

On a New Non-Geometric Element in Gravity

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Abstract

In this essay a generalized notion of flavor-oscillation clocks is introduced. The generalization contains the element that various superimposed mass eigenstates may have different relative orientation of the component of their spin with respect to the rotational axis of the the gravitational source. It is found that these quantum mechanical clocks do not always redshift identically when moved from the gravitational environment of a non-rotating source to the field of a rotating source. The non-geometric contributions to the redshifts may be interpreted as quantum mechanically induced fluctuations over a geometric structure of space-time.

Empirically observed equality of the inertial and gravitational masses leads to the theory of general relativity [1]. This theory of gravity lends itself to a geometric interpretation. In the framework of this theory, all clocks, independent of their workings, redshift in exactly the same manner for a given source. It is further assumed that when these clocks move from the gravitational environment of one source to another, identically running clocks run identically.

The primary purpose of this essay is to introduce the notion of the *generalized flavor-oscillation clocks*, and study their evolution in a weak gravitational environment of a *rotating* source. In the process we will uncover an inherently non-geometric aspect in gravity. Specifically, we will construct two intrinsically quantum mechanical clocks that do not redshift identically when introduced in the gravitational environment of a *rotating* source.

It is a direct consequence of (a) the 1939 paper of Wigner [2], and (b) the fact that *locally* space-time carries Poincaré symmetries; that a general quantum test particle, for evolution over “sufficiently small” space-time distances,² can be described by

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² For a global evolution one may need to consider test particles that are characterized by Casimir invariants associated with global space-time symmetries of the

$$|Q\rangle \equiv \sum_{k,j} A_{kj} |m_k, s_j; \vec{p}_k\rangle, \quad s_j = -s\hbar, (-s+1)\hbar, \dots, (s-1)\hbar, +s\hbar \quad ,$$

$$k = 1, 2, \dots, n \quad , \quad (1)$$

with A_{kj} as some appropriately chosen superposition coefficients. That is, a general quantum test particle is described by a linear superposition of the Casimir invariants associated with the Poincaré group — the m_k are the masses, s_j are the $(2s+1)$ spin projections (along the \vec{J} , for convenience) of spin s , and \vec{p}_k is the momentum of the k th mass eigenstate. To avoid certain complicated conceptual questions, I have refrained from summing over s (which takes integral and half-integral values).

The quantum test particle as introduced here is a slight generalization of the particles introduced by Wigner. Wigner's 1939 paper suggests that quantum particles are to be specified by the Casimir invariants associated with the Poincaré group. These are the $|m_k, s_j; \vec{p}_k\rangle$ that appear in Eq. (1). The notion of a quantum test particle as presented in Eq. (1) is a generalization based on the existence of empirically observed particles that are a linear superposition of mass eigenstates. The neutral $K^0-\bar{K}^0$ mesons, and the weak-flavor eigenstates of neutrinos as suggested by various data [4], fall in a subclass with, $\sum_{k,j} \rightarrow \sum_k$, in Eq. (1). With the existing laser technology, atomic systems can be easily constructed in a state similar to that given by Eq. (1), with m_k replaced by E_k (E_k being an atomic state). For the sake of simplicity, we will set $\vec{p}_k = \vec{0}$.

Definition of a general quantum test particle via Eq. (1) allows us to introduce two class of “flavors;” the first class for which the sum in Eq. (1) involves a *single* spin-projection independent of m_k . For the second class, the sum in Eq. (1) must contain at least *two distinct* spin-projections. Eqs. (5) and (6), below, provide an example of the first and second class, respectively.

It will be seen that by making appropriate “flavor measurements” one can use the flavor states to make flavor-oscillation clocks. We will show that flavor states of both classes redshift identically in the gravitational field of a spherically symmetric mass — i.e., as expected on the basis of the geometric interpretation of general relativity. However, when these identically redshifting clocks are introduced in the gravitational field of a rotating source, a splitting in the redshift of the flavor-oscillation clocks of the second class takes place with respect to the flavor-oscillation clocks of the first class.

We will consider the flavor states to be at rest at a fixed position in the

relevant gravitational source *a la* Feza Gürsey [3].

gravitational environment.³ Therefore, for the situation under consideration one need not worry about whether the particles are described by Klein–Gordon equation, or Dirac equation, or an equation for some higher spin [6].⁴ It will suffice to know that each mass eigenstate has $(2s + 1)$ spinorial degrees of freedom, and that each of these degrees of freedom evolves in time as

$$|m_k, s_j\rangle \rightarrow \exp\left[-\frac{iHt}{\hbar}\right] |m_k, s_j\rangle \quad . \quad (2)$$

The redshift of the flavor–oscillation clocks is determined by the gravitationally induced relative phases between various mass eigenstates. Each of the mass eigenstates, $|m_k, s_j\rangle$, picks up a gravitationally induced phase. This phase, in general, depends on m_k , and the relative orientation of s_j with respect to \vec{J} and \vec{r} (\vec{r} = position of the test particle). Various mass eigenstates develop relative phases as the quantum test particle evolves in a given gravitational environment. *These relative phases depend not only on the gravitational source but also on the specific quantum mechanical characteristics of the quantum test particle as contained in A_{kj} . This introduces the essential non-geometrical element when gravitational and quantum phenomena are considered simultaneously.* Something quite close to this was already realized by Sakurai [7, p. 129] when, in the context of the celebrated Colella, Overhauser, and Werner experiment on neutron interferometry [8] he wrote “This experiment also shows that gravity is not purely geometric at the quantum mechanical level because the effect depends on $(m/\hbar)^2$,” but noting immediately, “However, this does not imply that the equivalence principle is unimportant in understanding an effect of this sort. If the gravitational mass (m_{grav}) and inertial mass (m_{inert}) were unequal, $(m/\hbar)^2$ would have to be replaced by $m_{grav} m_{inert}/\hbar^2$.”

We take the following as working definitions. *Geometrical elements* are those that are completely specified by the gravitational source. *Non-geometrical el-*

³ See, for example, Ref. [5, Sec. 3.6] for the usual operational procedure for such a general relativistic setup. In particular, note [5, pp. 166,167]:

“Since the behavior of freely falling clocks is completely predictable from the principle of equivalence, we will use freely falling clocks for all our measurements in the gravitational field, even measurements at a fixed position, for instance, a measurement at a fixed position on the surface of the Earth. For this purpose, we use a freely falling clock, instantaneously at rest at the fixed position. As soon as the clock has fallen too far from our fixed position and acquired too much speed, we must replace it by a new clock, instantaneously at rest. Whenever we speak of the time as measured by “a clock located at a fixed position” in a gravitational field, this phrase must be understood as shorthand for a complicated measurement procedure, involving many freely falling, disposable clocks, used in succession.”

⁴ The essential elements of the structure that are needed are not the vector, nor the spinor fields, but their spin–independent time evolution as $\exp[\pm i p_\mu x^\mu]$.

ements are those that crucially depend on the specific details of quantum test particles and do not follow from general relativity alone.

In a weak gravitational field of mass,⁵ M , with spin angular momentum \vec{J} ($= J\hat{z}$, for convenience), the evolution of the mass eigenstates is governed by the Hamiltonian [9]

$$H = \mathbf{m}c^2 - \frac{GM\mathbf{m}}{r} - \left(\frac{\vec{\mathbf{s}}}{2}\right) \cdot \vec{b} \quad , \quad (3)$$

and the non-relativistic Schrödinger equation (recall that $\vec{p} = \vec{0}$). In Eq. (3), the gravitomagnetic field \vec{b} is given by [10, Eq. 6.1.25]

$$\vec{b} \equiv \frac{2G}{c^2} \left[\frac{\vec{J} - 3(\vec{J} \cdot \hat{r})\hat{r}}{r^3} \right] \quad , \quad (4)$$

and $\vec{\mathbf{s}} = \mathbf{s}_z \hat{z}$. The quantum mechanical operators that appear in Eq. (3) are defined as follows: $\mathbf{m}|m_k, s_j\rangle = m_k|m_k, s_j\rangle$ and $\mathbf{s}_z|m_k, s_j\rangle = s_j|m_k, s_j\rangle$.⁶ M and \vec{J} will be treated as classical gravitational sources. This framework is a natural extension of arguments that were first put forward by Overhauser, and Colella [11], and Sakurai [7, pp. 126-129].

Consider the simplest case with two distinct mass eigenstates of spin one half. That is, set $n = 2$ and $s = 1/2$. Further, introduce two sets of “flavor” states, one set where both the mass eigenstates have s_j in the same relative orientation, and the other set where s_j are oriented in opposite directions. First set:

$$|Q_a\rangle \equiv \cos\theta|m_1, \uparrow\rangle + \sin\theta|m_2, \uparrow\rangle, \quad |Q_b\rangle \equiv -\sin\theta|m_1, \uparrow\rangle + \cos\theta|m_2, \uparrow\rangle. \quad (5)$$

Second set:

$$|Q_A\rangle \equiv \cos\theta|m_1, \uparrow\rangle + \sin\theta|m_2, \downarrow\rangle, \quad |Q_B\rangle \equiv -\sin\theta|m_1, \uparrow\rangle + \cos\theta|m_2, \downarrow\rangle. \quad (6)$$

It should be emphasized that $m_1 \neq m_2$. In addition, without a loss of generality, we take $m_2 > m_1$. For convenience and simplicity of the arguments, we have set $\vec{p}_k = \vec{0}$; \uparrow indicates $s_j = +\hbar/2$, and \downarrow represents $s_j = -\hbar/2$.

To arrive at the stated result we now proceed in three steps.

I. In the absence of gravity, let us, at time $t = 0$, prepare a system in state $|Q_a\rangle$. The flavor–oscillation probability at a later time t that the system is

⁵That is, we keep terms that are of first order in the dimensionless parameter $-GM/c^2 r$.

⁶We follow the notation that boldface letters represent quantum mechanical operators.

found in state $|Q_b\rangle$ is:

$$\begin{aligned}
P_{a\rightarrow b}(t) &= \left| \langle Q_b | \left\{ \exp \left[-\frac{im_1c^2t}{\hbar} \right] \cos \theta |m_1, \uparrow\rangle + \exp \left[-\frac{im_2c^2t}{\hbar} \right] \sin \theta |m_2, \uparrow\rangle \right\} \right|^2 \\
&= \sin^2(2\theta) \sin^2[\varphi^0] \quad , \quad (7)
\end{aligned}$$

where the kinematically induced phase, φ^0 , is

$$\varphi^0 \equiv \frac{(m_2 - m_1)c^2t}{2\hbar} \quad . \quad (8)$$

The similarly defined probability of flavor oscillation for $|Q_A\rangle \rightarrow |Q_B\rangle$ is the same as above:

$$P_{A\rightarrow B}(t) = P_{a\rightarrow b}(t) \quad . \quad (9)$$

The characteristic time of flavor-oscillations, $a \rightleftharpoons b$ and $A \rightleftharpoons B$, is

$$T^0 = \frac{2\hbar}{(m_2 - m_1)c^2} \quad . \quad (10)$$

Thus, the phenomenon of the flavor-oscillation provides a quantum mechanical clock. In the absence of gravity, the flavor-oscillation clocks, $\{a \rightleftharpoons b, A \rightleftharpoons B\}$, are characterized by the same characteristic time of flavor-oscillations.⁷

II. Next, we study the test particle evolution in the vicinity of a non-rotating source of gravity.⁸

The above defined probabilities are now modified by the gravitationally induced relative phases (each of the m_k picks up a *different* phase from the gravitational field):

$$\begin{aligned}
P_{a\rightarrow b}(t) &= P_{A\rightarrow B}(t) \\
&= |\langle Q_b | \{ \exp[-i\varphi_1] \cos \theta |m_1, \uparrow\rangle + \exp[-i\varphi_2] \sin \theta |m_2, \uparrow\rangle \} |^2 \quad , \quad (11)
\end{aligned}$$

⁷ Without the requirement $m_1 \neq m_2$, φ^0 would identically vanish and no flavor-oscillation clock shall exist. The flavor-oscillation clocks have no classical counterpart. If $m_1 = m_2$, then the nearest classical counterpart is a gyroscope. For instance with $m_1 = m_2 = m$ and $\theta = \pi/4$, $|Q_A\rangle$ becomes $|m, \rightarrow\rangle$ and $|Q_B\rangle$ becomes $|m, \leftarrow\rangle$. These are the spin polarized states along the positive and negative x-direction.

⁸ We shall assume that various parameters are so chosen that the time scale, $T(|m_k, \uparrow\rangle \rightleftharpoons |m_k, \downarrow\rangle)$, associated with the gravitationally induced transitions, is much larger compared with the characteristic time of flavor-oscillations, i.e., $T(|m_k, \uparrow\rangle \rightleftharpoons |m_k, \downarrow\rangle) \gg T^0$, and that clocks can be discarded and replaced with the new ones following a simple extension of the operational procedure [5] outlined in footnote 3. I am thankful to Dr. A. Mondragón (IFUNAM) for a remark on this matter.

where

$$\varphi_1 = \left(m_1 c^2 - \frac{GMm_1}{r} - \frac{\hbar \hat{z} \cdot \vec{b}}{4} \right) \frac{t}{\hbar}, \quad \varphi_2 = \left(m_2 c^2 - \frac{GMm_2}{r} - \frac{\hbar \hat{z} \cdot \vec{b}}{4} \right) \frac{t}{\hbar}. \quad (12)$$

The gravitationally induced phases in φ_1 and φ_2 that arise from the $\vec{s} \cdot \vec{b}$ part of H , Eq. (3), are identical. Therefore, they do not contribute to the redshift-giving relative phase. The only contribution to the redshift-giving relative phase comes from the part of the phases that are proportional to $[GMm_k/r]t/\hbar$; $k = 1, 2$. A straightforward calculation yields:

$$P_{a \rightarrow b}(t) = P_{A \rightarrow B}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M]. \quad (13)$$

The gravitationally induced phase, φ^M , reads:

$$\varphi^M \equiv \frac{GM}{c^2 r} \varphi^0. \quad (14)$$

In φ^M , $GM/c^2 r$ is the dimensionless gravitational potential, $-\Phi$, due to a gravitational source of mass M . The flavor-oscillation clocks $\{a \rightleftharpoons b, A \rightleftharpoons B\}$ redshift, and redshift identically, as expected in the geometric interpretation of general relativity. The phase φ^M is similar to the one first considered by Good for the $K^0-\bar{K}^0$ mesons [12] and studied in greater detail by Aronson, Bock, Cheng, and Fischbach [13], and Goldman, Nieto, and Sandberg [14].⁹ The gravitationally induced neutrino-oscillation phase given in Eq. (12) of Ref. [16] is a generalization of φ^M to the relativistic case. However the gravitationally induced fractional change in the kinematic phase φ^0 , $-\varphi^M/\varphi^0$, is still found to be the same, i.e., equal to $-GM/c^2 r$, for both the relativistic and non-relativistic cases. This equality assures that in the environment of a spherically symmetric non-rotating gravitational source, flavor-oscillation clocks, $\{a \rightleftharpoons b, A \rightleftharpoons B\}$, redshift identically.

III. Finally, let us consider the test particle evolution in the vicinity of a rotating source of gravity.

The above defined probabilities are now modified by the gravitationally induced relative phases, and these phases depend not only on m_k but also on the s_j structure of the test particle (as contained in A_{kj}). In the first case, i.e., $a \rightleftharpoons b$ oscillation, the phase due to the $\vec{s} \cdot \vec{b}$ interaction is same for both the m_k and hence does not contribute to the flavor-oscillation probability. In the second case, i.e., $A \rightleftharpoons B$ oscillation, the phase due to the $\vec{s} \cdot \vec{b}$ interaction is

⁹ However, based on the general considerations of Grossman and Lipkin (where they consider neutrino oscillations), note should be made that the sign of the phase is determined by whether one is considering oscillations in time or distance [15].

opposite for the two m_k 's (and hence becomes relative) and contributes to the flavor-oscillation probability. The gravitationally modified flavor-oscillation probabilities are obtained to be:

$$P_{a \rightarrow b}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M] \quad , \quad (15)$$

$$P_{A \rightarrow B}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M - \varphi^J] \quad . \quad (16)$$

The new gravitationally induced phase that appears in $A \rightleftharpoons B$ flavor oscillations via Eq. (16) is

$$\varphi^J = \left(2 \frac{\vec{s}_z}{2} \cdot \vec{b} \right) \frac{t}{2\hbar} = \frac{G}{c^2} \left(\frac{J - 3(\vec{J} \cdot \hat{r})(\hat{r} \cdot \hat{z})}{r^3} \right) \frac{t}{2} \quad . \quad (17)$$

This new contribution, φ^J , to the gravitationally induced phases is a natural, but conceptually important, extension of Good's original considerations.¹⁰ Comparison of Eqs. (7) and (9) with Eqs. (15) and (16) yields the result that the flavor-oscillation clock $a \rightleftharpoons b$ redshifts as if \vec{J} were absent, while the redshift of the flavor-oscillation clock $A \rightleftharpoons B$ depends on both M and \vec{J} . If the clocks $a \rightleftharpoons b$ and $A \rightleftharpoons B$ are reintroduced in the environment of a non-rotating source they will run in synch, i.e., identically. This is the central result of this essay. Conceptually, this situation may be considered as a rough gravitational analog of the Zeeman effect of atomic physics.¹¹

The quantum-mechanically induced gravitational phase φ^J does not depend on \hbar . This \hbar independence is a generic feature of all interaction Hamiltonians that depend linearly on the Planck constant. However, what is remarkable here is that the relevant interaction Hamiltonian that gives rise to the non-geometric element obtained here, turns out, as a consequence of the equality of the inertial and gravitational masses, to be precisely of the form that removes \hbar dependence in the redshift-splitting phase φ^J .

We now discuss in a little greater detail the origin of the gravitationally induced phases. First, consider a non-rotating gravitational source. For a single mass eigenstate the classical effects of gravitation may be considered to depend on a force, \vec{F} , while the quantum-mechanical effects are determined by the gravitational interaction energy, H_{int}^M . In the weak field limit, the interaction energy and the force are given, respectively, by

$$H_{int}^M = m \phi, \quad \vec{F} = -\vec{\nabla} H_{int}^M \quad , \quad (18)$$

¹⁰ For a detailed discussion of the gravitational phase φ^M and its relationship with the pioneering work of Colella, Overhauser, and Werner on neutron interferometry [8], and its extension to neutrino oscillations in astrophysical contexts, the reader is referred to Refs. [16,17].

¹¹ I thank Dr. Nu Xu (LBNL) for bringing this analogy to my attention.

where the gravitational potential $\phi = -GM/r$. Along an equi- ϕ surface the \vec{F} vanishes and there are no classical effects in this direction. The constant potential along a segment of an equi- ϕ surface can be removed by going to an appropriately accelerated frame. Quantum mechanically, the mass eigenstate picks up a global phase factor $\exp[-im\phi t/\hbar]$. Again, there are no physical consequences.

If we now consider a physical state that is in a linear superposition of different mass eigenstates, then relative phases are induced between various mass eigenstates. This happens because the phase, $\exp[-im\phi t/\hbar]$, depends on mass, m , of the mass eigenstate, and by construction each mass eigenstate carries a different mass. These relative phases are observable as flavor-oscillation phases.¹² Specifically, along an equi- ϕ surface the gravitational force \vec{F} vanishes, while the relative quantum mechanical phases induced in the evolution of a linear superposition of mass eigenstates do not. For the flavor-oscillation clocks, $a \rightleftharpoons b$ and $A \rightleftharpoons B$, the general expression of the gravitationally induced flavor-oscillation phase, φ^M , is given by Eq. (14).

For a rotating gravitational source, all of the above observations still remain valid. However, in addition, one must now consider the gravitomagnetic interaction energy and the torque, given, respectively, by

$$H_{int}^J = - \left(\frac{\vec{s}}{2} \right) \cdot \vec{b}, \quad \vec{\tau} = \left(\frac{\vec{s}}{2} \right) \times \vec{b} \quad . \quad (19)$$

For the set of flavor states $\{|Q_A\rangle, |Q_B\rangle\}$, where $\vec{s} = (\sigma_z \hbar/2)\hat{z}$ and $\vec{J} = J\hat{z}$, if, to emphasize the differences between quantum and classical considerations, we study the evolution at $\vec{r} = r\hat{z}$, it is immediately clear that there is no classical effect (as far as their precession is concerned¹³) on the individual spins because of the vanishing $\vec{\tau}$, whereas quantum mechanically the flavor-oscillation evolution is determined by H_{int}^J -dependent phases, and these phases are non-zero (and opposite for the two spin configurations superimposed in the set $\{|Q_A\rangle, |Q_B\rangle\}$). The general expression for the gravitationally induced flavor-oscillation phase φ^J is given by Eq. (17).¹⁴

¹² The observability of these phases is not for a local observer, but for an observer making measurements stationed at a different equi- ϕ surface.

¹³ Note, the force exerted on the spin

$$\vec{F} = \left(\frac{1}{2} \vec{s} \cdot \nabla \right) \vec{b} \quad ,$$

is non-zero.

¹⁴ For a detailed discussion of gravitational redshift in the presence of rotation, though confined to classical test particles, the reader is referred to Ciufolini and Wheeler [10].

In this essay we presented a generalized notion of flavor–oscillation clocks. The generalization contains the element that various superimposed mass eigenstates may have different relative orientation of the component of their spin with respect to the rotational axis of the the gravitational source. It is found that these quantum mechanical clocks do not always redshift identically when moved from the gravitational environment of a non–rotating source to the field of a rotating source. The gravitationally induced non–geometric phase φ^J is independent of \hbar despite its origins in quantum mechanical phases. Finally, by interchanging the spin–projections associated with m_1 and m_2 in the the states $|Q_A\rangle$ and $|Q_B\rangle$ we introduce a *third* set of flavors:

$$|Q_{A'}\rangle \equiv \cos\theta|m_1, \downarrow\rangle + \sin\theta|m_2, \uparrow\rangle, \quad |Q_{B'}\rangle \equiv -\sin\theta|m_1, \downarrow\rangle + \cos\theta|m_2, \uparrow\rangle. \quad (20)$$

This results in replacing Eqs. (15) and (16) by:

$$P_{a\rightarrow b}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M] \quad , \quad (21)$$

$$P_{A\rightarrow B}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M - \varphi^J] \quad , \quad (22)$$

$$P_{A'\rightarrow B'}(t) = \sin^2(2\theta) \sin^2[\varphi^0 - \varphi^M + \varphi^J] \quad . \quad (23)$$

The flavor–oscillation clocks $\{a \rightleftharpoons b, A \rightleftharpoons B, A' \rightleftharpoons B'\}$ form the maximal set of flavors for spin one half.¹⁵ The sign of the gravitational phase induced by the $\vec{s}\cdot\vec{J}$ term in the Hamiltonian carries *opposite* signs for the $A \rightleftharpoons B$ and $A' \rightleftharpoons B'$ flavor–oscillations. If one agrees to measure redshift average with respect to an equally weighted ensemble of the three flavor types, then the non–geometric element averages out to zero. Extension to higher spins being obvious, we propose that the non–geometric element in redshifts may be interpreted as a quantum mechanically induced fluctuation over a geometric structure of space–time. In the weak field limit, the amplitude of these fluctuations is directly proportional to the product of the rotation, as measured by \vec{J} , and the spin of the test particle.

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¹⁵ The reversing of spin–projections in states $|Q_a\rangle$ and $|Q_b\rangle$ may be used to include a fourth flavor. Such an addition to the maximal set does not alter our conclusions.

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