Can Inflation be Falsified?

John D. Barrow and Andrew R. Liddle

*Astronomy Centre, University of Sussex,
Falmer, Brighton BN1 9QH, U. K.*

Abstract

Despite its central role in modern cosmology, doubts are often expressed as to whether cosmological inflation is really a falsifiable theory. We distinguish two facets of inflation, one as a theory of initial conditions for the hot big bang and the other as a model for the origin of structure in the Universe. We argue that the latter can readily be excluded by observations, and that there are also a number of ways in which the former can find itself in conflict with observational data. Both aspects of the theory are indeed falsifiable.

2 Inflation as a Theory of Initial Conditions

Given that inflation was intended to supply a homogeneous, flat, monopole-free Universe, one might hope that any observation contradicting this would also exclude inflation. Unfortunately, as far as the observations go, we are no closer to knowing whether the Universe is or is not very close to flatness, nor have the observational constraints on the admissible monopole density been significantly tightened. Instead, the main developments here have been theoretical, and have served to blunt the sharpness of inflation's predictions. There now exist perfectly valid inflation models which predict a significantly open Universe [2]; ironically, Linde has even argued [4] that an open Universe is just as strong a piece of evidence in favour of inflation as is an almost flat Universe, because inflation is the only known way of creating an open Universe that is also homogeneous. Even the monopole problem is no longer clear cut. There exist perfectly valid inflation models (e.g. Ref. [3]) which can predict any monopole density from zero up to the present observational upper limits (and above). Discovery of a monopole density somewhere *below* current observational limits would also support inflation, because low monopole number densities imply a violation of causality in the absence of inflation (a minimum number density well above the present observational limits is required by the Kibble mechanism and the known properties of monopoles [5]). It is clear that the observation of a hyperbolic geometry, or a non-zero monopole density, will not falsify inflation.

String theorists expect that there might exist long-lived Planck mass relics from the Planck or string scale. Clearly, unless their production were extraordinarily suppressed below thermal expectations, these relics would soon come to dominate the density of the Universe and create a similar problem to that posed by monopoles. If such superheavy particles were found they would tell us that inflation had not occurred to dilute their abundance. Likewise, any specific quantum cosmological signature of boundary conditions at the 'initial' quantum state of the Universe would be erased and overwritten by inflation, but could persist to the present if inflation did not occur. Thus inflation, whilst good news for cosmologists, is extremely bad news for the study of Planck scale physics: any information of cosmological

events at the Planck scale is degraded to unobservably low levels by the occurrence of inflation. If you seek to provide an explanation of the Universe's structure independent of initial conditions, you cannot expect to learn anything of those initial conditions from observations of its structure. Testing any theory of Planck scale quantum gravity is always going to be harder than testing inflation.

The most decisive observational evidence against inflation would be provided by evidence that the Universe possesses large-scale rotation. Any rotation existing prior to inflation is reduced to unobservably low levels by inflation, as the various cosmic no-hair theorems demonstrate. The same is true for pre-inflationary density irregularities or gravitational-wave distortions, but whereas density perturbations (scalar modes) and gravitational waves (tensor modes) can be generated during inflation to seed the structure of the Universe in the future, rotational perturbations (vector modes) cannot. The scalar nature of the source of inflation is needed to ensure that slow evolution (slower than the Hubble rate) is possible. Moreover, the motion of scalar fields is necessarily irrotational. Only upper limits on cosmic rotation exist at present [6], but if characteristic signatures of large-scale vorticity were to be found in microwave background maps then this would be incompatible with inflation. Vorticity cannot exist without shear distortion and observable levels of shear anisotropy would also be embarrassing for inflationary theories. Interestingly, it has been shown that physically realistic initial conditions of the sort used to study inflationary universe models predict a maximum level of microwave background temperature anisotropy that is very close to the observed level on large angular scales [7]. If the signature of these homogeneous anisotropies were detected it would rule out the occurrence of inflation in the past. When the expansion is isotropic, these anisotropies, which are contributed by very long-wavelength gravitational waves, will be absent and there should only exist a thermal graviton background as a relic of the Planck scale. This would not survive inflation, but if inflation did not occur it would still lie many orders of magnitude below the sensitivity of planned gravitational-wave detectors.

The discovery of observational evidence for any non-trivial topology of the Universe at or below the horizon would indicate that inflation has not occurred in the past, as the

periodicity scale would be pushed beyond the horizon if inflation solved the homogeneity problem. Current microwave observations require any periodic topology scale to be at least of order the horizon scale [8]. The existence of complex ('fractal') topology of space-time foam on sub-Planck scales ($\ll 10^{33}$ cm) would also probably make it difficult for inflation to supply large-scale smoothness.

3 Inflation as a Theory of the Origin of Structure

In the 1980s, it became the accepted folklore that inflation predicts a scale-invariant spectrum of density perturbations, and inflation might appear to have been undermined by the discovery that it can predict a range of different spectra. In fact, the original prediction was of only approximate scale invariance (Bardeen et al.'s paper [9] was entitled *Spontaneous Creation of Almost Scale-Free Density Perturbations in an Inflationary Universe*), commensurate with the quality of the data at that time. The remarkable improvement in the quality of the astronomical data has made such generalized predictions inadequate. A higher order of predicted detail is now required. Unfortunately, at this higher level of precision, the realization that inflation makes a broader set of predictions has led to an impression that inflation can in fact predict anything at all. This is far from the truth.

As Albrecht has emphasised [10], the fact that inflationary perturbations are laid down in the distant past and evolve passively, through linear theory, up to the present endows them with distinctive generic features. Since there is invariably a growing mode which dominates long before a given scale enters the horizon, those scales short enough to exhibit oscillatory behaviour when they enter the horizon will display oscillations with fixed temporal phases. This leads ultimately to the familiar oscillatory character of the radiation angular power spectrum, such as that in Figure 1. The oscillatory prediction has been pushed strongly as a test of inflation in Ref. [11]. If the power spectrum observed by the MAP or Planck Surveyor satellites fails to display these oscillatory features (it seems likely, for instance, that the cosmic string theory predicts their absence [12]) then inflation will be in conflict with observation.
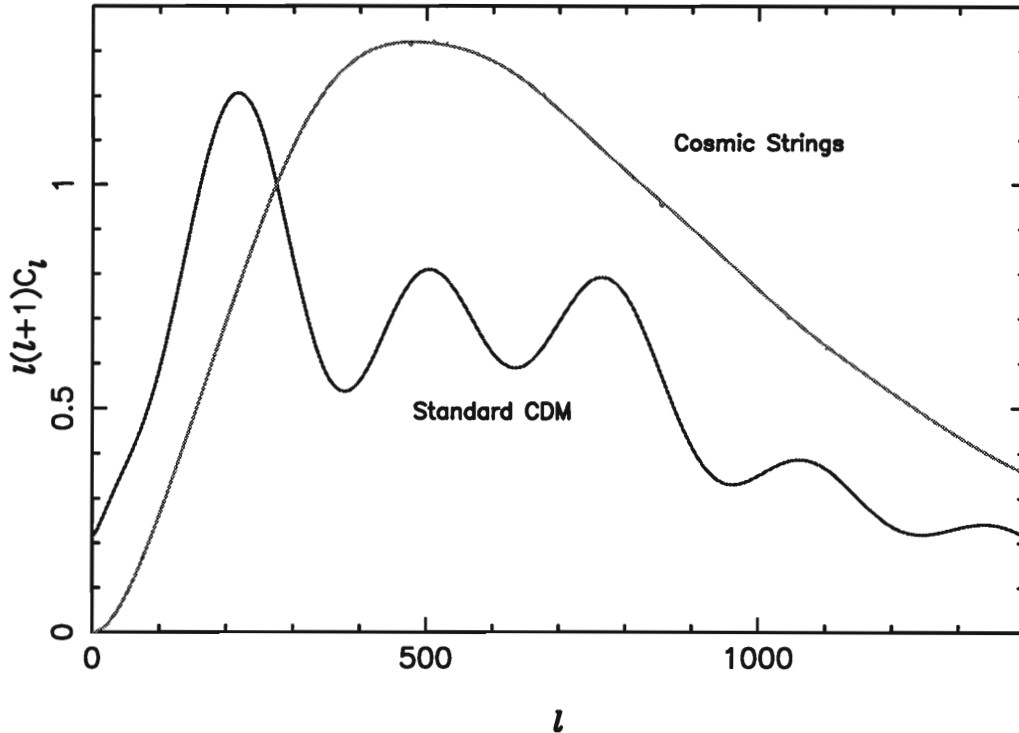
Figure 1: The radiation angular power spectrum for standard CDM, based on inflation, and that estimated for a cosmic string scenario [12]. The relative normalization is arbitrary; C_ℓ and ℓ have their usual meaning.

Ultimately, the angular power spectrum arises from linear theory, so one might question whether or not it is possible to choose a very complicated initial power spectrum, so as to mimic precisely the observed properties, at the percent or so level, of any observations that might be made in the future. Leaving aside all aesthetic prejudices about such theoretical over-engineering of the initial conditions, it is actually impossible in the case of inflation. One cannot insert arbitrarily sharp features into the power spectrum if inflation is occurring. As an example, Figure 1 shows a standard cold dark matter (CDM) spectrum, generated by a scale-invariant inflationary spectrum, alongside a cosmic string spectrum from Ref. [12]. In order to produce the latter from an inflationary model, the input power spectrum from inflation would have to look like that shown in Figure 2 (which covers ℓ from 50 upwards, the region where the string spectrum is valid). The lower panel shows that the required spectral index would have to fluctuate dramatically between approximately -4 and 6. Such values are incompatible with inflation taking place: it is impossible to tinker with the initial power spectrum sufficiently to reproduce observations of this sort. They would be fatal to
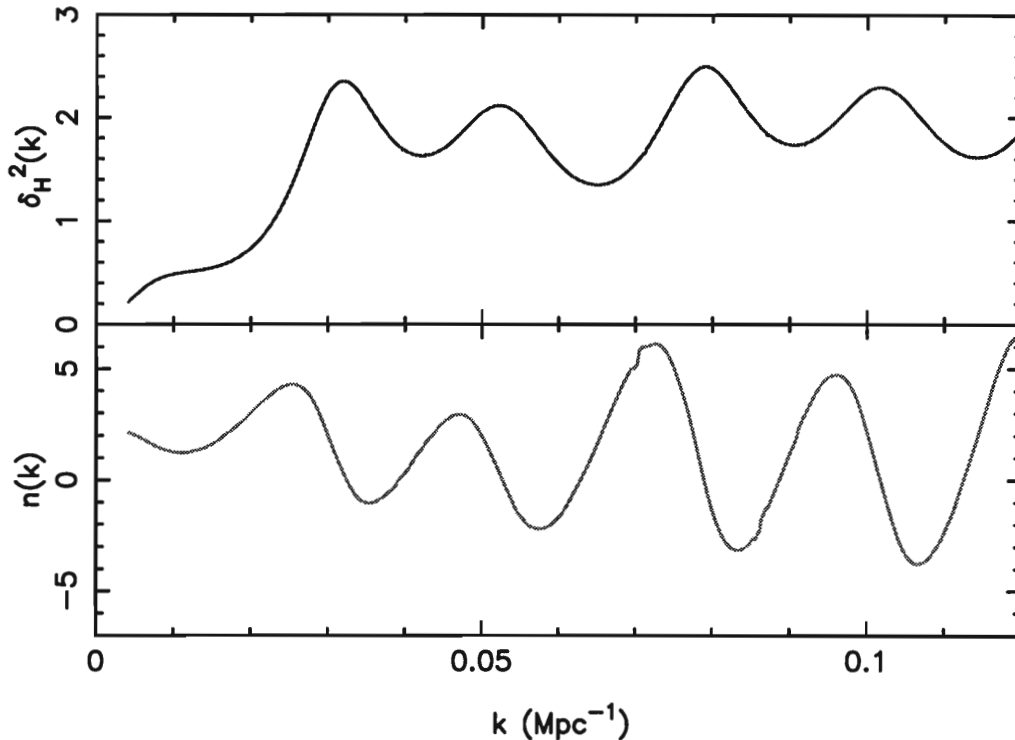
Figure 2: The required power spectrum for the inflation model to mimic the string model (arbitrarily normalized). The upper panel shows the power spectrum, defined as in Ref. [13], and the lower one the spectral index (the minor glitches are from rounding errors in the uncertain string prediction).

inflation. One cannot transform a standard CDM spectrum into a cosmic string spectrum. Moving to a low-density Universe will match the position of the peak better, but the need to erase the oscillations by introducing extra out-of-phase oscillations in the initial power spectrum to cancel them still remains.

Any proof of the existence of large-scale perturbations on scales well above the Hubble radius has already been cited as a strong indicator of inflation [14] — one can demonstrate that their existence (not yet required by existing data) would imply either that perturbations were generated acausally, or that a period of inflation occurred. Note that this is true regardless of whether or not inflation produces the perturbations. This would be supporting evidence for inflation, but it can also be turned on its head and used to rule out inflation as the explanation of structure formation. While we see that inflation must generate perturbations on scales close to the present horizon (we exist!), inflation cannot know where *our* present horizon will be. Therefore, a generic prediction of inflation is that there *must* exist

perturbations on scales larger than the Hubble length. This can be tested at decoupling using the microwave background, and conceivably at earlier times using nucleosynthesis. If these can be shown not to exist, inflation as a model of structure formation will be falsified.

The details of the perturbations may also inveigh against inflation. In single-field inflation models, there is a relation between the scalar and tensor modes that inflation produces [15]. Confirmation of this relation would be strong evidence in favour of single-field inflation. Unfortunately this relation does not survive as an equality in the case where there are many fields driving inflation, but it does survive as an inequality [16] which could certainly be violated by actual observations. The existence of vortical or magnetic field perturbations on very large scales would also be impossible to accommodate in present inflationary models, which are unable to support perturbations of a vector nature, though it is possible that models could be constructed giving magnetic perturbations by introducing new couplings [17].

The general statistics of inhomogeneous perturbations do not seem so useful as a test of inflation. In the simplest models, one has the powerful limitation that the perturbations are gaussian and adiabatic, both eminently testable, but each of these possibilities can be violated in more complex models. However, the only 'reasonable' non-gaussian models presently in existence are simple transformations of a gaussian (typically a chi-squared distribution). Observational evidence for an initial statistical distribution of fluctuations that is significantly more complex (such as that expected from topological defects) could still eliminate inflation as an explanation for structure formation.

4   Verdict

In summary, we believe that as a model of initial conditions, inflation is a very adaptable theory, but there remain several ways in which it might be ruled out. It remains to be seen whether any of these tests become decisive. But as a model of structure formation, inflation lives much more dangerously. Future observations offer the prospect of a critical test. Whether inflation created the large-scale structure of the Universe is at present not

proven, but will eventually be decided, one way or the other, beyond all reasonable doubt.

Acknowledgments

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