Gravitationally Induced Neutrino–Oscillation Phases

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In this essay, we introduce a new effect of gravitationally induced quantum mechanical phases in neutrino oscillations. These phases arise from an hitherto unexplored interplay of gravitation and the principle of the linear superposition of quantum mechanics. In the neighborhood of a 1.4 solar–mass neutron star, gravitationally induced quantum mechanical phases are roughly 20% of their kinematical counterparts. When this information is coupled with the mass square differences implied by the existing neutrino–oscillation data we find that the new effect may have profound consequences for type-II supernova evolution.

Neutrinos [1,2] play a profound role on almost all length scales. These scales range from the evolution of the nuclei to the evolution of biological structures [3], to the evolution of the stars, the galaxies, and the universe [4,5]. The detection [6] of an ancient neutrino burst on February 23, 1987 provided a dramatic confirmation of the essential elements of the physics of type-II supernovae [7] and placed an upper limit of about 10 eV on the mass of the electron neutrino [8]. The detection of solar neutrinos in terrestrial detectors provides a similarly convincing confirmation of our understanding of the solar evolution [9]. These important observational triumphs mark the beginning of a new era: The era of neutrino astronomy. The observations on the solar neutrinos have already given birth to the more controlled accelerator– and reactor–based neutrino oscillation experiments.

For decades the solar neutrino anomaly [10] has indicated that the neutrino flavor eigenstates may be a linear superposition of mass eigenstates [11]. This

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view has been further strengthened by the data on atmospheric neutrinos [12], and most recently by the observation of excess $\overline{\nu}_e$ observed at the Liquid Scintillator Neutrino Detector (LSND) Neutrino Oscillation Experiment (NOE) at LAMPF [13]. The excess events observed at LSND NOE have been tentatively interpreted as $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$ oscillations [13].

At the present time, (a) the question of neutrino masses [14], (b) the relation of the neutrinos to the spacetime symmetries as manifested in Dirac [15], or Majorana [16–18], or other fundamental constructs appropriate to the neutral particles [19], (c) the question of CP violation in the leptonic sector [21], and (d) other related issues that extend the physics beyond the standard model [22], are being probed with intense theoretical and experimental vigor.

Gravitation plays no direct role on neutrino oscillations in the existing literature.[3] In this essay we shall introduce the notion of gravitationally induced quantum mechanical neutrino–oscillation phases and discuss the possibility of their observation for type-II supernova.

A bit of history is necessary at this point. In one of the classic experiments of physics, Colella, Overhauser, and Werner (COW) established that quantum mechanics and gravitation, despite the well–known conceptual problems, behave in a manner expected for any other interaction [26]. Given the fact that the experiment involved thermal neutrons (non-relativistic quantum realm) and the Earth's gravitational field (weak gravity), this may not be too unexpected. Nevertheless, theoretical investigation of this elegant experiment provides a deep understanding of gravitation in the context of quantum mechanics [27]. From a formal point of view the COW experiment studies the effect of gravitation on the quantum mechanical evolution of a single–mass eigenstate. For neutrinos, when we extend the COW–like considerations to the linear superposition of mass eigenstates, a new gravitationally induced quantum mechanical effect emerges. Despite the similarities between the COW effect and the new effect introduced here, there are important conceptual differences between the two effects. These differences shall also be enumerated at the appropriate place in this essay. In a certain sense (to become precise below), the two effects shall be seen to be complementary.

Let us assume that in the "creation region," $\mathcal{R}_c$, located at $\vec{r}_c$, [4] a weak eigenstate with energy E_ν, denoted by $|\nu_\ell,\, \mathcal{R}_c\rangle$, is produced with the clock set to $t = t_c$. Each of the three neutrino mass eigenstates shall be represented by

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[3] However, see (a) Ref. [23], where Violation of Equivalence Principle (VEP) in the context of neutrino oscillation experiments is considered; (b) the work of Goldman *et al.* on "Kaons, Quantum Mechanics, and Gravity" in Ref. [24] (Ref. [24] also contains references to many classic works on the subject); and (c) the work of J. Anandan on the gravitational phase operator [25].

[4] The creation region $\mathcal{R}_c$ is assumed fixed in the global coordinate system.

$|\nu_\imath\rangle$; $\imath = 1, 2, 3$. So we have the linear superposition:

$$|\nu_\ell,\, \mathcal{R}_c\rangle = \sum_{\imath=1,2,3} U_{\ell\imath}\, |\nu_\imath\rangle \quad , \tag{1}$$

where $\ell = e, \mu, \tau$ represents the weak flavor eigenstates (corresponding to electron, muon, and tau neutrinos respectively).[5] The $|\nu_1\rangle$, $|\nu_2\rangle$, and $|\nu_3\rangle$ correspond to the three mass eigenstates of masses m_1, m_2, and m_3, respectively. Under the already–indicated assumptions the 3×3 unitary mixing matrix $U_{\ell\imath}$ may be parameterized by three angles and reads [20, Eq. 6.21 with the CP phase $\delta = 0$]:

$$U(\theta,\, \beta,\, \psi) = \begin{pmatrix} c_\theta\, c_\beta & s_\theta\, c_\beta & s_\beta \\ -\, c_\theta\, s_\beta\, s_\psi \;-\; s_\theta\, c_\psi & c_\theta\, c_\psi \;-\; s_\theta\, s_\beta\, s_\psi & c_\beta\, s_\psi \\ -\, c_\theta\, s_\beta\, c_\psi \;+\; s_\theta\, s_\psi & -\, s_\theta\, s_\beta\, c_\psi \;-\; c_\theta\, s_\psi & c_\beta\, c_\psi \end{pmatrix} \quad , \tag{2}$$

where $c_\xi \;=\; \cos(\xi)$, $s_\xi \;=\; \sin(\xi)$, with $\xi = \theta,\, \beta,\, \psi$.

At a later time $t = t_d > t_c$, we wish to study the weak flavor eigenstate in the "detector region," $\mathcal{R}_d$, located at $\vec{r}_d$.[6] The neutrino evolution from $\mathcal{R}_c$ to $\mathcal{R}_d$ is given by the expression:

$$|\nu_\ell,\, \mathcal{R}_d\rangle = \exp\left(-\frac{i}{\hbar} \int_{t_c}^{t_d} H \mathrm{dt} \;+\; \frac{i}{\hbar} \int_{\vec{r}_c}^{\vec{r}_d} \vec{P} \cdot d\vec{x} \right) |\nu_\ell,\, \mathcal{R}_c\rangle \quad . \tag{3}$$

Here H is the time translation operator (the Hamiltonian) associated with the system; $\vec{P}$ is the operator for spatial translations (the momentum operator), and $\left[H(t, \vec{x}),\, \vec{P}(t, \vec{x})\right] = 0$. Consider that both $\mathcal{R}_c$ and $\mathcal{R}_d$ are located in the Schwarzschild gravitational environment [4] of a spherically symmetric object of mass M. The direction of neutrino propagation is along $\vec{L} = \vec{r}_d - \vec{r}_c$.

We now confine to the weak field limit, and neglect the spin–dependent terms for the present, as we do not wish to study effects of the astrophysical magnetic

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[5] We shall assume that CP is not violated for the purposes of this analysis. Neutrinos shall be assumed to be of the Dirac type (for a recent analysis of various quantum field theoretic possibilities for the description of neutral particles of spin–1/2 and higher, and their relation with space–time symmetries, see Ref. [19] and references therein). In addition, we shall assume that both ν_ℓ and ν_m are relativistic in the frame of the experimenter.

[6] Like the creation region, the detection region $\mathcal{R}_d$ too is fixed in the global coordinate system.

fields, or the effects of interaction between spin (of neutrino) and angular momentum (of the astrophysical object). Under these conditions, the work of Stodolsky implies [28]:

$$\exp\left(-\frac{i}{\hbar}\int_{t_c}^{t_d} H\mathrm{d}t + \frac{i}{\hbar}\int_{\vec{r}_c}^{\vec{r}_d} \vec{P}\cdot d\vec{x}\right)|\nu_\imath\rangle$$

$$= \exp\left[-\frac{i}{\hbar}\int_{\mathcal{R}_c}^{\mathcal{R}_d}\left(\eta_{\mu\nu}+\frac{1}{2}h_{\mu\nu}\right)p_\imath^\mu\,\mathrm{d}x^\nu\right]|\nu_\imath\rangle \quad , \qquad (4)$$

where $h_{\mu\nu} = g^W_{\mu\nu} - \eta_{\mu\nu}$; $g^W_{\mu\nu}$ is the Schwarzschild space–time metric in the weak field limit and $\eta_{\mu\nu}$ is the flat space–time metric. In addition, $h_{\mu_\nu} = 2\,\phi\,\delta_{\mu\nu}$ with the dimensionless gravitational potential $\phi = -\,GM/(c^2\,r)$. Further, it shall be noted that $p^\mu_\imath$ is the four–momentum of special relativity with $p^\mu \equiv m\,dx^\mu/ds_0$ in the notation of Ref. [28]. The limits of applicability of this formalism are further enumerated in the above–cited paper of Stodolsky. To avoid notational confusion we remind the reader that $p_\imath \equiv |\vec{p}_\imath|$ — the subscript $\imath$ identifies the mass eigenstate, and does not refer to $\imath$th component of the momentum vector.

We now calculate the "neutrino oscillation probability" from a state $|\nu_\ell,\,\mathcal{R}_c\rangle$ to another state $|\nu_{\ell'},\,\mathcal{R}_d\rangle$ following closely the standard arguments, appropriately adapted to the present situation [20,29,30]. The oscillation probability is obtained by calculating the projection $\langle\nu_{\ell'},\,\mathcal{R}_d|\nu_\mu,\,\mathcal{R}_c\rangle$, i.e., the amplitude for $|\nu_\ell,\,\mathcal{R}_c\rangle \rightarrow |\nu_{\ell'},\,\mathcal{R}_d\rangle$, and then multiplying it by its complex conjugate. An algebraic exercise that (a) exploits the unitarity of the neutrino mixing matrix $U(\theta,\,\beta,\,\psi)$, (b) exploits orthonormality of the mass eigenstates, (c) exploits certain trigonometric identities, and (d) takes care of the fact that now dx and dt are related by

$$dx \simeq \left[1-\left(\frac{2GM}{c^2 r}\right)\right]c\,dt \quad , \qquad (5)$$

yields:

$$\begin{aligned}\mathcal{P}\Big[|\nu_\ell,\mathcal{R}_c\rangle \rightarrow |\nu_{\ell'},\mathcal{R}_d\rangle\Big] = \delta_{\ell\ell'} &-4\,U_{\ell'1}\,U_{\ell 1}\,U_{\ell'2}\,U_{\ell 2}\,\sin^2\Big[\varphi^0_{21}+\varphi^G_{21}\Big]\\ &-4\,U_{\ell'1}\,U_{\ell 1}\,U_{\ell'3}\,U_{\ell 3}\,\sin^2\Big[\varphi^0_{31}+\varphi^G_{31}\Big]\\ &-4\,U_{\ell'2}\,U_{\ell 2}\,U_{\ell'3}\,U_{\ell 3}\,\sin^2\Big[\varphi^0_{32}+\varphi^G_{32}\Big] \quad .(6)\end{aligned}$$

The arguments of $\sin^2(\cdots)$ in the neutrino oscillation probability now contain two types of phases. The usual *kinematic phase*, denoted here by $\varphi^0_{\jmath\imath}$, and defined as

$$\varphi^0_{\jmath\imath} \equiv \frac{c^3}{2\hbar}\frac{|\vec{r}_d - \vec{r}_c|\Delta m^2_{\jmath\imath}}{E} = \frac{c^3}{2\hbar}\frac{L\,\Delta m^2_{\jmath\imath}}{E} \quad ; \tag{7}$$

and the new *gravitationally induced quantum mechanical phase*, denoted here by $\varphi^G_{\jmath\imath}$, and defined as

$$\varphi^G_{\jmath\imath} \equiv \frac{GMc}{2\hbar}\left[\int_{\vec{r}_c}^{\vec{r}_d} \frac{\mathrm{dL}}{r}\right]\frac{\Delta m^2_{\jmath\imath}}{E} \quad . \tag{8}$$

It is readily seen that the gravitationally induced phases and the kinematic phases, for neutrino oscillations, are related via the identity

$$\varphi^G_{\jmath\imath} = \langle\phi\rangle\,\varphi^0_{\jmath\imath} \quad , \tag{9}$$

where $\langle\phi\rangle$ is the average dimensionless gravitational potential over the semi-classical neutrino path

$$\langle\phi\rangle \equiv \frac{1}{L}\int_{\vec{r}_c}^{\vec{r}_d} dL\,\frac{GM}{c^2 r} \quad . \tag{10}$$

Two immediate questions seem relevant. First, what are the conceptual similarities, and differences, between the neutron interferometer experiment of COW and the phenomenon of neutrino oscillations in the presence of gravity? Second, what are the astronomical, or experimental chances, of detecting the gravitationally induced modification to neutrino oscillations? We shall discuss these two questions in turn, and then proceed to explore the astrophysical consequences of our study.

The COW experiment studies the effect of gravitation on a *single mass eigenstate* (i.e., neutron of mass $m_n \simeq 940$ MeV). The observable physical effect arises because *spatially distinct* parts (spatial spread $\simeq$ a few cm) of the wave function pick up different gravitationally induced quantum mechanical phases. In the case of neutrino oscillations the gravitationally induced modification to the $|\nu_\ell\rangle \rightarrow |\nu_{\ell'}\rangle$ oscillations arises from the difference in phase that *each of the different mass eigenstates* picks in a given gravitational environment. *No spatial spread* of the wave function is required. Thus, the formal difference between the effect induced by gravity in the COW experiment, and the one induced in neutrino oscillations, is that in the former the different strengths of the gravitational field at different locations (and interacting with the different superimposed amplitudes of the wave function associated with a neutron mass eigenstate) manifest into a physically observable result; whereas in the latter the different gravitational interaction energies–momenta of the respective mass eigenstates leave their trace in the gravitationally induced quantum mechanical phases. The magnitude of the two effects is also affected by the non-relativistic nature of the neutrons in the COW experiment, and the extreme relativistic nature of the neutrinos.

An important question in the context of our present discussion is related to the observability of the new effect. Let us again note that in the COW experiment the relevant quantity, apart from Earth's gravitational field, is the product of mass of neutron and the (vertical) spatial spread of the wave function. For neutrino mass eigenstate of the order of an eV (nine orders of magnitude below the neutron mass) one may expect that if the flavor eigenstate of a neutrino is allowed to travel a (vertical or horizontal!) distance of the order of 10^9 cm (actually ten to a hundred times less than this will do because of the fact that the COW experiment saw a shift in fringes by a rather large number) one may see gravitationally induced effect on neutrino oscillations in terrestrial experiments (such as those involving atmospheric neutrinos [12]). However, this is not so. The reason is that the COW experiment used thermal neutrons (i.e., non-relativistic particles) and hence the effect was proportional to the *mass* of the neutron. For the MeV–GeV neutrinos (i.e., relativistic particles), as is the case for the accelerator and the atmospheric neutrinos, the effect is proportional to *mass squared differences.* As a result, the effect may only be seen in astrophysical environments.

The above considerations complete the formal aspects of this paper. We now turn to what is essentially a back–of–the–envelope exploration of the astrophysical consequences.

In the context of supernova explosions, and the problem of obtaining successful explosions, we follow Colgate *et al.* [7] and assume that the matter next to the neutron star is heated by neutrinos from the cooling neutron star. Colgate *et al.* note that in some models "this results in strong, large scale convective flows in the gravitational field of the neutron star that can drive successful, albeit weak, explosions." Now we recall that the energy flux in each of the electron neutrinos and antineutrinos is about $L_{\nu_e} \approx L_{\overline{\nu}_e} \approx \text{few} \times 10^{52}$ ergs s^{-1}, with comparable fluxes of ν_μ, $\overline{\nu}_\mu$, ν_τ, $\overline{\nu}_\tau$ [31]. A particularly relevant fact about these fluxes is that while average energy of ν_e is about 10 MeV, the average energy of other neutrinos may be higher by a factor of about 2 to 3. Because of the extremely large fluxes, and different energies (and hence different cross sections) associated with the neutrinos, even a small variation in the neutrino oscillation probabilities may profoundly affect the success of the supernova explosion provided at least one of the $\lambda_{\jmath\imath}^{\rm osc}$ has appropriate length scale.[7]

The existing experiments imply that the two independent mass square differences [32] that define $\varphi_{\jmath\imath}^0$ are as follows: $\Delta m_{21}^2 \simeq 10^{-2}$ eV^2 if the zenith-angle dependence of the atmospheric neutrino anomaly is explained by neutrino oscillations (or [33], $\Delta m_{21}^2 \simeq 10^{-5}$ eV^2 if the energy dependence of the solar neutrino deficit is explained by invoking the MSW phenomenon [34]) and $\Delta m_{32}^2 \simeq 0.5$ eV^2 (and opposite these values if the inverted mass hierarchy is considered [35]). Taking the typical energy of a supernova neutrino to be 10 MeV, these mass square differences yield oscillation lengths $\lambda_{21}^{\rm osc} \simeq 5$ km (or, $\lambda_{21}^{\rm osc} \simeq 5 \times 10^3$ km) and $\lambda_{32}^{\rm osc} \simeq 100$ m. These length scales are certainly relevant to the supernova–evolution processes of *neutrino diffusion*, *neutrino trapping*, and *neutrino heating.*[8] These length scales, and hence the associated supernova processes (and very importantly the compatibility arguments between terrestrial neutrino oscillations and supernova evolution [33]), will be altered if we find $\varphi_{\jmath\imath}^G$ equals a few percent of $\varphi_{\jmath\imath}^0$.

To see if the neutrino oscillation phases can be altered at a level of a few percent in the neighborhood of the neutron star we consider the radially outward motion of a neutrino, and set $\vec{r}_d = \alpha\,\vec{r}_c$, $1 < \alpha \leq \infty$, then we find

$$\left(\varphi_{\jmath\imath}^G\right)_{\parallel} = \frac{GMc}{2\hbar}\frac{\Delta m_{\jmath\imath}^2}{E}\ln(\alpha) = \left[\frac{GM}{c^2 r_c}\,\frac{\ln\alpha}{\alpha - 1}\right]\varphi_{\jmath\imath}^0 \quad . \tag{11}$$

For motion transverse to the radial direction (and in the *vicinity* of $\vec{r}_c$), the corresponding expression is

$$\left(\varphi_{\jmath\imath}^G\right)_{\perp} = \frac{GMc}{2\hbar}\frac{\Delta m_{\jmath\imath}^2}{E}\frac{|\vec{r}_d - \vec{r}_c|}{r_c} = \left[\frac{GM}{c^2 r_c}\right]\varphi_{\jmath\imath}^0 \quad . \tag{12}$$

Taking M to be 1.4 solar mass, $1.4\,M_\odot$, and $r_c = 10$ km, we have

$$\left(\varphi_{\jmath\imath}^G\right)_{\parallel} = 0.21\left(\frac{\ln(\alpha)}{\alpha - 1}\right)\varphi_{\jmath\imath}^0, \quad \left(\varphi_{\jmath\imath}^G\right)_{\perp} = 0.21\,\varphi_{\jmath\imath}^0 \quad . \tag{13}$$

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[7] In neutrino oscillation literature “oscillation length” is defined as

$$\lambda_{\jmath\imath}^{\rm osc} = \frac{2\pi}{\eta}\frac{E}{\Delta m_{\jmath\imath}^2} \quad ,$$

where $\Delta m_{\jmath\imath}^2$ are measured in eV^2, E in MeV, and $\eta = 1.27$.

[8] For details on neutrino heating in the context of supernova explosion we refer the interested reader to Ref. [7], and on supernova–evolution processes of neutrino diffusion and neutrino trapping the reader may find Ref. [36] very valuable.

We thus find that for a 1.4 solar mass neutron star, with a radius of ten kilometers, $\varphi_{\jmath\imath}^{G}$ is about twenty percent of the $\varphi_{\jmath\imath}^{0}$. In astrophysical situations matter effects, and the presence of magnetic fields (if neutrinos have non–zero magnetic moments), will further alter neutrino oscillations [34,37,38]. The gravitationally induced neutrino–oscillation phases arise from an hitherto unexplored interplay of gravitation and the principle of the linear superposition of quantum mechanics and cannot be ignored in many astrophysical environments.

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