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The Gravitationally-Stabilized Hydromagnetic Model of the Elementary Particle

by

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Abstract

An electrohydromagnetic torus model of the elementary particle, in which only the classical electromagnetic fields contribute to the observable mass, spin, and magnetic moment of the particle, is formed from a self-gravitationally-stabilized, electrically-charged, electrically-conducting thread. This thread is coiled and circulates in such a way as to produce the dimensions of the toroidal model appropriate for an elementary particle. A small amount of the self-gravitational energy of the thread may be used to account for the necessary binding energy E_b for a stable particle. The electromagnetic field energy E_f of such a particle is related to E_b and mc^2 (the energy of experience) by $E_f - E_b = mc^2$.

The Einstein cosmological solution which compresses the electromagnetic field into elementary particles with a smeared cohesive pressure $p = -\frac{\rho c^2}{2}$, where ρ is the average density of the universe gives $E_b = \frac{-mc^2}{2}$. Thus E_f should be $\frac{3}{2} mc^2$.

The field energy E_f is computed, along with the magnetic moment μ, the spin, the g factor (and its correction Δg), for two sample electrohydromagnetic torus models. The calculations show that although the models bear some resemblance to the properties of the elementary particles, there is at present no simple way to have the values of E_{f}, μ, spin, g, and Δg all correspond closely with those of the electron or the proton for a simple torus model. For a more complex model approximate agreement can be obtained.

These models have considerable conceptual merit, and work toward their improvement should continue. Their conceptual merit lies in the absence of self-energy infinities, their stabilization by self-gravitational fields, and the recognition that their stabilization is influenced by the universe as a whole through Ein-

stein's relation $p = -\frac{\rho c^2}{2} = -\frac{nmc^2}{2} = -nE_b$. The recognition that mass is entirely electromagnetic and is corrected by self-gravitation is of great importance.

Energy Considerations in Continuous Creation

The Einstein cosmological solution, which assumes that the entire mass M of the universe (with average, smeared density ρ and radius a) is electromagnetic in origin and that an average cohesive pressure p binds the electromagnetic energy into the elementary particles, states that

$$c^2 = \frac{2}{\pi}\frac{MG}{a}, \quad \text{and} \quad p = -\frac{\rho c^2}{2}. \tag{1}$$

It is legitimate to visualize this universe from a Newtonian point of view where the concept of continuous creation of matter proceeds naturally from this solution: The expanding universe requires that the radius a increase, and if a increases M must increase. Indeed, since $a^2 = \frac{3}{8}\frac{c^2}{G\rho}$,

$$\frac{da}{dt} = -\frac{3}{16}\frac{c^2}{a\rho G}\left(\frac{1}{G}\frac{dG}{dt} + \frac{1}{\rho}\frac{d\rho}{dt}\right).$$

If ρ is constant in time $\frac{dG}{dt} < 0$, since $\frac{da}{dt} > 0$. This universe, depicted in Figure 1a, may be thought of as a transient stage in the build up of Hoyle's steady-state universe. A more reasonable variation of $\rho(r,t)$ for such a transient stage would be

$$\rho(r,t) = \rho_0 e^{-(r/a + t/\tau)},$$

where a increases with t and exceeds the radius of the observable universe at $t \geqq \tau$. This distribution of $\rho(r,t)$ is depicted in Figure 1b, where it can be seen that as $t \gg \tau$, the steady state universe is approached.

Most of the mass of the universe is in the form of protons of mass m_p. Thus, to a good approximation $\rho = m_p n$, where n is the average number of protons per cm^3.

Since the time of Ernst Mach it has been recognized that any mass within the universe owes its inertial properties to the presence of all the masses within the universe. Thus the self energy of any elementary particle must be described within the framework of the universe as a whole. The energy of a proton observed in this universe is thus

$$m_p c^2 = E_f - E_b, \tag{2}$$

where E_f is the total positive electromagnetic field energy in the proton, and E_b is the cohesive or binding energy which holds the electromagnetic field into the particle.

If we consider the creation process whereby a neutron $(m_n \cong m_p)$ is born into the universe, conservation of energy must prevail during the act. Thus, for example, if a proton is born at a radius r from the center of the Newtonian universe depicted in Figure 1a, the sum of the positive energy and the negative energy produced in the event must be equal to zero. That is

$$E_f \quad - \quad E_b \quad - \quad \frac{Gm_pM}{a}\left[1 + \frac{1 - r^2/a^2}{2}\right] = 0. \tag{3}$$

E_f	E_b	$\frac{Gm_pM}{a}[\dots]$
positive electro-magnetic field energy	negative energy which binds the field into proton	negative gravitational energy of the proton in the universe

The last term averaged over this Newtonian universe of constant density is $-\frac{7}{5}\frac{Gm_pM}{a}$. Hence, with equation (2) we can write

$$m_p c^2 \cong \frac{7}{5}\frac{m_pMG}{a}. \tag{4}$$

On the other hand equation (1) multiplied by m_p gives $m_p c^2 = \frac{2}{\pi}\frac{m_pMG}{a}$, which differs from equation (4) by about a factor of $\frac{1}{2}$. Thus this simple Newtonian picture of energy conservation in continuous creation is only approximately consistent with the Einstein relation in equation (1). Nevertheless, E_f can be computed from equation (2). For, if we note from the second of equations (1), that

$$-nE_b = p = -\frac{\rho c^2}{2} = -\frac{nm_pc^2}{2},$$

$$\text{or} \quad E_b = \frac{m_pc^2}{2},$$

equation (2) becomes

$$E_f = \frac{3}{2} m_p c^2. \tag{5}$$

A similar relationship is to be expected for the electron.

The Particle Model:

We now introduce a rudimentary model of an elementary particle in which it is attempted to calculate E_f. More refined models will be examined later.

The recently developed subject of hydromagnetics has taught us a great deal concerning the stability of various electrically conducting shapes constructed of plasma (a fluid) and magnetic fields. Indeed the experimental effects, such as the shapes assumed by plasmoids are unexpectedly rich and have provided information considerably in advance of the ability of theory to predict.

Any model of an elementary particle which we try to construct should be guided by some of the stability criteria which can be drawn from both the experimental and theoretical efforts in hydromagnetics. A commonly observed hydromagnetic property of plasmas is the tendency of the plasma spontaneously to form shreds or thin filaments. The obvious first step in the construction of an electrohydromagnetic model of particle is to start with a filamentary toroidal model where the electric and magnetic fields are those arising from the charge and the motion of charge within the particle. The simplest model is that of Figure 2a, where the charge e is assumed to be continuously distributed as a toroidal shell, so that as it circulates around the torus with a velocity of $v=c$ it does not radiate. In the range $r_0 < r < R$

$$|\mathbb{E}| = |\mathbb{H}| \cong \frac{e}{\pi r R}. \qquad (6)$$

Thus the hoop stress due to $\mathbb{H}(=H_\theta)$ exactly balances the radial bursting stress due to $\mathbb{E}(=E_r)$, and sausage type instabilities should not occur. However, the pressures tending to elongate the circumference $2\pi R$ are unbalanced and the particle will therefore tend to increase its R. As will be seen later the model can be improved to remedy this defect. In this simple model $E_b = 0$.

The value of the magnetic moment μ may be obtained by multiplying the current $e/2\pi R$ by πR^2.

Thus
$$\mu = \frac{eR}{2}. \qquad (7)$$

If the empirical values for μ are taken

$$\mu = \frac{k^1 e\hbar}{2mc}, \qquad (8)$$

where $k^1 \cong 1$ for the electron and 2.8 for the proton. Thus R can be computed to be

$$R = \frac{k^1\hbar}{mc} \qquad (9)$$

The value of the electromagnetic field energy, E_f, is

$$E_f = \frac{1}{8\pi}\int_{\substack{\text{all}\\ \text{space}}} (|\mathbb{E}|^2 + |\mathbb{H}|^2)\, dV \cong \frac{e^2}{2R}\Bigg|_{\sim 2R}^{\infty} + \frac{1}{8\pi}\frac{2e^2}{\pi^2R^2}\, 2\pi R \int_{r_0}^{R} \frac{2\pi r\, dr}{r^2} \qquad (10)$$

$$\cong \frac{e^2}{R}\left(\frac{1}{4} + \frac{1}{\pi}\ln\frac{R}{r_0}\right).$$

The angular momentum, i.e., the spin, as carried classically by the Poynting vector is computed in erg sec to be

$$|S| \cong 2\pi R^2 \int_{r_0}^{R} \frac{\mathbb{E}\times\mathbb{H}}{4\pi c}\, 2\pi r\, dr = \frac{2\pi R^2}{4\pi c}\int_{r_0}^{R} \frac{e^2}{\pi^2R^2}\, \frac{2\pi r\, dr}{r^2}$$

or
$$|S| = \frac{e^2}{\pi c}\ln\frac{R}{r_0}. \qquad (11)$$

If the spin for such a particle is taken as 1,

$$|S| = \hbar = \frac{e^2}{\pi c}\ln\frac{R}{r_0}, \qquad (11a)$$

and
$$\ln\frac{R}{r_0} = \pi\frac{\hbar c}{e^2} = 137\pi. \qquad (11b)$$

From equation (10), E_f becomes

$$E_f \cong \frac{e^2}{R}\left(\frac{1}{4} + \frac{\hbar c}{e^2}\right) = \frac{e^2}{R}\left(\frac{1}{4} + \frac{1}{137}\right) = \frac{e^2}{R}\frac{\hbar c}{e^2}\left(1 + \frac{1}{4\times 137}\right) \qquad (12)$$

$$\cong \frac{mc^2}{k^1}\left(1 + \frac{1}{4\times 137}\right)$$

If k^1 is taken as $k^1 = 1 + \frac{1}{4\times 137}$, $E_f = mc^2$ for this particle and equation (2) agrees with our assumption that the binding energy for this model, E_b, is zero.

The magnetic flux (strictly poloidal flux) which threads this torus is

$$\Theta \cong 2\pi R \int_{r_0}^{R} \frac{e\, dr}{\pi R r} = 2e\ln\frac{R}{r_0} = 2e\pi\frac{\hbar c}{e^2} = \frac{2\pi\hbar c}{e} \qquad (13)$$

Thus $\Theta = 2\pi$ fluxons, where a fluxon is equal to $\frac{\hbar c}{e}$. Note that the assumption of quantized charge e, and quantized angular momentum (equation 11a) leads to quantization of the flux in this model.

We can now compute the gyromagnetic ratio g of such a particle in terms of $e/2mc$:

$$g \equiv \frac{\mu}{|S|} = \frac{\frac{e\hbar k^1}{2mc}}{\hbar} = \frac{e}{2mc}\left(1 + \frac{1}{4 \times 137}\right), \tag{14}$$

and $$\Delta g = e^2/4\hbar c = \frac{1}{4 \times 137}.$$

This crude model which has been used only to introduce the filamentary concept obviously does not correspond to either the electron or the proton: it has spin 1; g value $\cong 1$ instead of $2\left(1 + \frac{1}{2\pi \times 137}\right)$; it has no wavelength. However, it does have a Δg of the right order of magnitude for the electron. Physically the Δg for this crude model can be visualized as coming from the first term in equation (10) which contributes a small amount to the mass of the particle but does not contribute to the angular momentum.

If the value of the spin $|S|$ had been taken as $\sqrt{1(1+1)}\,\hbar$, instead of $\hbar$, equation (10) becomes $E_f = \frac{mc^2\sqrt{2}}{k^1}\left(1 + \frac{1}{4\sqrt{2} \times 137}\right)$

If, somewhat similarly as before, $k^1 = 1 + \frac{1}{4\sqrt{2} \times 137}$, $E_f = \sqrt{2}mc^2$, which is a little less than the value $E_f = \frac{3}{2}mc^2$ of equation (5).

The Gravitationally Stabilized Electrohydromagnetic Thread

Before we attempt to show how such a model as that of Figure 2a can be modified to correspond more closely to an elementary particle, such as the electron, let us discuss the stabilization of such a thin electrohydromagnetic filament by self-gravitational forces, and thereby inquire into the origin of E_b.

Let us start with a simple thread of the general form of Figure 2a, although eventually threads of the field configurations of types of Figures 2b, c and d should be considered. Let this loop of thread have a large radius R_g, and a small radius r_{0g}, and the radius variable r_g is such that $r_{0g} < r_g < R_g$. The objective now is to compute the negative gravitational energy associated with the mass density distribution

$$\rho(r_g) = \frac{1}{8\pi c^2}(|\mathbb{E}|^2 + |\mathbb{H}|^2) = \frac{2e^2}{8\pi c^2\pi^2 R_g^2 r_g^2} = \frac{e^2}{4\pi^3 c^2 R_g^2 r_g^2}. \tag{15}$$

The gravitational potential energy associated with this mass distribution is

$$E_g = -2\pi R_g G \int_{r_{0g}}^{R_g} 2\pi r_g \rho\, dr_g \int_{r_g}^{R_g} \frac{2dr_g}{r_g} \int_{r_{0g}}^{r_g} 2\pi r_g \rho\, dr_g = -\frac{e^4 G}{\pi^3 c^4 R_g^3} \ln^3 \frac{R_g}{r_{0g}}. \tag{16}$$

When $|E_g|$ is comparable to the field energy, $E_{fg} \cong \frac{e^2}{R_g}\left(\frac{1}{4} + \ln\frac{R_g}{r_{0g}}\right)$, of such a super thin filament (much thinner than that of equation 12) it is possible for the thread to be gravitationally stabilized. The virial theorem would suggest that stability results when $|E_g| = |2E_{fg}|$, but this would leave a very large negative contribution to the mass of the particle, i.e. $|E_b|$ would be far too large. It is more appropriate here to let $|E_g| \cong |E_{fg}|$ and the small residue of E_g can be used to provide the necessary E_b for the particle. Thus

$$\frac{e^2}{\pi R_g} \ln \frac{R_g}{r_{0g}} \cong \frac{e^4 G}{\pi^3 c^4 R_g^3} \ln^3 \frac{Rg}{r_{0g}}, \quad \ln \frac{R_g}{r_{0g}} \cong \frac{\pi c^2 R_g}{eG^{1/2}} \cong 4 \times 10^{21}. \tag{17}$$

This value of $\ln \frac{R_g}{r_{0g}}$ is thus much greater than $\ln \frac{R}{r_0}$ for a toroidal elementary particle, given by equation (12). Therefore the charge and current distributions for toroidal elementary particles cannot consist of a single straight strand of such a super thin gravitational filament but must be established by the coiling of such a filament or filaments into a solenoid or radius r_0. This solenoid can then possess a toroidal H_ϕ field as well as the poloidal H_θ field. If the solenoidally-wound thread is then projected in the ϕ direction at speeds up to $v=c$ the appropriate values of H_θ can be obtained.

The model of the gravitational thread or fiber we have used here is that with the field configuration of Figure 2a, but where r_{0g} is employed instead of r_0. Here the Poynting vector circulates only in the ϕ direction and hence would contribute angular momentum to prevent a gravitational collapse that would shorten the circumference $2\pi R_g$. It would be well to refine the gravitational fiber to include the field configurations of Figures 2b, 2c and 2d where the current circulates with a θ component and thus produces an H_ϕ. Again it must be remembered that the appropriate dimension for the gravitational fiber is r_{0g}, not r_0. The Poynting vector can now circulate with a θ component and gravitational collapse about r_{0g} can be prevented. Further details of gravitationally stabilized fibers cannot be treated here. We shall note, however, that these filaments are the only form in which stabilized matter arising from the electromagnetic field can exist (aside from the large geon structures).

The construction of elementary particles from these elemental fibers gives the possibility of the circulation of the fibers about the toroidal particle in both the θ and the ϕ directions in which these two circulations must be *integrally commensurable*.

Thus, the construction of particles from such fibers may cause quantum effects to arise naturally, even as integral numbers arise naturally, in mathematics out of topology. It may be possible eventually to demonstrate how quantization of charge, spin, and mass proceed naturally from the integrally commensurable circulation of such fibers in toroidal particles. A more modest program is to attempt to show how various models of toroidal particles can correspond to the electron and proton.

The Helical Model:

The helical model with a radius of R as shown in Figure 3 if spun around its axis will exhibit de Boglie waves such that $v_g v_{\text{phase}} = c^2$ if $R = \hbar/mc$ and if the angular velocity of spin, $\omega = c/R$. However this particle, if assigned a spin value of $\frac{1}{2}$, will not exhibit the correct values for E_f, μ, and Δg for the electron.

Complex Models:

It is possible however by introducing additional circulating positive and negative charges, as shown in Figure 4b, and superimposing these over either a torus with small kinks as shown in Figure 4a or over the torus of Figure 2a, to obtain enough current, with the appropriate spin value, to give the correct values of μ and E_f not only for the electron, but for the proton as well.

It is even possible to obtain the correct value for Δg. A specific example of such a model is as follows: Let there be one negative and one positive charge, each of value e circulating as shown in Figure 4b about a torus of the type of Figure 2a (lined up along the curved center line of Figure 4b). The value E_f is then given by

$$E_f \cong \frac{e^2}{4R} + \frac{e^2}{\pi R}\ln\frac{R}{r_0} + \frac{2e^2 \ln \frac{R}{r_0^1}}{\pi R(\tan^2\alpha^1 + 1)^{1/2}} + \frac{2e^2 \sin^2\alpha^1}{4\pi R} \qquad (18)$$

energy in spherical field beyond $2R$	Field energy in torus of Figure 2a	field energy of + and − torus with helical angle	energy in toroidal field, if + and − charges are both left hand or right hand helices.

The last term is zero if one of the two charges of Figure 4b is a left-handed helix and the other is a right-handed helix. The net spin is due only to the torus of Figure 2a. If this spin is $|S| = \hbar/2$, equation (11) gives

$$\ln\frac{R}{r_0} = \frac{137\pi}{2} \qquad (19)$$

The angle α^1 may be chosen so that

$$\mu = \frac{eR}{2} + 2\,\frac{eR\cos\alpha^1}{2} = \frac{e\hbar k^1}{2mc},$$

and

$$g = \frac{\mu}{|S|} = \left(\frac{e}{2mc}\right)2k^1.$$

It might seem that a good value for α^1 for the electron would be given by $\cos\alpha^1 = 1/2$. The value of $E_f = (3/2)\,mc^2$ can then be obtained with the proper choice of $\ln\frac{R}{r_0^1}$. For

$$\frac{3}{2}mc^2 = \frac{mc^2}{k^1}\left(\frac{1}{2} + \cos\alpha^1\right)\left[\frac{1}{2\times 137} + 1 + \frac{4}{137\pi}\,\frac{\ln\frac{R}{r_0^1}}{(\tan^2\alpha^1 + 1)^{1/2}} + \frac{\sin^2\alpha^1}{137\pi}\right], \qquad (20)$$

and this equality can be satisfied by proper choice of $\ln\frac{R}{r_0^1}$, for $\cos\alpha^1 = 1/2$.

However, for the electron k^1 should equal $1 + \frac{1}{2\pi\times 137}$ in order that Δg should be the correct value. From equation (20), in the present model

$$k^1 = \frac{2}{3}\left[\frac{1}{2\times 137} + 1 + \frac{4}{137\pi}\,\frac{\ln\frac{R}{r_0^1}}{(\tan^2\alpha^1 + 1)^{1/2}} + \frac{\sin^2\alpha^1}{137\pi}\right]. \qquad (21)$$

If, for example, $\ln \frac{R}{r_0^1}$ is chosen so that the third term in equation (21) is $\frac{1}{2}$, then $k^1 = 1 + \left(\frac{1}{3} + \frac{1}{2\pi}\right)\frac{1}{137}$ and $\Delta g = \left(\frac{1}{3} + \frac{1}{2\pi}\right)\frac{1}{137}$ which, for the electron, is too large by about a factor of 3. However, if $\ln \frac{R}{r_0^1}$ is chosen so that the third term in equation (21) is $\frac{1}{2}\left(1 - \frac{1}{137}\right)$, then k^1 and Δg become the correct value for the electron.

For the proton, in equation (20) $\cos \alpha^1$ can be chosen to be approximately 1, and if $\ln \frac{R}{r_0^1}$, is chosen so that the third term in equation (20) is approximately 1.8, then k^1 will be the correct value, 2.8.

Here the parameters of the models have been adjusted to fit the experimentally determined constants. In a more fully developed form the models should be able to predict, from stability and topological considerations, the values of the experimentally determined constants.

The electrohydromagnetic model should thus be further examined because of its potentialities. In helical form, as in Figure 3, it can produce de Broglie waves.

The introduction of extra charges positive and negative, but equal in number, as suggested in Figure 4b, is not inconsistent with the other attributes exhibited by the electrohydromagnetic model: the net energy E_b of the gravitationally stabilized fiber is the residue of large positive and negative components. There is hidden or uncounted magnetic moment in the toroidal H_ϕ field. There is hidden or uncounted angular momentum in the θ component of the Poynting vector. It is thus not strange that there should be hidden or uncounted extra positive and negative charges which contribute to the magnetic moment and to E_f, but not to the spin.

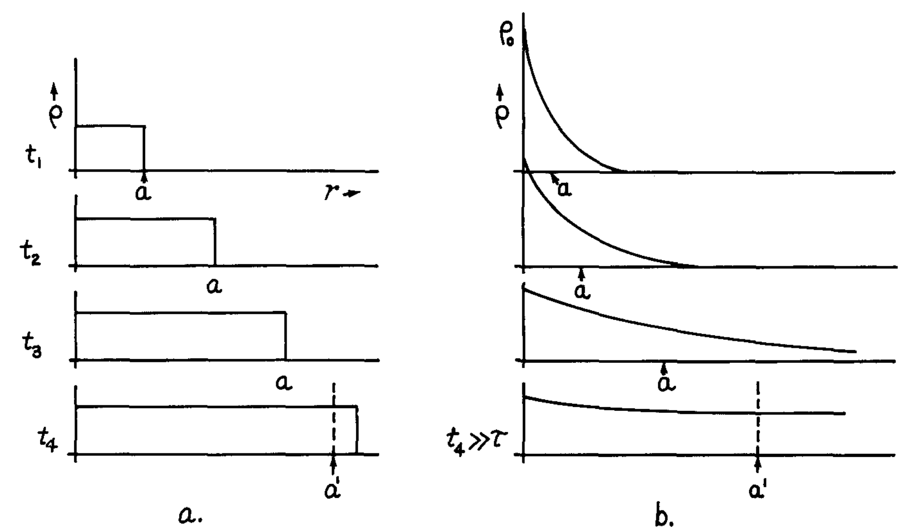

Fig. 1
Two possible density distributions $\rho(r,t)$ in the transient stage of a continuous-creation universe depicted at various times t_1 through t_4. a' is the radius of the observable universe.

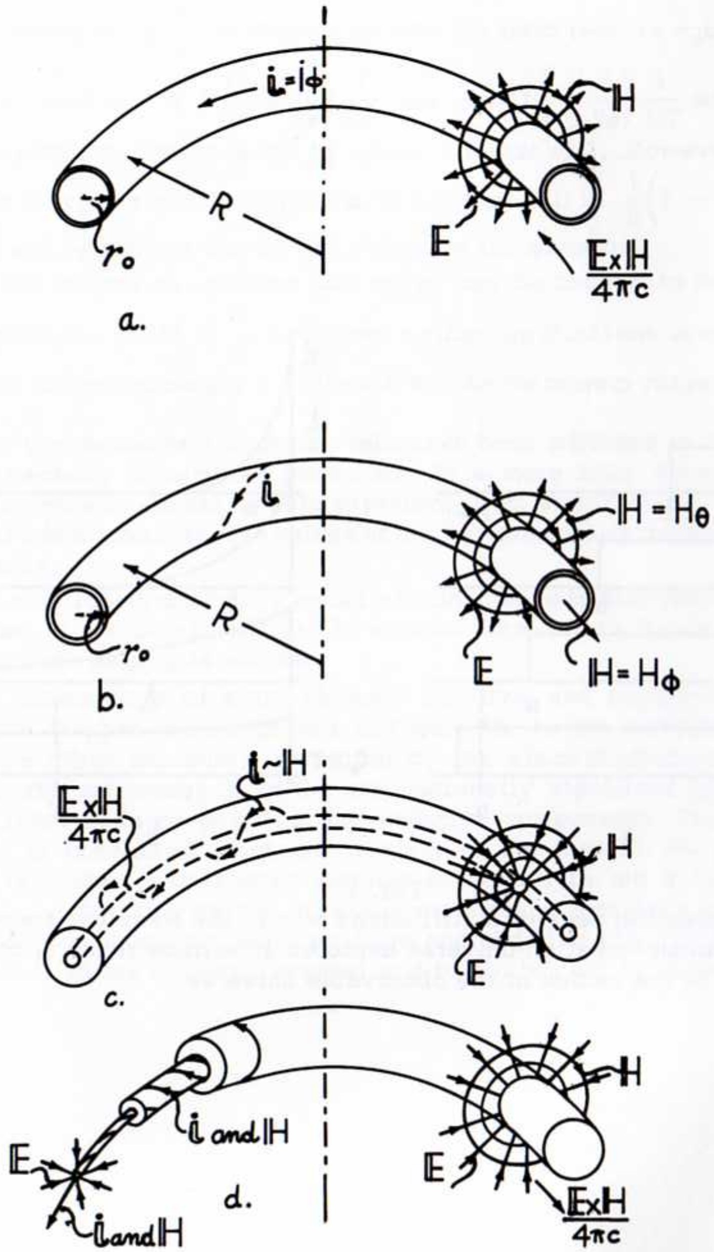

Fig. 2
Various types of toroidal models. These configurations can also be used for gravitational fibers if the values of r_{0g} and R_g are used instead of r_0 and R.

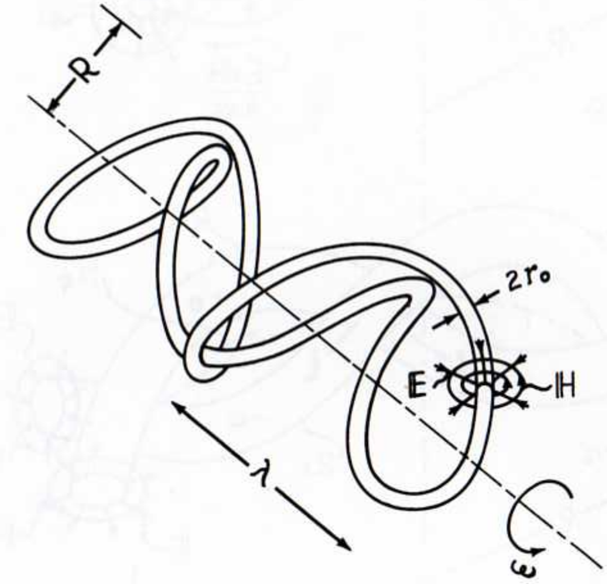

Fig. 3
A helical model which can exhibit de Broglie waves.

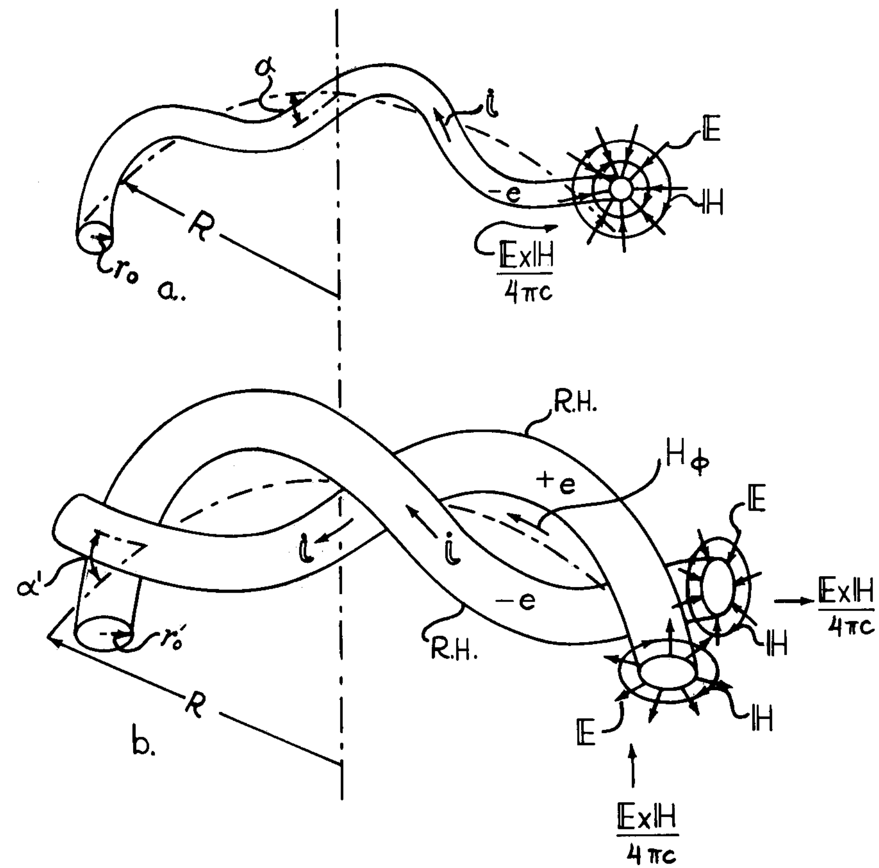

Fig. 4
A complex toroidal model. The extra charges in 4b contribute to μ but not to the spin.