IS THERE A UNIQUE CONSISTENT THEORY OF QUANTUM GRAVITY?

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IS THERE A UNIQUE CONSISTENT THEORY OF QUANTUM GRAVITY ?*

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ABSTRACT

Superstrings have been proposed as a quantum-theoretical framework for unifying all the fundamental forces, including gravity. We consider the question of whether there might be more general supersymmetric possibilities, based on higher extended objects such as membranes, jellies, etc. We argue that all the possible extended objects in all possible spacetime dimensions are quantum-mechanically inconsistent except for the ten-dimensional superstring and the eleven-dimensional supermembrane. These are also the only two such theories that contain massless gravitons, and thus that can describe gravity at low energies. It is remarkable that the range of possibilities can be narrowed down to this extent. Whether these can be further narrowed down to just one consistent theory remains open to further research.

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In recent years we haved learned that superstring theory might provide a framework for the quantum-mechanical unification of all forces, including gravity. The probable finiteness of the theory is a consequence of the 'smearing out' of the usual short-distance behaviour of quantum gravity, while at the same time infra-red problems are avoided because of supersymmetry. These observations immediately raise the question of whether other supersymmetric extended objects, such as membranes, might share similar properties to the string. It is important to study this question if one seriously believes that superstrings may provide *the* fundamental unified theory. For example, showing that nothing other than superstrings can work would be very significant from the fundamental point of view. On the other hand, if some higher-dimensional extended object could solve the main problems of quantum gravity and finiteness, we would then need to find justifications, on the basis of its predictions, for favoring one extended object over the others. The discussion of such issues is clearly beneficial from the point of view of fundamental physics.

Following the discovery of the supermembrane [1,2], generalizations to higher extended objects have been found and classified. The variables in these theories comprise d-dimensional spacetime bosonic coordinates $X^{\mu}(\tau, \vec{\sigma})$ and N spacetime spinors $\psi(\tau, \vec{\sigma})$, where $\vec{\sigma}$ denotes the p spatial coordinates of the extended object or 'p-brane' and τ is a time coordinate that parametrizes the evolution of the p-brane. Thus $p = 1, 2, 3 \cdots$ correspond to strings, membranes, jellies, etc. Classically, a bosonic p-brane can be formulated in any spacetime dimension d. However, it is remarkable that their supersymmetric extensions in the Green-Schwarz formulation that exhibits manifest *spacetime* supersymmetry is severely limited, and only the following values of the spacetime dimension d, the brane dimension p and number of supersymmetries N are allowed at the classical level:

1) $(d,p)_N = (10,1)_2, \quad (11,2)_1$

2) $(d,p)_N = (6,1)_4, \quad (7,2)_2, \quad (8,3)_1, \quad (9,4)_1, \quad (10,5)_1$

3) $(d,p)_N = (4,1)_2, \quad (5,2)_2, \quad (6,3)_2$

4) $(d,p)_N = (3,1)_2, \quad (4,2)_1$

Table 1

These restrictions follow from the requirement that the action exhibit a local fermionic 'Siegel-symmetry' which is possible only if certain spacetime Γ-matrix identities are satisfied [3]. However, the same restrictions can also be derived in a more physical way by demanding an equal number of bosonic and fermionic physical degrees of freedom (after gauge fixing). One of the consequences of this equality is the appearance of a zero-mass vacuum state through the cancellation of the vacuum energy of the bosons against that of the fermions when the vacuum is in a spacetime supersymmetric configuration. This is a prerequisite if massless states are to appear in these theories, which is necessary if the theory is to have a low-energy limit that connects with physical observations. We shall return to the question of whether these theories can describe gravity at the end of this paper.

The number of bosonic degrees of freedom is given by $(d-p-1)$, where the $p+1$ reparametrizations have been used to eliminate $p+1$ bosonic functions. The N spinors ψ are taken to be Majorana unless $d = (5,6,7)$ mod 8, in which case N must be even and the spinors satisfy pseudo-Majorana conditions. Each spinor has $D(d,n) = 2^{[d/2]}/n$ real components, where $[d/2]$ is the integer part of $d/2$ and $n=1$ unless $d = (2,6)$ mod 8, in which case we may simultaneously enforce a Weyl condition and hence $n=2$. In these cases N may be written as $N_- + N_+$, where N_- and N_+ are the numbers of left- and righthanded spinors. The number of fermionic degrees of freedom is $ND(d,n)/4$, where one factor of 1/2 arises from gauge fixing and the other 1/2 from the fact that the Majorana spinor ψ satisfies a first-order equation and hence is its own canonical momentum. Thus to achieve equal numbers of bosonic and fermionic degrees of freedom, we must satisfy $d-p-1 = ND(d,n)/4$ with only positive integers d, p, N . The only solutions turn out to be those appearing in Table 1. The numbers of degrees of freedom are thus 8, 4, 2 and 1 for the four series 1), 2), 3) and 4) respectively. It is interesting to note that these are the same as the dimensions of the division algebras for octonions, quaternions, complex numbers and real numbers, although the possible physical implications of this observation have not been understood.

Further restrictions on d and p can be expected from quantum considerations, as is familiar from string theory. In the Green-Schwarz formalism the only known approach to quantization is in the light-cone gauge. This gauge breaks Lorentz covariance, since the two spacetime coordinates $X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^{d-1})$ are singled out and eliminated through gauge choices and the solution of constraints. The full Lorentz group is, of course, still a true symmetry of the physical variables in the classical theory, albeit a non-linearly realized one. However, after quantization this may no longer be true; in other words the quantum theory might suffer from anomalies. These anomalies, which would be signalled in the light-cone approach as a failure of the closure of the Lorentz algebra, would lead in a covariant approach to the appearance of unphysical ghosts that do not decouple after quantization. This would be disastrous for the quantum theory.

Remarkably, it is possible to rule out most of the super p-brane theories listed in Table 1 even without performing a complete analysis to check the closure of the Lorentz algebra. We do this by noting that a *necessary* condition that must be satisfied by a quantum theory in which there are no Lorentz anomalies is that the degeneracies of the massless and massives states in the theory must be such as to be compatible with their being assembled into full representations of the appropriate little group. In m-dimensional Minkowski space (possibly with $d-m$ additional compactified dimensions) the little groups for massless and massive states are $SO(m-2)$ and $SO(m-1)$ respectively.

It is a standard calculation for the Green-Schwarz formulation of the ten-dimensional Type IIA superstring in light-cone gauge to show that the vacuum state of the theory comprises the supersymmetric set of $SO(8)$ *massless* multiplets $(1+8_v+28+35_v+56_v)_{\rm Boson}$ plus $(8_+ + 8_- + 56_+ + 56_-)_{\rm Fermion}$, and furthermore that the excitations fall into $SO(8)$ representations that can be assembled into full representations of the $SO(9)$ little group for *massive* states. That this can be done, even though the *manifest* symmetry in the light-cone gauge is only $SO(8)$, is a reflection of the fact that the $SO(9,1)$ Lorentz generators do indeed form a closed algebra in the quantized theory.

Less well known is that the analogue of the consistency check that we have just described (when supplemented with supersymmetry) is sufficient to establish that the superstring is inconsistent at the quantum level in its other classically-allowed dimensions, namely 6, 4 and 3. In other words one finds that the degeneracies of the massive states in these cases are not such that they can be assembled into full supersymmetric representations of the appropriate little groups. This provides a quick way to rule out these dimensions for superstrings without getting involved in the full complexities of checking the closure of the Lorentz generators [4]. This approach is very useful when we turn to the consideration of the higher-dimensional super p-branes, because now we are dealing with theories where no one has yet been able to perform the full computation of the algebra of the Lorentz generators.

To study the quantum consistency of the super p-brane theories, we exploit the fact that even though the topological possibilities for closed p-branes are much more diverse than for closed strings, it is sufficient to exhibit a failure of the Lorentz algebra for just one topology in order to rule out a particular p-brane. In each case, we choose to consider the case where the p-brane has the topology of a p-torus, residing in a spacetime that is the product of $(d-p+1)$-dimensional Minkowski space and a $(p-1)$-torus [5,6], so that $p-1$ of the p-brane dimensions are wrapped around compactified spacetime dimensions. Thus for the sequences 1), 2), 3) and 4) in Table 1, closure of the $SO(d-p,1)$ Lorentz algebra would mean that the massive states should assemble into representations of the same little groups that we discussed above for the string member of each sequence. In fact the group theory analysis for the states of any member of a given sequence contains that for its string member as a subset, so the entire sequence stands or falls with the string, as far as this consistency check is concerned. Since the strings in the sequences 2), 3) and 4) all fail, this means that *all* the super p-branes in Table 1 must fail except for the ten-dimensional superstring and the eleven-dimensional supermembrane [4,6].

Of course, showing that a theory survives this test of consistency is no substitute for doing the full analysis of verifying that the Lorentz algebra closes. However, the remarkable point is that these considerations alone are enough to rule out all except the ten-dimensional superstring and the eleven-dimensional supermembrane as possible quantum-consistent theories. So far, the eleven-dimensional supermembrane has survived all the consistency checks that we have been able to perform; further work is needed in order to establish whether it really is quantum consistent. Should it turn out that it is, the fact that the Type IIA superstring at least can be obtained from the supermembrane by a process of 'double dimensional reduction' [7] raises the intriguing possibility that the superstring may be but the tip of vastly more complex supermembrane iceberg.

There is another reason why the two members of sequence 1) seem to be preferred. When one examines the structure of the massless vacuum state itself, one finds that it is supersymmetric, with quantum numbers determined by the fermionic zero modes. Only for sequence 1) is the vacuum a supergravity multiplet; for the other sequences the multiplet describes either super Yang-Mills or super matter. In particular, for the $d = 11$

supermembrane the massless states form the $SO(9)$ multiplets $(44+84)_{\mathrm{Boson}} + 128_{\mathrm{Fermion}}$, which correspond to the fields of $d = 11$ supergravity [8]. For the $d = 10$ superstring the $SO(8)$ multiplets described earlier correspond to $d = 10$ Type IIA supergravity. Thus the sequence 1) theories are the only ones that include a graviton, and so it seems that gravity and quantum consistency go hand in hand. This is a fortunate circumstance, since our goal is to find a unified quantum theory incorporating gravity. It is indeed remarkable that the number of possible theories is so small, and that apparently not only does gravity need extended objects, but extended objects need gravity.

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