# NEW RELATIVISTIC GRAVITATIONAL EFFECTS USING CHARGED PARTICLE INTERFEROMETRY

by

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## SUMMARY

A new class of gravitational effects, in the quantum interference of charged particles, are studied in electron interferometry and superconducting Josephson interferometry. These include phase shifts due to the gravitationally induced Schiff-Barnhill field, rotationally induced London moment and the modification of the Aharonov-Bohm type of phase shifts, due to the the general relativistic coupling of the electromagnetic field to the gravitational field. These effects are interesting, even from a purely theoretical point of view, because they involve an elegant interplay between gravitation, electromagnetism and quantum mechanics. But new predictions are also made which, if confirmed, would provide the first observation of relativistic gravitational effects, involving the electric charge, at the quantum mechanical level. The possibility of using these effects to detect gravitational waves is also discussed.

A beautiful experiment that demonstrated the effect of the gravitational field in interferometry, for the first time, was performed by Colella, Overhauhauser and Werner (COW)[1]. If g is the acceleration due to gravity, m the mass of the interfering particle, k the wave-number of the incoming beam and A the area enclosed by the interfering beams, then the phase shift due to the gravitational field between the upper and the lower beams which are in a vertrical plane that is predicted by non-relativistic quantum mechanics, is

$$\Delta\phi_{COW} = - m^2 g A / \hbar^2 k \tag{1}$$

This was confirmed, for the special case of neutrons, by the COW experiment.

This effect (1) is probably the only observed effect that depends on the gravitational constant <u>and</u> the Planck's constant h. We shall show in this essay that there exists, in charged particle interferometry, gravitational effects which depend, in addition to the above parameters, on the velocity of light c and the electric charge, and which may be observable. These novel effects are purely relativistic and cannot be understood non-relativistically, unlike the COW phase shift. Although the relativistic corrections to (1) have been studied previously[2,3,4] they are too small to be detected for <u>neutral</u> particles. But, as will be seen, charged particle interferometry provides us with the important advantages of choosing an electromagnetic configuration such that

(i) the non-relativistic gravitational effect, that would normally mask the relativstic effects, is cancelled and/or

(ii) the relativistic gravitational effect is enhanced by a strong electromagnetic field.

By charged particle interference we mean here, not only the interference of two coherent electron beams[5], but also the Josephson effect[6] in superconductors. The former has the advantage over neutron interferometry that, for the typically 100 keV electrons used[5], $v/c \sim 0.55$. Since for the thermal neutrons used in the COW experiment $\frac{v}{c} \sim 10^{-5}$, electron interferometry should be much more sensitive to relativistic effects. Also the Josephson effect has enabled the construction of the SQUID, which is probably the most sensitive device available now: its sensitivity, now, is limited only by the quantum uncertainty principle.

Consider first an interferometric experiment in which a horizontal, monochromatic beam of electrons is coherently split into two at O (Fig. 1) and the resulting two beams are led around the sides of a rectangle OACB, which is in a vertical plane, and are made to interfere at C. The horizontal beams OA and BC travel through identical, isolated, hollow, conducting cylinders $K_1$ and $K_2$. Both conductors are neutral so that, the only phase shift, the electron experiences, is due to the gravitational field.

Suppose now that the conductors $K_1$ and $K_2$ are joined by a neutral conducting wire W. There must exist, in any conductor that has no currents, an electric field $\vec{E}_s$, to prevent a drift of electrons due to the gravitational field $\vec{g}$.

This $\vec{E}_s$ must therefore satisfy

$$m\vec{g} + e\vec{E}_s = \vec{0}, \tag{2}$$

in the non-relativistic limit, as pointed out by Schiff and Barnhill[7]. In the present case, the field $\vec{E}_s$ in the wire would result in an electrostatic potential difference $\delta A_o$ between the conductors $K_1$ and $K_2$. Indeed,

$$\delta A_o = -\int \vec{E}_s \cdot d\vec{r} = \frac{m}{e} \int \vec{g} \cdot d\vec{r} = -\frac{mgr}{e} \tag{3}$$

where r is the distance between the conducting cylinders.

Therefore, the phase shift due to the electromagnetic field given by[8]

$$\Delta\theta = -\frac{e}{\hbar} \oint A_\mu dx^\mu , \tag{4}$$

where the integral is along the unperturbed classical trajectories along the beams[9], is non-zero. Using (3), this phase shift is

$$\Delta\theta_{SB} = -\frac{e}{\hbar} \delta A_o T = \frac{mgrs}{\hbar v} \tag{5}$$

where T is the original time of flight along BC, v the velocity and s = BC. On using the non-relativistic De Broglie relation $mv = \hbar k$, and A = rs, $\Delta\theta_{SB} = -\Delta\phi_{COW}$, i.e. the phase shift (5) due to the Schiff-Barnhill field exactly cancels the COW phase shift (1) for the electron!

Hence only the relativistic corrections to these two phase shifts will now be observable. To determine this, first note that the relativistic generalization of (2) in a stationary situation, is[10]

$$m\vec{g}\left(1 + \frac{\bar{u}^2}{2c^2}\right) + e\vec{E}_s = \vec{0} , \tag{6}$$

where $\bar{u}$ is the "root mean square velocity" of the electrons which participate in the conduction process, neglecting higher order terms. Using an argument similar to the derivation of (5), and the relativistic De Broglie relation $mv/\sqrt{1-v^2/c^2} = \hbar k$ the corresponding relativistic phase shift is, neglecting higher order terms,

$$\Delta\theta^r_{SB} \simeq -\Delta\phi_{COW}\left(1 + \frac{v^2}{2c^2} + \frac{\bar{u}^2}{2c^2}\right)$$

Also the relativistic generalization of (1) is[3] $\Delta\phi^r \simeq \Delta\phi_{COW}\left(1 + \frac{v^2}{c^2}\right)$. Therefore the total phase shift is

$$\Delta\phi^r + \Delta\theta^r_{SB} \simeq \Delta\phi_{COW}\left(\frac{v^2}{2c^2} - \frac{\bar{u}^2}{2c^2}\right). \tag{7}$$

Since $\bar{u}^2$ is temperature dependent, so is the phase shift (7)! But at room temperature, $\bar{u} \simeq$ Fermi velocity = $1.57 \times 10^6$ m/s, for copper. Then $\frac{\bar{u}^2}{c^2}$ is negligible. On the other hand for 100 keV electrons, $\frac{v^2}{c^2} \simeq 0.3$, so that (7) is an appreciable fraction of the COW phase shift for electrons. If slower electrons are used so that (7) is negligible, then the disappearance of the COW phase shift, when the two conductors are joined with a wire, which is predicted above, would be a "null" experiment to detect the gravitationally induced Schiff-Barnhill field in interferometry for the first time.

For the magnetic analog of this experiment, note that when a superconductor has angular velocity $\vec{\Omega}$ relative to local inertial frames, it develops the London moment[11], i.e. a magnetic field $\vec{B}_L$ which satisfies,

$$e\vec{B}_L + 2m\vec{\Omega} = \vec{0}, \tag{8}$$

inside the superconductor. Consider, now, electron interference in which the interferometer has angular velocity $\vec{\Omega}$, relative to a local inertial frame. There is then a phase shift due to the Sagnac effect, analogous to the phase shift due to the earth's rotation in neutron interferometry that has been detected by Werner et al.[12]. This phase shift is[13]

$$\Delta \phi_s = \frac{2 m \Omega A}{\hbar}$$

Now suppose a superconductor is inserted between the interfering beams, so that it is at rest relative to the interferometer and fills almost the entire space between the beams (Fig. 2). Then, $\vec{B}_L$ gives a phase shift

$$\Delta \theta_L = \frac{e}{\hbar} B_L A = -\frac{2 m \Omega A}{\hbar} = -\Delta \phi_s$$

using (4) and (8). Hence the two phase shifts add to zero in the non-relativistic limit! This experiment is probably best done with a superconducting ring with a Josephson junction as the interferometer. The Josephson effect due to rotation alone, in such an interferometer, has been observed.[14]

De Witt[15] has pointed out that a superconductor, at rest with respect to the distant stars, when near a rotating body, develops this London moment, because of the local Lense-Thirring precession of inertial frames. Hence the above analysis would apply when the apparatus is at rest relative to distant stars[16], with $\vec{\Omega}$ now being the Lense-Thirring precession of the local inertial frames due to the earth's rotation. In this case the special relativistic corrections to (8) and $\Delta \phi_s$ are negligible and the above experiment would be a "null" experiment to detect the general relativistic Lense-Thirring field, in principle.

We consider now the phase shift in charged particle interferometry due to the modification of an external electromagnetic field because of its coupling to the gravitational field. Since the electromagnetic field $F^{\mu\nu}$ is a purely relativistic system, its coupling to the gravitational field has to be described general relativistically. Hence it is necessary, in general, to solve the general relativistic Maxwell equations,

$$\partial_\nu ( \sqrt{-g}\, F^{\mu\nu}) = \sqrt{-g}\, j^\mu , \qquad \partial_{[\mu} F_{\nu\rho]} = 0, \tag{9}$$

subject to appropriate boundary conditions, where $g = \det(g_{\mu\nu})$, $g_{\mu\nu}$ being the metric.

Consider the special case of electron interference in the electric field produced by isolated charges on three parallel, equally spaced conducting plates, as shown in Fig. 3. The charge on the central plate is +Q, while each of the other two plates has charge -Q/2. This configuration has been chosen so that the phase shift in the absence of the gravitational field is zero, which is important for coherence and stability. The experiment consists of rotating the entire apparatus about OA from the horizontal to the vertical position and measuring the phase shift.

To determine the new field, note first that the conservation law $\partial_\mu(\sqrt{-g}j^\mu) = 0$ implies that the total charge $\int d^3x \sqrt{-g}j^0$ on each plate is unchanged. Hence, neglecting curvature effects, in a coordinate system fixed to the apparatus, the uniform charge density $\sqrt{-g}j^0$ is also unchanged. Since the situation is also stationary after the interferometer is turned, $j^i = 0$ and $\partial_0 g_{\mu\nu} = 0$. Then the solution of (9) is[17]

$$\sqrt{-g}\, F^{oi} = F^{oi}_{(o)} , \qquad F^{ij} = 0 \tag{10}$$

where $F^{oi}_{(o)}$ is the electric field in the horizontal position, whereas now the electic field is vertical. Assuming $g_{\mu\nu} = (g_{oo}, -1, -1, -1)$, and the z-axis in the vertical direction, so that $g_{oo} = (1+2\frac{gz}{c^2})$, the phase shift is

$$\Delta\theta_{\mathcal{E}} = -\frac{e}{\hbar}\int F_{oz}\, dt\, dz = -\frac{1}{4}\frac{e\mathcal{E}gTZ^2}{\hbar c^2} \qquad (Z = OB)$$

$$= -\frac{1}{4}\frac{e\mathcal{E}gAZ}{\hbar c^2 v} = -\frac{1}{4}\frac{e\mathcal{E}Z}{mc^2}\Delta\phi_{COW}$$

where $\mathcal{E} = F_{(o)oz}$ is the original electric field. Since $e\mathcal{E}\, Z/mc^2$ can be made comparable to one, $\Delta\theta_{\mathcal{E}}$ which is a purely relativistic effect, can be comparable to the COW phase shift for electrons!

There is also, in addition, the COW phase shift in this experiment. But this can be canceled using the Schiff Barnhill field, as done previously, by simply connecting the top and bottom plates in Fig. 3 by a wire. $\Delta\theta_{\mathcal{E}}$ can also be isolated by varying $\mathcal{E}$.

We consider now the problem of gravitationally coupled magnetic field, due to a steady current in a stationary, rigid solenoid. Choose a coordinate system, fixed with respect to the solenoid, in which all fields introduced here are time independent and $g_{\mu\nu} = \mathrm{diag}\,(g_{oo},-1,-1,-1)$. Hence, the conservation law $\partial_\mu(\sqrt{-g}\,j^\mu) = 0$ implies $\partial_i(\sqrt{-g}\,j^i) = 0$. Therefore, by Gauss' theorem, $\tilde{I} \equiv \int_S \sqrt{-g}\,\vec{j}.\vec{ds}$, where S is a cross section of the wire, is constant along the wire. Now $\tilde{I}$ is the rate of flow of charge per unit coordinate time. Hence, I, the rate of flow of charge per unit proper time, is not constant along the wire. Suppose the wire is thin and has uniform cross section, then $\tilde{j} \equiv \sqrt{-g}\,j$ is constant along the wire, where $j = (\delta_{ij}\; j^i\, j^j)^{1/2}$

Now, from (9), the magnetic field $B^i = \frac{1}{2}\,\epsilon^{ijk}F^{jk}$ satisfies

$$\mathrm{Curl}\,\vec{\tilde{B}} = \vec{\tilde{j}}\,, \qquad \mathrm{div}\,\vec{\tilde{B}} = \vec{g}\cdot\vec{B}_o\,, \tag{11}$$

where $\vec{\tilde{B}} = \sqrt{g_{oo}}\,\vec{B}$, $\vec{B}_o$ is the unperturbed magnetic field, and $O(g^2)$ terms are neglected. For a horizontal toroidal solenoid, or a horizontal open ended solenoid which is very long, $\vec{g}\cdot\vec{B}_o \simeq 0$. Then (11) are the usual flat space-time Maxwell equations, which give $\vec{\tilde{B}}$, if $\vec{\tilde{j}}$ is known. Suppose that the current in the coil is fed by a battery of constant e.m.f. V. By a suitable general relativistic generalization of the nonrelativistic Ohm's law $\vec{j} = \sigma\,\vec{E}$, it can be shown that $V = I_B R = \dfrac{\tilde{I}_B}{\sqrt{(g_{00})_B}} R$

where the subscript B denotes value at the battery and R is the total resistance in the circuit, including the internal resistance. Therefore $\tilde{I} = \tilde{I}_B = \sqrt{(g_{00})_B} I_0$, where $I_0 = V/R$ is the old current in the absence of the gravitational field, assuming that the e.m.f. is unaltered by gravity. Therefore $\tilde{j} = \sqrt{(g_{00})_B} j_0$. So the solution to (11) is $\vec{\tilde{B}} = \sqrt{(g_{00})_B}\ \vec{B}_0$ or

$$\vec{B} = \sqrt{\frac{(g_{00})_B}{g_{00}}}\ \vec{B}_0 = \left(1 - \frac{gz}{c^2}\right) \vec{B}_0 \qquad (12)$$

where z is the height above the battery. Hence the phase shift for an Aharonov-Bohm[8] type experiment, in which the magnetic flux between the interfering beams is provided by the flux in the above solenoid (Fig. 4), is

$$\Delta\theta_G = \frac{eB_0A}{\hbar}\left(1 - \frac{g\bar{z}}{c^2}\right) = \Delta\theta_{AB}\left(1 - \frac{g\bar{z}}{c^2}\right)$$

where $\bar{z}$ is the height of the centroid of the cross section of the coil, in the plane of the interferometer, from the battery, A is the area of cross section and $\Delta\theta_{AB}$ is the usual Aharonov-Bohm effect in the absence of gravity.

It follows, from (12), that if the vertical distance between the battery and the coil is changed by H, then the magnetic field changes by $(gH/c^2)\ B_0$. It is possible to make $B_0 \sim 2 \times 10^4$ Gauss. If $H \sim 10$ m then the change in B is $\sim 2 \times 10^{-11}$ Gauss, which should be detectable by means of a SQUID[18].

If the curvature effects on the apparatus are non-negligible, then we cannot set $g_{ij} = -\delta_{ij}$ in a coordinate system fixed to the apparatus, in general. Assuming then the more general condition $g_{\mu\nu} = \mathrm{diag}\ (1 + h_{00}, -1+h_{11}, -1+h_{22}, -1+h_{33})$ in (10), appropriately lowering the indices and neglecting $O(h^2)$,

$$F_{0z} = \left(1 + \frac{h_{00}}{2} + \frac{h_{11}}{2} + \frac{h_{22}}{2} - \frac{h_{33}}{2}\right)\mathcal{E}$$

This suggests that a gravity wave may produce an oscillating phase shift in a device of this type. A careful analysis, taking into account the time dependent deformations of the apparatus, is needed to determine this effect.

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16 This experiment may be done, in principle, by placing the interferometer on a platform which is made non rotating relative to the distant stars by means of telescopes and observing the phase shift, due to the Lense-Thirring precession of local inertial frames, which would disappear when the superconductor is inserted. But it does not seem to have the necessary sensitivity, at present, to detect the Lense-Thirring field due to the earth.

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18 It would be necessary to shield the SQUID, from the earth's magnetic field $B_e$ by superconductors, because of fluctuations in $B_e$. Also fluctuations in the battery e.m.f. can be eliminated by using two solenoids at different heights, carrying currents from the same power source, and taking the ratio of their simultaneous magnetic fields. I thank Andre Drukier for suggesting this to me.

## Figure Captions

### Figure 1

When the neutral, hollow, cylindrical conductors $K_1$ and $K_2$ are joined by a conducting wire W, there must be a transfer of charge between them to account for the Schiff-Barnhill electric field in W. Then $K_1$ and $K_2$, and therefore the beams OA and BC through them, will be at different electrostatic potentials. The resulting phase shift, exactly cancels the phase shift due to the different gravitational potentials at OA and BC, in the non relativistic limit. The variations of each potential along OB and AC are the same, due to symmetry.

### Figure 2

The superconductor nearly fills the space between the interfering electron beams, and the entire system is given an angular velocity. Then the Sagnac effect in interference, due to the rotation is entirely canceled by the Aharonov-Bohm phase shift due to the London moment, in the non relativistic limit.

### Figure 3

The electric field, due to the charges shown, is modified by the general relativistic coupling to the gravitational field. This gives a phase shift between the beams OBC and OAC, which is purely relativistic and has no Newtonian analog.

### Figure 4

Horizontal toroidal solenoid carrying current from a battery of constant e.m.f. and the Aharonov-Bohm experiment, with the magnetic flux between the interfering beams provided by this solenoid. There is a phase shift due to the general relativistic coupling of the magnetic field and the current to the gravitational field, which has no Newtonian analog.

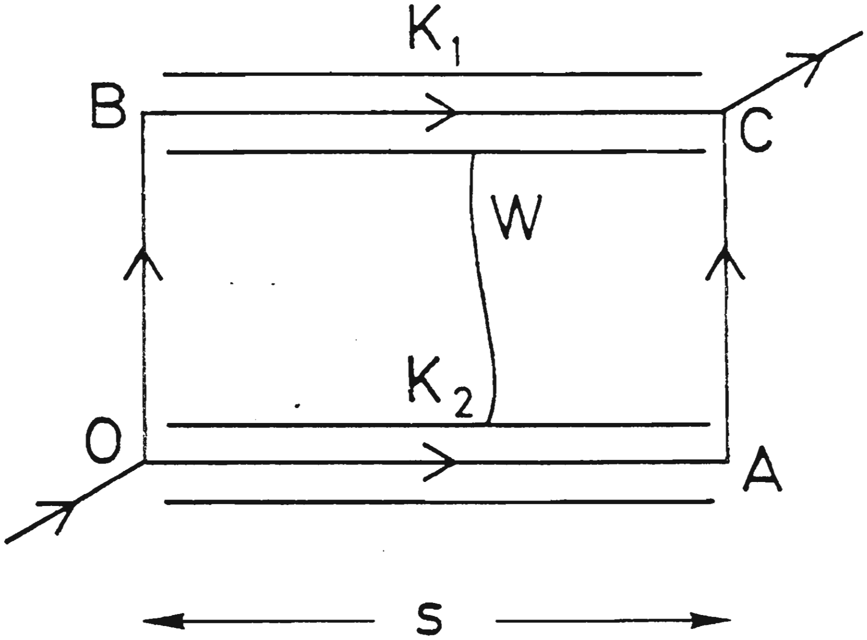


Fig. 1

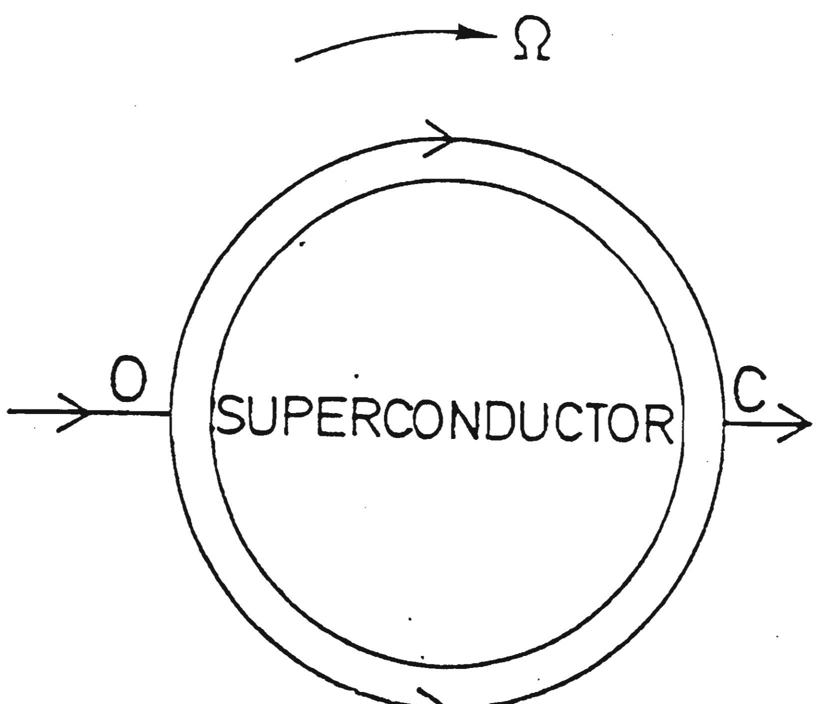


Fig. 2

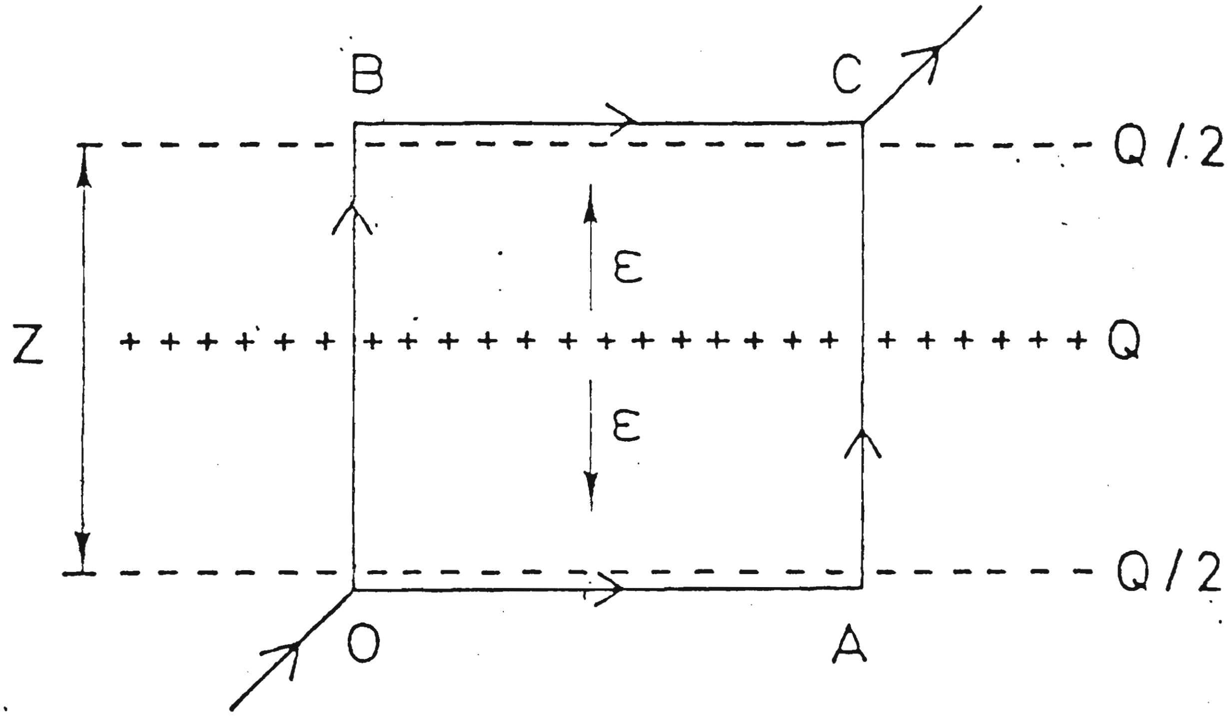


Fig. 3

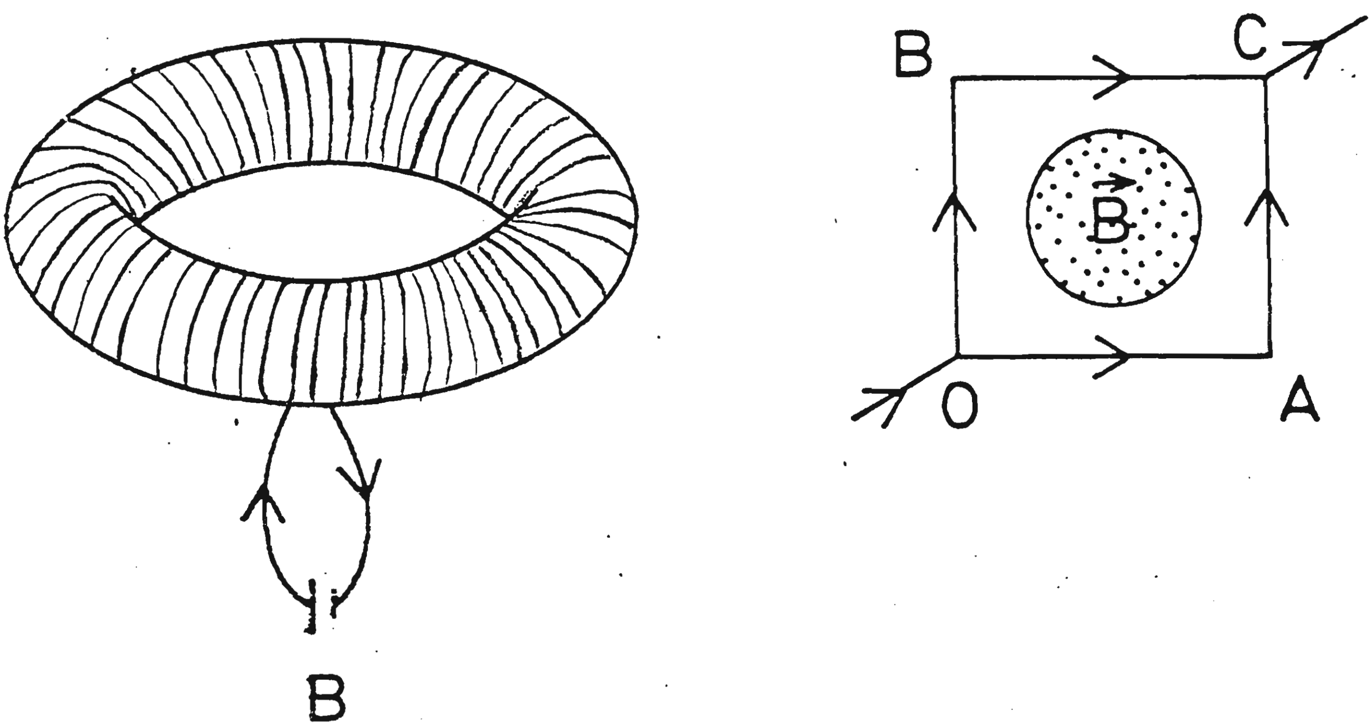


Fig. 4