GRAVITATIONAL INTERACTION AND SPONTANEOUS BREAKING OF SYMMETRIES

Essay on the Theory of Gravitation

by

Abhay Ashtekar

Enrico Fermi Institute, 5630 S. Ellis Avenue, Chicago, Il 60637.

1. G
2. G+
3. G+

Summary

An explicit example of spontaneous symmetry breaking due to gravitational interaction is given. It is shown that, in the framework of quantum field theory in curved space-times, the "dragging of inertial frame" effects lead to a spontaneous symmetry breaking in the ultra-relativistic regime. This situation is compared with those which arise in other interactions. It is pointed out that spinning-up of relativistic configurations is analogous to cooling-down of systems in solid state physics -especially in magnetism- and in high-energy physics. This analogy may turn out to be significant in the investigation of thermodynamical properties of the gravitational field.

Ideas associated with the phenomenon of spontaneous symmetry breaking have become increasingly more useful in many branches of physics over the past few years. Indeed, by now, they dominate much of the theoretical work in weak and strong interactions. Perhaps the essential reason behind the current popularity of of these ideas is the fact the basic phenomenon is <u>universal</u>: it manifests itself repeatedly at a variety of levels and for a wide spectrum of physical systems. The purpose of this essay is to point out that the gravitational interaction is not an exception. More precisely, we wish to present an explicit example in the framework of quantum field theory in curved space-times in which a symmetry is spontaneously broken and to point out similarities and differences between this example and the more familiar ones from solid-state and high energy physics. The example to be discussed has two curious features. First, it requires strong gravitational fields; the full general relativistic effects are crucial. Second, a complete quantum field theoretic analysis is possible in this case; no ad-hoc insertions from phenomenology are necessary at any stage.

We begin by recalling the basic mathematical setting in which spontaneous symmetry breaking is generally discussed. Fix a physical system. Consider the space V of solutions to the classical dynamical equation of this system. Let us suppose that the system possesses a symmetry, i.e., that there is a natural action of some Lie-group G on the space V. Consider, next, the quantum description of the system. At the kinematical level, one has simply an algebra of quantum operators and a Hilbert space of quantum states. Now, it turns out [1] that for a wide class of physical systems, the algebra of quantum operators -to be denoted by $\mathcal{A}$ - can be constructed (-using, e.g., the canonical quantization procedure-) directly from the space V. Consequently, for these systems, the action of G can be extended to $\mathcal{A}$ in a straightforward fashion. The result is a group $G(\mathcal{A})$ of automorphisms on $\mathcal{A}$. Thus, for these systems, classical symmetries always find their way to the algebra of quantum operators. However, in general, it may happen that the action of these symmetries can not be extended to the space of quantum states: the group $G(\mathcal{A})$ of automorphisms need not arise from a group of <u>unitary</u> transformations on the Hilbert space $\mathcal{H}$ of quantum states. If $G(\mathcal{A})$ can not be unitarily implemented on $\mathcal{H}$, one says that the symmetry represented by G is spontaneously broken.

Note that the mathematical description of symmetry breaking is intimately intertwined with one's choice of the Hilbert space of states, i.e., of the representation of $\mathcal{A}$. In quantum field theory, the choice of the vacuum state - or, rather, of the expectation value functional defined by the vacuum state on the algebra of quantum operators - determines this representation completely. Hence, in this case, one can discuss symmetry breaking in terms of the action of the group G on the vacuum: a given symmetry g in G is said to be spontaneously broken if and only if the vacuum expectation-value of some quantum operator fails to be invariant under the action of the automorphism induced on $\mathcal{A}$ by g . Finally, we note that, if a given symmetry is not broken, the vacuum state is necessarily left invariant by the action of the resulting group of unitary transformations on the Hilbert space $\mathcal{H}$. This fact gives rise to the more well-known statement that a symmetry is broken if the vacuum state fails to transform according to the trivial representation of the given group G .

We are now equipped to discuss our specific example. Consider a sequence of relativistic, uniformly rotating configurations labelled by, say, their total angular momentum J . Suppose, furthermore, that at a certain point J_o in the sequence, an ergo-region develops in the corresponding space-time + ~~in the corresponding space-time~~ and continues to exist in all space-times for which $J > J_o$. Physically, this corresponds to the situation in which the rotation of the source is so rapid for $J \geqslant J_o$, that the dragging of inertial frame effects are predominant in some neighborhood of the source; every observer in this neighborhood (with a time-like 4-velocity) must rotate with respect to infinity. Thus, for $J \geqslant J_o$, we are necessarily in the ultra-relativistic regime.

Fix in this sequence, a space-time with angular momentum J , and consider (test) Klein-Gordon (and/or Maxwell) fields on this space-time. Our system will consist of these fields. The space V is now just the (vector) space of solutions to the classical (Klein-Gordon and/or Maxwell) equation. The group G is now the two-parameter group (isomorphic to $SO(2)\times\mathbb{R}$) of isometries on the given space-time. Clearly, G has a natural action on the space V . Consider, next, the quantum description of fields. In this case, it is possible [2] to construct the algebra $\mathcal{A}$ of quantum operators explicitly and to show that the group G does indeed induce a (two-parameter) group $G(\mathcal{A})$ of automorphisms on this algebra - irrepective of the value J of angular momentum. The key question is of-course

---

\+ It is assumed that all space-times in the sequence are asymptotically simple. This means, in particular, that formation of horizona is excluded; we are considering rapidly rotating bodies other than black-holes.

whether or not $G(\mathcal{A})$ can be unitarily implemented on the Hilbert space $\mathcal{H}$ of quantum states.

How is this Hilbert space to be selected ? In ordinary quantum field theory in Minkowski space, one regards the classical field as an assembly of simple harmonic oscillators, introduces the (quantum) vacuum as the state in which all oscillators are in their ground state and obtains the full space of quantum states by operating repeatedly with creation operators. It turns out that for all space-times with angular momentum $J < J_0$, this procedure can in fact be repeated.[2]. Furthermore, the group $G(\mathcal{A})$ of automorphisms can indeed be unitarily implemented and the vacuum state in the resulting quantum description is invariant under the action of these unitary transformations.++ Thus, in absence of ergo-regions, there is no spontaneous breaking of symmetries.

However, the situation is drastically different when $J > J_0$, i.e., in the ultra-relativistic regime. In this case, the (conserved) energy associated with classical fields is no longer positive-definite. Hence, even at the classical level, the field can not be decomposed into simple harmonic oscillator-type modes : since negative energies are permissible, at least some of the modes must resemble oscillators in <u>repulsive</u> potentials (RHO's) rather than the usual simple harmonic oscillators (SHO's). One can quantize SHO-type modes in a straightforward manner. For other modes, however, one can not repeat the standard procedure : these must be quantized following Schrodinger quantization scheme for <u>repulsive</u> oscillators. We have carried out this quantization in detail. As a direct consequence of the fact that for repulsive oscillators the quantum Hamiltonian fails to be bounded below, it follows that there is <u>no stable</u> ground state available for RHO-type quantized modes. That is, in the resulting quantum description of fields, the vacuum state fails to be invariant under the (induced) action of the one-parameter family of automorphisms corresponding to time-translation. More precisely, on the resulting Hilbert space $\mathcal{H}$ of quantum states, the entire group $G(\mathcal{A})$ ($\approx SO(2)\times\mathbb{R}$) of automorphisms of $\mathcal{A}$ can not be unitarily represented: only the $SO(2)$ part of the group appears as the symmetry group in the full quantum picture. Thus, in presence of an ergo-region, the symmetry group $G \approx SO(2)\times\mathbb{R}$ is broken

---

++ In fact the requirement that G be unitarily implementable determines the quantum description <u>completely</u>. For details, see, e.g., ref.2

down to just SO(2).

Finally, we remark that although for concreteness we have discussed symmetry breaking using explicit Hilbert spaces of states, a more abstract statement is also possible: one can show that [3] for $J > J_0$, there exists no representation of $\mathcal{A}$ (satisfying certain weak and physically motivated conditions) by operators on a Hilbert space on which the full group G can be unitarily implemented!

How does this breaking of symmetries compare with those arising in other branches of physics? Note, first, an important difference: whereas in the present case we have used space-time symmetries, in other interactions one generally uses internal symmetries. As a consequence, further differences arise. Thus, for instance, the standard Goldstone theorem [4] is not directly applicable to the present example. Indeed, a detailed examination shows that, in the present case, symmetry breaking gives rise to only zero-energy excitations, rather than, to the usual zero rest-mass Goldstone bosons. Note, however, that, inspite of these differences, the gravitational interaction does provide as genuine an example of symmetry breaking as any other interaction: the Lagrangian for the quantum field is invariant under the action of the full two-parameter (isometry) group while the vacuum is invariant under only a one parameter sub-group of this group. Furthermore, the fact that (for the given family of space-times) there is a critical value J_0 of angular momentum below which symmetries are not broken, and, above which they are, is very reminiscent of the situation in other branches of physics. Thus, for example, in models which arise in high energy physics, it is often the case that, beyond a certain threshold energy, all symmetries are unitarily implementable and as the energy decreases below this threshold, they commence breaking one by one. [5].
Similarly, in solid-state physics -particularly, in magnetism- it is often the case that symmetries are manifest until a critical temperature and begin to break below this temperature. Thus, the spinning-up of relativistic configurations is very analogous to the cooling down of systems in solid-state physics or to the losing of energy in high energy physics.

It would be fruitful to investigate in detail whether the fact that spinning-up -rather than spinning-down- corresponds to cooling is related to the fact that, in gravitational interactions, (effective) specific heats tend to be negative. Perhaps ideas associated with spontaneous symmetry breaking will lead us to a better understanding of the thermodynamics of the gravitational field, and, more generally, provide us with a greater insight in to the nature of the gravitational interaction, just as they have for other interactions.

ACKNOWLEDGEMENTS : It is a pleasure to thank Robert Geroch, Anne Magnon-Ashtekar and Burt Ovrut for discussions.

REFERENCES

[1]. A. Ashtekar & R.Geroch, Rep. Prog. Phys. 37, 1211, (1974), especially pages 1242-1250.

[2]. A. Ashtekar & A. Magnon, Proc. Roy. Soc. (London) 346A, 375 (1975).

[3]. A. Ashtekar & A. Magnon-Ashtekar, C.R.Acad.Sc. (Paris) 280A, 875 (1975).

[4]. J. Goldstone, Nuovo Cimento 19, 154 (1961); H.Ezawa & J.A. Swieca, Comm. Math. Phys. 5, 330 (1967).

[5]. J. Weinberg, Phys. Rev. D9, 3357 (1974).

Biogrophical Data

Name : ASHTEKAR, Abhay Vasant

Address: Enrico Fermi Institute
The University of Chicago
5630 S. Ellis Ave
Chicago, Il 60637.

Education:

University of Bombay, Bombay, India.
Undergraduate student; Degree: B.Sc. (Hon). Received: (6th June 69)

University of Texas at Austin, Austin, Texas 78712. U.S.A.
Graduate student in Physics (September 69 to September 71)

University of Chicago, Chicago, Il 60637. U.S.A.
Graduate student in Physics, Field of Specialization: Relativity.
Degree: Ph.D. (Date of final oral 13 August 74)

Positions held:

Post-doctoral Research Assistant, Mathematical Institute, University of Oxford, Oxford, England; (October 74 to September 76)

Visiting Member, Max Planck Institüt für Physik und Astrophysik, München, FDR (May & June of 76)

Research Associate, Enrico Fermi Institute, 5630, S. Ellis Ave., Chicago, Il (From October 76)