BLACK HOLE MEMORY

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Summary

We discuss the formation of black holes during the very early stages of a universe in which the gravitational 'constant' evolves with time. We argue that black holes will retain 'memory' of the value of the gravitational 'constant', G, at the time of their formation. Their horizon size and their thermal characteristics are determined by the value of G when they form not by the value we measure in the external universe today. The observational effects of primordial black hole explosions are therefore radically altered.

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There are many motivations for considering the formation of small black holes during the early moments of the expansion of the universe. If variations in the gravitational potential were sufficiently large then black hole formation is inevitable in the early universe(1). Such variations can arise from inhomogeneous initial conditions(2), phase transitions(3), vacuum bubble collisions(4) or the gravitational decay of cosmic string loops(5). Any black holes formed in this manner can exert a significant influence upon the future evolution of the universe. However, interest in such relics of the very early universe was first aroused by the possibility of observing the final stages of the Hawking evaporation of a black hole with a lifetime of order the Hubble age of the universe(6). This would present astronomers with direct information about a local quantum gravitational event not dissimilar to the Big Bang itself. And even the non-observation of such effects would provide important information about the smoothness of the early universe. Here, we discuss some possible consequences of the time-variation of the gravitational coupling 'constant', G, for the scenario of black hole evaporation. In this case there are potentially very strong changes in the observable aspects of black hole explosions to be expected. We shall assume that if any period of inflation occurred during the early universe then it was completed before the formation of black holes with Hawking lifetime equal to the age of the universe(7).

In order to illustrate the effects of a varying-G cosmology it is most transparent to use a scalar-tensor gravity theory(8,9,10) with gravitational action ($c = \hbar = 1$):

$$S = (16\pi)^{-1} \int [\varphi R - \varphi^{-1}\omega(\varphi)g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + 2\varphi\lambda(\varphi)](-g)^{\frac{1}{2}}\, d^4x. \qquad (1)$$

Variation of S and the matter field action with respect to the metric $g_{\mu\nu}$ and the scalar field φ gives the field equations, linking the space-time Ricci tensor, $R_{\mu\nu}$, to the covariantly conserved energy-momentum tensor of the matter fields, $T_{\mu\nu}$:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R - \lambda(\varphi)g_{\mu\nu} = \varphi^{-1}\, T_{\mu\nu} + \varphi^{-2}\omega(\varphi)(\varphi_{,\mu}\varphi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}\varphi_{,\lambda}\varphi^{,\lambda}) + \varphi^{-1}(\varphi_{;\mu\nu} - g_{\mu\nu}\Box\varphi), \qquad (2)$$

$$(3+ 2\omega(\varphi))\Box\varphi + [2\varphi^2\lambda'(\varphi) - 2\varphi\lambda(\varphi)] = T - \omega'(\varphi)\varphi_{,\mu}\varphi^{,\mu} . \qquad (3)$$

The coupling parameter $\omega(\varphi)$ and the cosmological function $\lambda(\varphi)$ must be specified to complete the theory. In the case where $\lambda = 0$ and $\omega = \omega_0$ = constant we obtain the

Brans–Dicke theory[11]. In general, the weak-field corrections to general relativity tend to zero when $\omega \to \infty$ so long as $\omega'(\varphi)\omega^{-3} \to 0$. We shall cite the Brans–Dicke case as the simplest example of a cosmology in which $G \propto \varphi^{-1}$ in what follows. We know that the observational limits, approximately $\omega_0 > 500$, from solar system tests and cosmological nucleosynthesis are very strong in this case, but there exist many other scalar–tensor theories with $\omega'(\varphi) > 0$, which are compatible with observational constraints. In such theories $\omega(\varphi)$ can be very small in the early universe, so producing significant time–evolution of $G(\varphi)$ early on, but if $\omega(\varphi)$ grows larger than about 500 by the nucleosynthesis epoch ($t \sim 1-10^3$s) then observable deviations from general relativistic Friedman models will be negligible[10]. Since we shall be interested in variations in $G(\varphi)$ which occur during the first 10^{-20}s of the expansion, when black holes small enough to evaporate today can form, there can always be significant evolution of G during this very early phase without adverse solar system or nucleosynthesis effects and our arguments are not constrained to operate within the observation restrictions of Brans–Dicke models alone.

It was shown by Hawking[12] that static vacuum black hole solutions of the Brans–Dicke equations are identical to those of general relativity. We can easily generalise this result to all scalar–tensor theories governed by (1)–(3). This is evident from equations (2)–(3). If φ is constant then the field equations reduce to those of general relativity.

If primordial black hole formation occurred during a phase of the very early universe in which the equation of state was that of radiation then black holes could form exactly as in general relativity. For simplicity we shall assume these holes to be of Schwarzschild type.

We now pose the problem: what happens to black holes during the subsequent evolution of such a universe if the gravitational 'constant' evolves in time? In general, the scalar field evolution ensures that the present value of the gravitation 'constant', $G(t_0)$, will differ from its value, $G(t_f)$, at the time, t_f, when primordial black holes formed. The problem may be stated in this form irrespective of the precise scalar–tensor coupling or the details of the cosmological evolution between t_f and t_0. Of course, given particular models for this evolution (of which the Brans–Dicke theory is the simplest) one can use other observational constraints derived from nucleosynthesis and solar system observations to constraint the magnitude of the ratio $G(t_f)/G(t_o)$.

Suppose Schwarzschild black holes of size R_f form at t_f and $G(t_f) \neq G(t_o)$. Subsequently, the scalar field generating G must remain constant over the length scale R_f whilst it evolves on larger scales at the cosmological rate. At present the black hole will have a size that is determined by the value of $G(t_f)$ at the time of its formation whilst the background universe is characterized by the value $G(t_o)$. The scalar field φ is a function of space and time which does not vary within the black hole horizon on small scales, but varies in time over larger scales.

This scenario assumes that there is no non-quantum evolution of the black holes because the φ field remains constant over the scale of their event horizons. Hence, the hole behaves like a small gravitationally bound structure in an expanding universe. The novelty of this proposal is that a black hole carries with it a "gravitational memory" of the value of the gravitational coupling, $G(t_f)$, at the time of its formation. The Hawking temperature and lifetime, τ_{bh}, of a black hole formed in the early universe at time t_f will be determined by the value of $G(t_f)$, *at the time of its formation*, not by the value of $G(t_o = \tau_{bh})$ today. The lifetime of a black hole formed when $G = G(t_f)$ is $\tau_{bh} = \alpha G^2(t_f)M^3 \sim 3\times10^{-27}\alpha\{G^2(t_f)/G^2(t_o)\}(M/1g)^3$ s where α measures the effective number of spin states of the evaporated species. This lifetime equals the present age of the universe, t_o, for holes with initial mass

$$M = \left[\frac{t_o}{\alpha G^2(t_o)}\right]^{1/3} \left[\frac{G(t_o)}{G(t_f)}\right]^{2/3}; \qquad (4)$$

that is,

$$M \sim 4.4 \times 10^{14}\ \text{gm} \times \left[\frac{G(t_o)}{G(t_f)}\right]^{2/3}, \qquad (5)$$

where the numerical term on the right-hand side of (5) gives the usual Hawking mass assuming the Hubble constant is 100 $Kms^{-1}Mpc^{-1}$ and the density of the universe is equal to the critical density(13). The Hawking temperature of these black holes becomes:

$$T_{bh} = 24\ \text{MeV} \times \{G(t_o)/G(t_f)\}^{1/3}. \qquad (6)$$

In the standard evaporation picture[6], with constant G, black holes of this temperature initially emit photons, light neutrinos, electron–positron pairs and gravitons together with Boltzmann–suppressed abundances of more massive particles like muons, taus and more massive hadrons. The detailed spectrum has been analysed by MacGibbon and Carr[13] taking into account the detailed behaviour of quarks and gluons emitted in jet–like events at energies close to the QCD confinement scale $\Lambda_{QCD} \sim 250$–300 MeV. They discuss the detailed observational features expected of the emission over the 50MeV – 1GeV range.

However, we see that the effect of a "gravitational memory" upon the evaporation process is to alter the mass and temperature of primordial black holes that will be undergoing explosive evaporation today. The change in T_{bh} by the factor $\{G(t_o)/G(t_f)\}^{1/3}$ means that the observable effects of black hole evaporations may be very different to those normally envisaged even if there is only a very short period of $G(\varphi(t))$ evolution during the first 10^{-25} s of the universe's history. For example, if $G(t_f) > 10^6 G(t_o)$, where $t_f \sim 10^{-21}$ sec then the black hole evaporation temperature is reduced to less than 0.24 MeV, below the electron rest mass, and there would no longer exist any possibility of detecting black hole explosions via the observation of radio or γ–ray bursts created by relativistic electrons and positrons evaporated from the hole spiralling in the Galactic magnetic field[13,14]. Likewise the limits deduced from the x–ray and γ–ray backgrounds[13] are significantly affected. If $G(t_f) > 10^{12} G(t_o)$ then the black hole temperature is less than 2.4 KeV and the photon emission is primarily in the x–ray band with massive particle emission restricted to very light weakly inteacting particles. Thus the possibility of time variation of the gravitation constant during the very early stages of the universe may completely change the manifestations of black hole evaporation in the present day universe. Conversely, the quoted limits on the possible abundance of black hole explosions depend crucially upon the history of $G(\varphi)$ at very early times. As a corollary, the observation of black hole explosions would allow us to draw conclusions about the gravitational lagrangian at very high energies.

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References

[1] B.J. Carr & S.W. Hawking, Mon. Not Roy. astr. Soc. **168** 399 (1974).
[2] B.J. Carr, Astrophys. J. **201**, 1 (1975). J.D. Barrow, Nature **267,** 117 (1977).
[3] M. Crawford & D.N. Schramm, Nature **298,** 538 (1982).
[4] S.W. Hawking, I. Moss & J.M. Stewart, Phys. Rev. D **26**,2681 (1982). D. La & P.J. Steinhardt, Phys. Lett. B 220, 375 (1975). J.D. Barrow, E.J. Copeland, E.W. Kolb & A.R. Liddle, Phys. Rev. D **43**, 984 (1991).
[5] A.G. Polnarev & R Zemboricz, Phys. Rev. D **43**, 1106 (1991). S.W. Hawking, Phys. Lett. B **231**, 237 (1989).
[6] S.W. Hawking, Nature 248, 30 (1974). S.W. Hawking, Comm. Math. Phys. **43,** 199 (1975).
[7] J.D. Barrow, E.J. Copeland & A.R. Liddle, Phys. Rev. D **46,** 000 (1992).
[8] C. Will, *Theory and experiment in gravitation physics*, (CUP, Cambridge 1981).
[9] R.V. Wagoner, Phys. Rev. D 1, 3209 (1970). K. Nordtvedt, Astrophys. J. **161**, 1059 (1970).
[10] J.D. Barrow & K. Maeda, Nucl. Phys. B **341**, 294 (1990).
[11] C. Brans & R.H. Dicke, Phys. Rev. **124**, 925 (1961).
[12] S.W. Hawking, Comm. Math. Phys. **25**, 167 (1972).
[13] B.J. Carr & J. MacGibbon, Astrophys. J. **371**, 447 (1991).
[14] M.J. Rees, Nature **266**, 333 (1977). R. Blandford, Mon. Not. Roy. astron. Soc. **181**, 489 (1977). B.J. Carr, Astrophys. J. **206**, 8 (1976).