Axionic Black Holes and Wormholes

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Theories in which gravity is coupled to a Kalb-Ramond field are known to have black hole solutions characterized by the value of the conserved axion charge. The Kalb-Ramond field configuration for these black holes has vanishing field strength. The axion charge may be measured by an analog of the Aharanov-Bohm interference effect. The axion-charge to mass ratio may be arbitrarily large, as contrasted to the case of the Reissner-Nordstrom black hole where the electric-charge to mass ratio has an upper bound of one. The generic endpoint of semiclassical evaporation of an axionic black hole would therefore be an object of very large axion charge with mass of order the Planck mass. Axion charge also couples to Giddings-Strominger type instantons (wormholes) present in these theories. Instead of evaporating completely, therefore, it is likely that an axionic black hole will be swallowed by a wormhole, avoiding the appearance of a naked singularity. The loss of quantum coherence is a more subtle issue.

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There are very few stationary black hole solutions to Einstein's equation. For the vacuum case the unique stationary solutions form a 2-parameter family given by the Kerr metric and labelled by the total mass M and angular momentum J [1]. This remarkable result has strongly influenced our view of black holes. Although we are a long way from knowing the most general stationary solutions of Einstein's equation, the uniqueness theorems allow us to classify the final states of gravitational collapse of sufficiently massive objects such as stars with mass exceeding the Chandrasekhar limit. Black holes in the vacuum can carry only two kinds of charges (hair). Many of the characteristics of the original matter distribution are thus lost in gravitational collapse. The entropy of the black hole, which is proportional to the area of its event horizon, is one measure of this information loss, since it corresponds to the number of distinct configurations that give rise to the same final macrostate. In the vacuum case all the multipole moments of the original mass distribution are radiated away in collapse except the zeroth moment (mass) and the first moment (angular momentum). Mass and angular momentum cannot be radiated away by a spin-2 field.

One would like to be able to characterize all the possible hair that can be associated with a black hole. We know that elementary particles can carry a large number of different quantum charges. Are black holes really so different? If one considers gravity coupled to an electromagnetic field, then a black hole may also carry an electric and a magnetic charge. The coupled Maxwell-Einstein equations have a similar uniqueness theorem to the vacuum case—there is a unique 4-parameter family of black-hole solutions described by the Kerr-Newman metric [2].

It is not so clear if black holes can carry a non-abelian charge like QCD color and there is certainly no uniqueness theorem to cover this case. There is, however, another natural way to generalize the abelian U(1) theory. This is to consider p-form abelian gauge potentials with associated $p+1$-form field strengths. Such theories have many interesting features such as instantons in $p+2$ dimensions [3,4]. We will consider here the case of $p=2$. This antisymmetric second-rank tensor field will be referred to below as the Kalb-Ramond (KR) field [5] and denoted by $B_{\mu\nu}$. Its 3-form field strength will be denoted likewise $H_{\mu\nu\lambda}$. The KR field appears in string theory, supergravity theories and in the Peccei-Quinn proposal to deal with the strong-CP problem, so it is not unrealistic to suppose it will be relevant in the final formulation of quantum gravity. The coupling of gravity to the KR field in four dimensions is described by the Euclidean action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}(-R + H_{\mu\nu\lambda}H^{\mu\nu\lambda}), \tag{1}$$

where we work in natural units with $h = c = G = 1$. H satisfies the Bianchi identity

$$dH = 0 \tag{2}$$

and the equations of motion following from (1) are

$$R_{\mu\nu} = -6\ {}^*H_\mu {}^*H_\nu, \tag{3}$$

$$d\ {}^*H = 0, \tag{4}$$

where $*$ denotes the Hodge dual. By virtue of (2) there is a conserved current ${}^*H$ with conserved charge inside a two-surface Σ given by

$$q = \int_\Sigma B. \tag{5}$$

This will be called the axion charge below, in view of the dual description of the KR field as a massless pseudoscalar field (axion). Using $H = dB$, q may also be written as

$$q = \int_V H, \tag{6}$$

where V is a spatial 3-surface with boundary Σ.

It may seem unlikely that black holes can carry axion charge. In fact if one considers a KR field in the vicinity of a black hole, one finds that it is either radiated to infinity or falls down the black hole. This does not mean, however, that the black hole cannot carry axion charge, since $H_{\mu\nu\lambda}$ may be non-zero at the singularity. To be more precise, note first that if $H_{\mu\nu\lambda} = 0$, the axion stress-energy vanishes, so the Schwarzschild metric is still an exact solution. The Schwarzschild metric has topology $R^2 \times S^2$ and thus has a non-trivial generator of the second cohomology group. Let the KR field B be this generator, normalized so that

$$\int_{S^2} B = q \tag{7}$$

for some real number q. Explicitly, $B_{\mu\nu} = q\epsilon_{\mu\nu}/4\pi r^2$, where $\epsilon_{\mu\nu}$ is the induced volume form on the two-spheres of spherical symmetry in the Schwarzschild solution.

It was shown in [6] that the unique static solutions of the Einstein-KR field equations are given by the Schwarzschild metric

$$ds^2 = -(1 - 2M/r)dt^2 + \frac{dr^2}{(1 - 2M/r)} + r^2 d\Omega^2 \tag{8}$$

and KR field

$$B_{\mu\nu} = \frac{q\epsilon_{\mu\nu}}{4\pi r^2}. \tag{9}$$

This describes a black hole of mass M and axion charge q. Note that the field strength H vanishes for this solution.

There are three features of these axionic black holes that I will discuss in this paper. First of all they provide an example of black holes with a new kind of hair—axion hair. Furthermore this hair is a topological charge rather than the conserved charge of a local symmetry (as is usually claimed must be the case). Secondly the axionic charge on a black hole may be measured by a higher-dimensional analog of the Aharanov-Bohm effect [7]. Finally, in the late stages of its evaporation, the coupling of an axionic black hole to axionic instantons (wormholes) may be so strong that the black hole is swallowed by a wormhole, rather than evaporating away completely to leave a naked singularity.

Before elaborating on possible physical consequences of axionic black holes I review how the axion charge of a black hole may be detected [6]. Since the gauge-invariant couplings of the KR field to ordinary matter involve only the vanishing gauge-invariant field strength $H_{\mu\nu\lambda}$, the axion charge cannot be detected through ordinary scattering processes. The KR field does, however, have a gauge-invariant coupling to strings which takes the form [5]

$$S_B = \frac{T}{2}\int_\Sigma d^2\sigma\sqrt{h}B_{\mu\nu}(X^\lambda(\sigma))\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu, \tag{10}$$

where T is the string tension, Σ is the string world sheet, and h is the world-sheet metric. Such a coupling occurs for fundamental strings. It may also occur for axionic cosmic strings which arise from the breaking of global U(1) symmetries [8].

Consider the propagation of a string in the vicinity of an axion-charged black hole. If the string world sheet is the boundary of a three-surface V not intersecting the singularity of the black hole, then by the Poincaré lemma we can write $B = d\Lambda$ for a one-form Λ defined everywhere on V. In this case the coupling in (10) becomes a total derivative

$$S_B = \frac{T}{2}\int_\Sigma d^2\sigma\sqrt{h}\nabla_a(\epsilon^{ab}\partial_b X^\nu \Lambda_\nu) \tag{11}$$

and the axion charge has no effect on string propagation. To see the axion charge the string world sheet should encircle the hole. Mathematically the world sheet should be in a non-trivial second cohomology class. The axion charge on a black hole may be measured by creating a pair of strings at some point, allowing one of them to encircle the hole, and then detecting the interference at some other point. The action for such a process is just $S_0 + S_B$, where S_0 is the action for the corresponding process with no axion charge on the black hole. The relative change in phase between the two strings, when they are compared, is given by qT/M_p^2. This phase is clearly a higher-dimensional generalization of the classical Aharanov-Bohm effect.

A noteworthy feature of the above solutions is that the axion charge does not gravitate since the spacetime metric is independent of q. In turn this means there is no restriction on the magnitude of the axion charge for a given mass black hole. It is classically possible, therefore, to have a very small mass black hole with non-zero axion charge. This is in contrast to the situation for electrically charged Reissner-Nordstrom black holes. It is impossible to make the electric charge e large relative to the mass M since an unphysical naked singularity is encountered for $e > M$. Furthermore in astrophysical situations it is very difficult to have a charge to mass ratio exceeding 10^{-18} since a body with larger charge to mass ratio would selectively attract particles of opposite charge.

This situation may have qualitative implications for the process of Hawking evaporation. The semiclassical calculation of black hole radiation is unaffected by the axion charge since point particles do not couple to B but only to H. By contrast, electrically charged black holes preferentially radiate particles to neutralize the charge. Thus a black hole will evaporate until it reaches the Planck mass. At this point it will generically have an axion charge which is very large compared to its mass. Again in the electrically charged case this does not happen because the Hawking temperature falls to zero when the charge equals the mass. String or other quantum gravitational corrections to the evaporation process should then become important and may well be affected by the axion charge. For large enough axion charge, macroscopic causality and energy conservation will prevent all the charge being radiated away in a finite time. Thus a Planck mass black hole with a large axion charge cannot totally evaporate by the Hawking process. This invalidates the simplest arguments in favor of coherence loss by black holes since they assume the black hole evaporates completely and leaves no remnant in the final state.

Now it is also known [9–12] that in a theory with gravity coupled to a KR field there are generalized gravitational instantons i.e., solutions of the Euclidean equations of motion. These are referred to as wormholes. The line element [9,13] for the instanton is given by

$$ds^2 = \frac{|q|}{2\pi^2} cosh2x(dx^2 + d\Omega^2), \qquad (12)$$

with KR field strength

$$H = q\epsilon, \tag{13}$$

where the three form ϵ is the volume element for surfaces of constant x normalized to integrate to one. The two regions $x \to \pm\infty$ are asymptotically flat. Restricting to the coordinate region $x > 0$ gives a half-wormhole instanton which represents tunneling from $R^3 \to R^3 + S^3$. Since the H field on the S^3 boundary is proportional to the volume element on the boundary the data on this surface corresponds to initial data for a Robertson-Walker universe. The above tunneling process can thus also be interpreted as the nucleation of a baby Robertson-Walker universe. There is a one-parameter family of such instantons labelled by q, which represents the total axion charge which flows through the wormhole.

There is a more general notion of a wormhole as simply a Euclidean field configuration consisting of two asymptotically flat regions connected by a throat. These need not be solutions of the Euclidean equations of motion. There has been much interest recently in wormholes as sources for topology-change and loss of quantum coherence in quantum gravity [9,10,14–16] and as the origin of a dynamical mechanism for the vanishing of the cosmological constant [10,17,18].

Consider then an axionic black hole which is evaporating by Hawking radiation. From the preceeding discussion it is clear that the generic endpoint of the semiclassical description of this evaporation will be a black hole with very large axion-charge to mass ratio. It is necessary to know the correct short-distance theory of quantum gravity to understand the dynamics of the late stages of the evaporation but we have seen that all the axion charge cannot be radiated away in a finite time. Furthermore the axionic black hole will couple very strongly to wormholes because of its large charge. It is very likely then that such a black hole will be swallowed by a wormhole. An observer in our universe would see a net loss of axion charge which would flow through the wormhole throat into a baby universe. Of course the total axion charge is always conserved. This is exactly the mechanism by which wormholes give rise to an effective violation of global conservation laws and induce all possible local couplings. Wormholes thus allow an axionic black hole to evaporate without leaving a naked singularity. This contrasts with the usual Hawking view of black hole evaporation. A mechanism for avoiding the appearance of a naked singularity in black hole evaporation, which is similar in spirit to this one, was proposed shortly after Hawking's original work by Dyson [19,18].

Now consider the issue of quantum coherence. Look at particle-antiparticle pairs created in the vicinity of the event horizon by the large gravitational curvature. In some cases one member of the pair will fall inside the event horizon and the other member will be radiated to infinity where it is detected as radiation with a thermal spectrum at the Hawking temperature $T = \frac{\kappa}{2\pi}$, where κ is the surface gravity of the black hole. If the black hole evaporates completely the correlation information between these pairs will be lost forever and there is a consequent loss of quantum coherence. An original pure state has evolved into the thermal state detected by the observer at infinity. Above we have suggested that instead an axionic black hole will be swallowed by a wormhole when it has, say, mass M_f. One might argue the joint state of the radiation reaching infinity together with the baby universe remnant of the black hole is still pure. This is not so clear. The subtlety may be seen by an entropy argument [20]. The entropy of the black hole just before it is swallowed is of order M_f^2. This is much less than the entropy of the radiation which is of order M_i^2, where M_i is the initial mass of the black hole. Such a mismatch in the number of internal states ($expM_f^2$ versus $expM_i^2$) makes it difficult to see how coherence could be restored straightforwardly. Yet one has the feeling that the total system of our universe plus any number of baby universes into which the black hole has disappeared must contain all the correlation information necessary to reconstruct a pure state. It would be of much interest to work out the exact dynamical details of the process outlined above. Quantum coherence may be maintained if the baby universe effectively retains the entropy associated with particles whose world lines ended in the singularity. This would also mean that the entropy acquired by a baby universe in swallowing a black hole is larger than the entropy of the black hole itself. It also raises the question of the fate of the singularity when a black hole is swallowed by a wormhole. Finally, it might appear that an observer in our universe will see an effective loss of quantum coherence in processes like these, in which information disappears down baby universes, but Coleman has shown, with certain assumptions, that this is coherence that was never possessed to begin with [16].

The proposal made here has the advantage of dealing with a specific model and one should be able to make detailed calculations to prove or disprove the above ideas.

Acknowledgements

I wish to thank my collaborators Steve Giddings, Jeff Harvey, Gary Horowitz and Andy Strominger for earlier joint work on axionic black holes on which this paper is based. I would also like to thank Steve Giddings, Pawel Mazur and Lee Smolin for valuable discussions.

References

[1] D. C. Robinson, Phys. Rev. Lett. **34** (1975) 905.

[2] P. Mazur, J. Phys. **A15** (1982) 3173; Phys. Lett. **A100** (1984) 341.

[3] A.M. Polyakov, Phys. Lett. **B59** (1975) 82; Nucl. Phys. **B120** (1977) 429

[4] P. Orland, Nucl. Phys. **B205** (1982) 107

[5] M. Kalb and P. Ramond, Phys.Rev. **D9** (1974) 2273.

[6] M.J. Bowick, S.B. Giddings, J.A. Harvey, G.T. Horowitz and A. Strominger, Phys. Rev. Lett. **61** (1988) 2823.

[7] Y. Aharanov and D. Bohm, Phys. Rev. **119** (1959) 485.

[8] J. A. Harvey and S. G. Naculich, Phys. Lett. **B217** (1989) 231.

[9] S. B. Giddings and A. Strominger, Nucl. Phys. **B306** (1988) 890.

[10] S. B. Giddings and A. Strominger, Nucl. Phys. **B307** (1988) 854.

[11] K. Lee, Phys. Rev. Lett. **61** (1988) 263.

[12] J. E. Kim, Univ. of Michigan preprint UM-TH-88-09.

[13] A. Strominger, "Baby Universes," Lectures presented at the TASI summer school, Brown University, June 1988.

[14] S.W. Hawking, Phys. Lett. **B195** (1987) 337 ;
Phys. Rev. **D37** (1988) 904.

[15] G.V. Lavrelashvili, V.A. Rubakov and P.G. Tinyakov, JETP Lett. **46** (1987) 167.

[16] S. Coleman, Nucl. Phys. **B307** (1988) 867.

[17] S. Coleman, Nucl. Phys. **B310** (1988) 643.

[18] T. Banks, Nucl. Phys. **B309** (1988) 493.

[19] F. Dyson, "Breakdown of Physics in Gravitational Collapse?" Institute for Advanced Study Report (1975), unpublished.

[20] R. M. Wald, "Black Holes, Singularities and Predictability," in **Quantum Theory of Gravity**, ed. S. M. Christensen (Adam Hilger Ltd, Bristol, 1984).