of probability) are also essential features of such a theory. Yet these are not the only tests that a successful quantum theory of gravity must pass. It is equally important that such a theory resolve certain puzzles raised in the formulation of quantum field theory in the presence of strong background gravitational fields.

One of the most important of these puzzles is the ultimate fate of an evaporating black hole. The semiclassical description of black hole evaporation requires the breakdown of unitary evolution of quantum states^{1 2}. The radiation observed at infinity from a black hole is in a thermal (mixed) state. Yet a certain amount of information about the emitted radiation is contained inside the event horizon in the form of correlations between photons absorbed by the black hole and those emitted to infinity. If there were a way of ascertaining the quantum state inside the horizon, it could be combined with the thermal state of the emitted radiation to construct a pure state. As the black hole evaporates, therefore, the information content inside would have to increase with time. Yet the Bekenstein - Hawking entropy of the black hole, given by $4\pi M^2$, where M is the black hole mass, decreases continuously. If the information inside the event horizon is lost, upon complete evaporation for example, the initial pure state has evolved into a mixed state of thermal radiation as seen by an observer at infinity. This results in the loss of quantum coherence².

Yet another problem confronting black hole evaporation is the fact that evaporating black holes seem to leave behind naked singularities. This would entail a violation of the cosmic censorship hypothesis at the quantum level^{3 4}. This is a serious difficulty since it is not known how to assign boundary conditions to classical or quantum fields at a naked singularity. This compromises our ability to predict the future even more than the loss of quantum coherence discussed above.

Superstring theories are presently under intensive investigation as candidates for unified theories of all the interactions⁵. They are perturbatively finite and unitary at the one loop level and there are indications that this may be true to all orders. It is important to understand what these theories have to say about the above problems in semiclassical gravity. The present perturbative formulation of string theory is not adequate for a rigorous discussion of non-perturbative processes like black hole evaporation. The physical principle underlying the validity of perturbative string theory and an action describing the interacting theory (the analogues of the equivalence principle and the Einstein-Hilbert action for gravity) are not yet understood.

In place of a rigorous account of black hole evaporation based on a non-perturbative quantum string theory we turn to a thermodynamic analysis of an ideal gas of strings. We draw some hope that something useful may be learnt in this approach from the successes of thermodynamic reasoning applied in the past to gravitational systems - in particular to black holes. The laws of black hole physics derived from classical general relativity have the form of the basic laws of thermodynamics, with temperature corresponding to surface gravity and entropy to the area of the event horizon⁶. These correspondences are evidently more than an analogy

since black holes do radiate, when quantum fluctuations are taken into account, like black bodies with a temperature proportional to their surface gravity. Why average macroscopic laws (thermodynamics) should be the same as exact microscopic laws of general relativity is not at all understood, but it suggests that there is a profound underlying connection between thermodynamics, quantum physics and gravity. In this spirit we hope that a thermodynamic analysis of the role of string excitations in black hole evaporation will tell us something that will eventually be substantiated by full dynamical calculations.

Superstring theories are generalizations of point particle field theories. The fundamental object in such theories is a one dimensional extended structure (string) characterized by two parameters, a string tension $M_s^2 = 1/\alpha'$ and an interaction strength g . Their excitation spectrum includes massless spin two fields (gravitons) and massless spin one particles (Yang-Mills fields) as well as an infinite tower of massive states, with an exponentially rising level density, beginning at the Planck mass M_p . Superstring theories are only unitary and Lorentz covariant in ten space-time dimensions. The most promising candidate string theory is the heterotic string⁷, which is a theory of closed superstrings with the Yang-Mills charges distributed along the string.

The fact that string theory yields general relativity, at the linearized level, in the low energy limit^{8 9} suggests that Riemannian geometry is embedded in some more general geometric structure. This would be the natural setting for the formulation of string theories. When the curvature of space approaches Planck scales, such as near space-time singularities or in the final stages of black hole evaporation, we would expect to uncover this more fundamental geometry. The higher excitations of the string will

inevitably reveal the new geometrical degrees of freedom and will signal the breakdown of Riemannian geometry. Since a black hole is an excited state of the gravitational field in ordinary general relativity it is natural to ask whether it can make a quantum transition to a state of the new geometry. If so a complete understanding of black hole evaporation must incorporate the physics associated with this transition.

For simplicity we assume that $\gamma = M_p \alpha' \gg 1$. In the zero slope limit of closed superstring models γ is inversely proportional to g and for weak coupling γ can be large, making our assumption realistic. The entropy of a black hole of energy E is $S_b = 4\pi(E)^2$ and the entropy of massive string modes of energy E is $S_s = -a \ln E + bE$ where $a=10$ and $b=\pi(2+\sqrt{2})\sqrt{\alpha'}$ for the heterotic string^{10 11} and all quantities are measured in Planck units. Note that we are here using the entropies of a four dimensional black hole and a ten dimensional string. This is the correct thing to do because the black hole is assumed to have an initial mass much greater than the Planck mass, so that the only degrees of freedom of the gravitational field that are involved are the four dimensional ones. When it becomes probable to make a transition to a state in which the massive modes of the string are excited, however, the black hole has a radius on the order of the size of the extra compact dimensions, so that all the degrees of freedom of the ten dimensional string may be excited. As seen in fig.1 the entropy of a black hole becomes less than that of strings when the energy E goes below γM_p . Therefore a black hole can increase its entropy by making a quantum transition to a state of massive string modes provided the Bekenstein-Hawking entropy is the true measure of the number of accessible quantum states. Massive closed string modes have negative specific heat and cannot be in equilibrium with an infinite heat bath. In the infinite volume

limit these modes evaporate into massless modes of the string (thermal radiation of photons, gravitons and other light particles) ¹².

It is worth noting that the use of the Bekenstein-Hawking entropy , and the consequent loss of information, is essential for the scenario of black hole evaporation described here. For there to be no information loss the true degeneracy of a black hole which has evaporated down to the Planck mass would be of the order $\exp(4\pi M^2)$, where M is its original mass. This is much larger than the degeneracy of the massive string modes of energy E, which is given by $E^{-10} \exp(bE)$, when E is near the Planck mass. Thus it would be unlikely for the black hole to transform into massive string modes. If there is no information loss, therefore, the massive string modes will not play a significant role in the final stages of black hole evaporation.

In order to better understand the thermodynamic processes involving strings and black holes we next study configurations involving black holes, massive string modes, and radiation at fixed energy and volume. Since both black holes and massive string modes have negative specific heat ^{10 11} it is clear that all three phases cannot be simultaneously in thermal equilibrium. Thus if we have a black hole, massive string modes and radiation in a box, the equilibrium configuration will be black hole and radiation or string and radiation or radiation alone depending on the total energy density and volume.

First we compute the critical volume V_c , above which only radiation can be in thermal equilibrium, for both systems; black hole and radiation, and massive strings and radiation . The total entropy will be maximized if the second derivative of the total entropy is negative. For the black hole and radiation system this condition means that the energy in the radiation (E_r)

must be less than one fourth of the mass of the black hole¹. For the string and radiation system the corresponding condition is $E_r < E_s^2 / 4aT$, where E_s is the energy in massive string modes. Here we have used the entropies $S_b = 4\pi E_b^2$, $S_s = -a \ln E_s + bE_s$ and $S_r = 4/3(\sigma VT^3)$. These inequalities impose restrictions on V . For the black hole and radiation one finds $V_c = 9 \times 2^{20} \sigma E^5 / 125$ and for strings and radiation $\sigma V_c = (E + 3a/2b - D)b^4 (D - 5a/2b)^4 / (D - 3a/2b)^4$, where $D = (4Ea/b + (3a/2b)^2)^{1/2}$.

It is amusing and informative to establish now the various phases accessible to the three component system just as one would do for any statistical mechanical system. There are three distinct phases- 1) black hole and radiation, 2) string and radiation and 3) pure radiation. If the total energy is E and the volume V , phase 1 has energy $E = \sigma VT^4 + 1/8\pi T$ and entropy given by $S_{b+r} = 4/3(\sigma VT^3) + 1/16\pi T^2$. For phase 2 the analogous expressions are $E = \sigma VT^4 + aT/(bT-1)$ and $S_{s+r} = 4/3(\sigma VT^3) + baT/(bT-1) - (bT-1)/T$. Solving these equations numerically we have found the entropies S_{b+r} and S_{s+r} as functions of E and V . For a given E and V , the system with higher S will be preferred. This is plotted in the phase diagram Fig.2 for the values corresponding to the heterotic string and $\gamma=10$. The two critical volume curves intersect at $E=7M_p$ and $V=1.1 \times 10^5$ Planck units. Below this energy V_c for the string and radiation system is higher than V_c for the black hole and radiation system.

This phase diagram enables us to explore the different evolutions available to the combined system. For example take the quasistatic evaporation of an order Planck mass black hole. Suppose there is a black hole bathed in radiation with total energy $5M_p$. Starting at a point P in the phase diagram, imagine increasing the volume slowly. At point Q , the system will undergo a phase transition to enter the string and radiation

phase. Continuing to increase the volume, one sees that the massive strings will evaporate at point R into pure radiation. In this thermodynamic process the black hole has evaporated through its transition to a string phase. If the string theory is free of singularities- this is not at present known but is conceivable- there will be no singularity left.

In conclusion we have argued that if a superstring theory of quantum gravity and matter is the correct description of nature then it is necessary to include the degrees of freedom associated with the massive string excitations for a complete understanding of the final stages of black hole evaporation. Indeed it seems that these degrees of freedom eventually dominate statistically over those of the black hole, suggesting that the ultimate fate of the black hole is a quantum transition to massive string excitations and thereby to radiation. The proof of a no-singularity theorem in superstring quantum gravity would then show that the singularity inside the event horizon of a black hole never becomes naked. Loss of quantum coherence, however, seems to be an essential feature of black hole evaporation whether or not massive string degrees of freedom are excited.

For superstring theories to be complete quantum theories of gravity and matter it is essential that the problems we have addressed with a crude thermodynamic approach eventually be amenable to a dynamical analysis.

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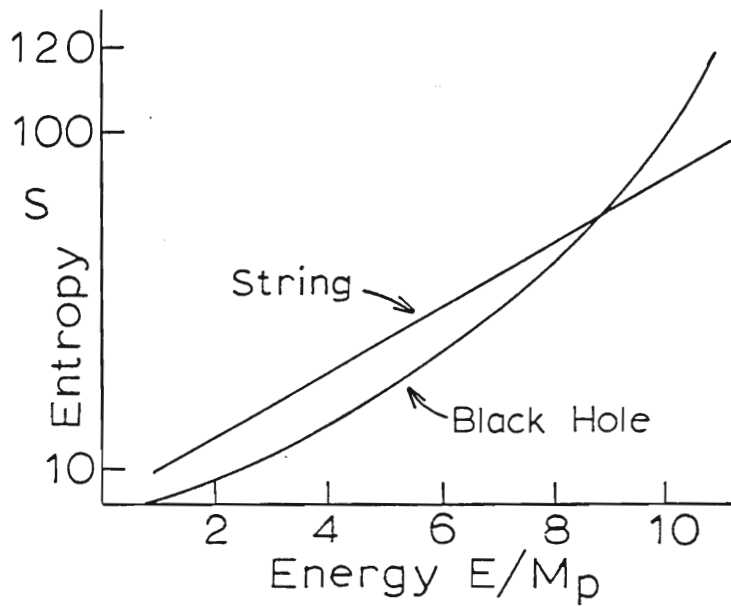


Fig.1 The entropy of black holes versus strings as a function of Energy for the heterotic string and $M_S = 0.1 M_p$

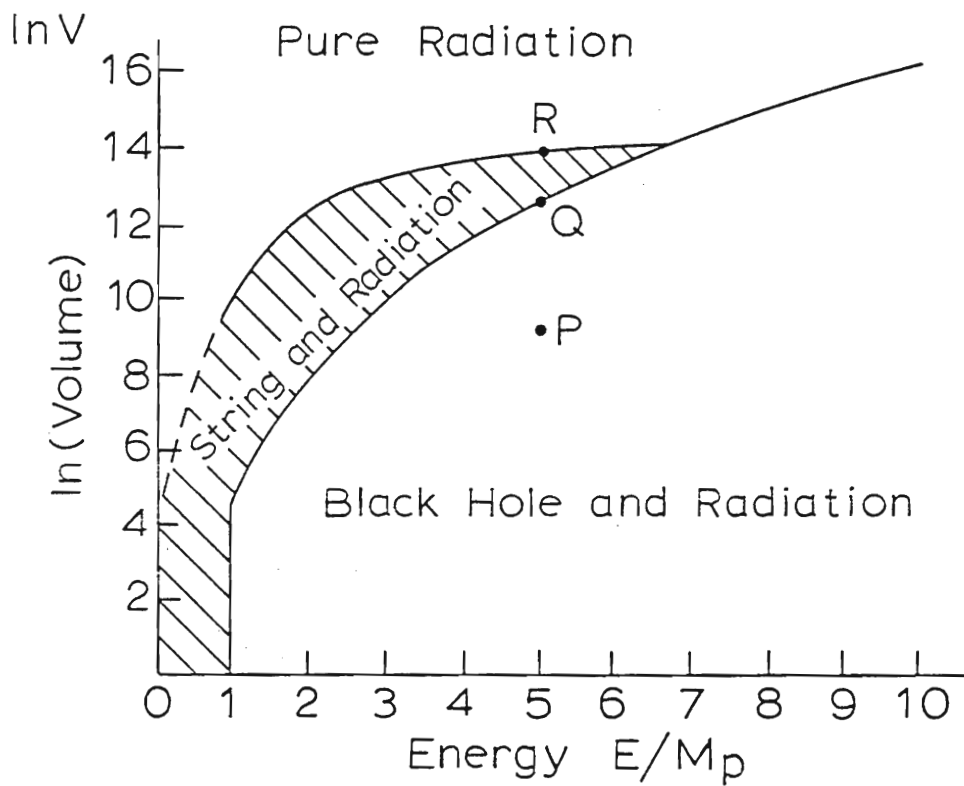


Fig. 2 Volume - Energy Phase Diagram for the heterotic String and $M_S = 0.1 M_p$