Charge conjugation and Lense-Thirring Effect
— A new Asymmetry —

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Abstract

This essay presents a new asymmetry that arises from the interplay of charge conjugation and Lense-Thirring effect. When applied to Majorana neutrinos, the effects predicts $\nu_e \rightleftharpoons \bar{\nu}_e$ oscillations in gravitational environments with rotating sources. Parameters associated with astrophysical environments indicate that the presented effect is presently unobservable for solar neutrinos. But, it will play an important role in supernovae explosions, and carries relevance for the observed matter-antimatter asymmetry in the universe.

Introduction.— Consider a rotating matter sphere charged with electrons. Then, consider a similarly rotating antimatter sphere charged with positrons. Formally, the latter is obtained from the former, and vice versa, by the action of an appropriate set of charge conjugation operators associated with matter and gauge fields. It is a purely general-relativistic prediction that in this transformation from matter to antimatter, the gravitational Lense-Thirring moment [1] — or, the so called gravitomagnetic moment — does not alter its direction, while the associated magnetic moments flip their directions. If for the matter sphere the magnetic and Lense-Thirring moments are chosen to be *parallel*, for the antimatter sphere they become *anti-parallel*. This suggests that gravity may carry an intrinsic charge conjugation asymmetry. This is a brief summary — missing operational considerations — of an argument which I presented in late 1983, or perhaps early 1984, in a conversation with Kip Thorne at Texas A&M University.

Now, independently, work of Singh and collaborators hints at a similar asymmetry [2,3]. If these gravitationally-induced asymmetries were to be true it would have profound consequences for theoretical physics as well as astrophysical and cosmological processes. I now pursue, and establish, one of these asymmetries in a manner which shall reduce it to its bare essentials without any reliance on a particular theory of quantum gravity.

Establishing the thesis.— Even though it is not a conventional wisdom for most general relativists, the primitive notion of test particles derives its definition from, (a) the Casimir invariants of the Poincare group, and (b) local gauge symmetries of the associated wave equations. [1] The interplay of these two, for instance, gives the possibility to have massive spin–1/2 test particles. There are two type of spin–1/2 particles: Those which are eigenstates of the charge operator, $\mathcal{Q}$, and those which are eigenstates of the charge conjugation operator, $\mathcal{C}$. The former are governed by the Dirac equation, while the latter eigenspinors — note, $[\mathcal{C},\,\mathcal{Q}]\neq 0$, in general — we must construct *ab initio.*

The eigenspinors of the $\mathcal{C}$ have the property that [6]: $\mathcal{C}\,\lambda(\mathbf{p}) = \pm\,\lambda(\mathbf{p})$. Those corresponding to the *plus* sign, we call self conjugate, and are given by:

$$\lambda^S(\mathbf{p}) = \begin{pmatrix} \sigma_2\phi_L^*(\mathbf{p}) \\ \phi_L(\mathbf{p}) \end{pmatrix}, \tag{1}$$

where $\phi_L(\mathbf{p})$ is a massive, left-handed, Weyl spinor. These are the standard Majorana spinors found in textbooks [7]. Those corresponding to the *minus* sign, we call anti-self conjugate, and these read:

$$\lambda^A(\mathbf{p}) = \begin{pmatrix} -\sigma_2\phi_L^*(\mathbf{p}) \\ \phi_L(\mathbf{p}) \end{pmatrix}. \tag{2}$$

These are new. The two sets differ by a physically important relative phase between the right-handed transforming components, $\sigma_2\phi_L^*(\mathbf{p})$, and the left-handed transforming components, $\phi_L(\mathbf{p})$. The $\lambda^A(\mathbf{p})$ must complement $\lambda^S(\mathbf{p})$ in order to have a mathematically complete representation space. Without $\lambda^A(\mathbf{p})$ no correct physical interpretation exists for the massive case. [2]

To arrive at the asymmetry induced by gravitational Lense-Thirring field, let $\phi_L(\mathbf{p})$ be eigenstates of the helicity operator, $h \stackrel{\mathrm{def}}{=} \boldsymbol{\sigma}\cdot\widehat{\mathbf{p}}$. That is,

$$h\,\phi_L^\pm(\mathbf{p}) = \pm\,\phi_L^\pm(\mathbf{p})\,. \tag{3}$$

With these definitions, it is now a simple mathematical exercise to show that,

$$h\,\sigma_2\phi_L^{*\,\pm}(\mathbf{p}) = \mp\,\sigma_2\phi_L^{*\,\pm}(\mathbf{p})\,. \tag{4}$$

[1] In the conventional general relativistic framework it is implicitly assumed that the nature of test particle is hardly of significance as long as it satisfies some very primitive requirements of size, etc. The only exception that we know are inclusion of spin/internal-structure effects on geodesic deviations where the work of Anandan *et al.* and Mohseni provide latest developments on the subject [4,5].

[2] See, Appendix for more details and brief interpretational remarks.

Essay for Gravity Research Foundation, 2004

Stated in words: *The left- and right- transforming spinor components of* $\lambda(\mathbf{p})$, *irrespective of self/antiself conjugacy, carry* <u>*opposite*</u> *helicities. An eigenspinor of* $\mathcal{C}$ *is a dual helicity object.*

It is this circumstance which profoundly distinguishes the eigenspinors of $\mathcal{C}$ from those of $\mathcal{Q}$. The eigenspinors of the latter are definite — i.e., single, as opposed to dual — helicity objects. There are four eigenspinors of $\mathcal{C}$. These can be enumerated as:

$$\lambda^S_{\{-+\}}(\mathbf{p}),\ \lambda^S_{\{+-\}}(\mathbf{p}),\ \lambda^A_{\{-+\}}(\mathbf{p}),\ \text{and}\ \lambda^A_{\{+-\}}(\mathbf{p})\,, \tag{5}$$

where the first symbol in the subscript refers to the helicity of the right-transforming spinor component; while the second refers to the helicity of the left-transforming spinor component. Now if we introduce, say the state associated with $\lambda^S_{\{-+\}}(\mathbf{p})$, in the gravitational environment of a (roughly)spherically symmetric astrophysical source of mass, M, and angular frequency ω — which contributes Lense-Thirring component to the gravitational field — then there are two physically relevant phase factors that such a state picks up:

α. An overall gravitationally-induced phase which depends on source mass, M, and energy, E, of the test particle.

β. A gravitationally-induced *relative* phase between the right- and left- transforming components of the test particle.

The first of these enumerated phases has no effect for a single mass eigenstate. However, it is precisely this phase when considered in the context of neutrinos — which the solar, atmospheric, and other neutrino oscillation data [8–12] establishes to be linear superposition of three different mass eigenstates in a leptonic-flavor dependent manner — that flavor oscillation clocks gravitationally redshift as required by general relativity. [13–19].

The second of the enumerated phases — which we emphasize constitutes a new discovery presented in this essay — does not contribute to the the redshift of the flavor oscillation clocks, and it carries observable effects even for a single mass eigenstate. It arises because the Lense-Thirring gravitational interaction energy associated with the positive and negative helicity components of $\mathcal{C}$ eigenspinors has opposite signs, and hence induces opposite phases. Such an effect is entirely absent from eigenspinors of the $\mathcal{Q}$ operator where both the right- and left- transforming components carry the same Lense-Thirring gravitational interaction energy, and hence the same phases.

The former, i.e. the effect enumerated as α, is generic to both the eigenstates of the $\mathcal{Q}$ and $\mathcal{C}$, while the latter, i.e. the effect arising from item β, is a unique characteristic of the $\mathcal{C}$ eigenstates.

As a result, if the initial eigenstate was associated with the spinor $\lambda^S_{\{-+\}}(\mathbf{p})$, then — if the test particle is in the immediate vicinity of the surface — it oscillates with frequency

Essay for Gravity Research Foundation, 2004

$$\Omega = \frac{G}{2c^2}\left|\frac{\vec{\mathcal{J}}\cdot\widehat{h} - 3\left(\vec{\mathcal{J}}\cdot\widehat{r}\right)\left(\widehat{r}\cdot\widehat{h}\right)}{R^3}\right|, \tag{6}$$

to an eigenstate associated with the spinor $\lambda^A_{\{-+\}}(\mathbf{p})$. The result (6) is valid in weak field limit with involved speeds $v \ll c$, i.e., all the way to the vicinity of neutron stars. In Eq. (6), $\widehat{h}$ represents the direction to which helicity eigenstates in $\lambda(\mathbf{p})$ refer to (and coincides with $\widehat{\mathbf{p}}$), $\widehat{r}$ is a unit position vector pointing from center of the idealized astrophysical sphere to a general location on the sphere, and the magnitude of $\vec{\mathcal{J}}$ is given by: $\mathcal{J} \approx \frac{2}{5}MR^2\omega$. If the state is considered stationary, i.e at rest, on the surface, then the probability of oscillation is $\sin^2(\Omega t)$. This expression readily generalizes to relativistic case, but then spatial variation of Ω must be accounted for.

For accessing rough magnitude of the effect let us consider polar region of the gravitational source, with a test particle prepared in such way that: $\widehat{h} = \widehat{r}$. Then, we have

$$\Omega = \left[\frac{2}{5}\left(\frac{GM}{c^2R}\right)\right]\omega\,. \tag{7}$$

The above results easily extend to a linear superposition of different mass eigenstates, specifically to neutrinos. In that context (see Appendix for some technical details), rotation in gravitational environments induces Majorana neutrinos to oscillate from ν_e to $\bar{\nu}_e$. In reference to Eq. (7), note that

$$\frac{GM}{c^2R} \approx \begin{cases} 2.12\times10^{-6}, & \text{for Sun} \\ 0.2, & \text{for Neutron stars}, \end{cases} \tag{8}$$

while

$$\omega \approx \begin{cases} 3\times10^{-6}\ \mathrm{s}^{-1}, & \text{for Sun} \\ 6.3\times10^{3}\ \mathrm{s}^{-1}, & \text{for millisecond Neutron stars}. \end{cases} \tag{9}$$

Therefore, should neutrinos be Majorana (for strong indications in that direction, see below), the new effect is unobservable for solar neutrinos with present experiments [20]. But the noted orders of astrophysical parameters indicate the effect may be significant in early universe and it carries importance for supernovae explosions (where neutron stars are a significant part of the gravitational environment). This is so because $\nu_e \rightleftharpoons \bar{\nu}_e$ oscillations modify energy transport.

Concluding remarks.— We end this essay with the observation that after decades of pioneering work, the Heidelberg-Moscow collaboration has, in the last few years, presented first experimental evidence for the Majorana nature of ν_e and $\bar{\nu}_e$. The initial 3-σ signal now has better than 4-σ significance [21–25]. Our essay shows that Majorana neutrinos carry unsuspected sensitivity to gravitational environments with rotational elements (which induce a new asymmetry). This asymmetry causes $\nu_e \rightleftharpoons \bar{\nu}_e$ oscillations with a frequency determined by the gravitational environment. A neutrino

Essay for Gravity Research Foundation, 2004

sea composed entirely of ν_e shall in time develop a $\bar{\nu}_e$ component inducing a matter-antimatter asymmetry. It is worth noting that Raffelt [26] has emphasized that core of a collapsing star, where matter density can reach as high as 3×10^{14} g cm^{-3} — with neutrino trapping density of about 10^{12} g cm^{-3} for 10 MeV neutrinos — is the only known astrophysical site, apart from early universe, where neutrinos are in thermal equilibrium. It is here that the predicted $\nu_e \rightleftharpoons \bar{\nu}_e$ oscillations may induce significant matter-antimatter asymmetry while at the same time affecting the entire evolution of the supernovae explosions.

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Essay for Gravity Research Foundation, 2004

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Essay for Gravity Research Foundation, 2004

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Appendix: A set of linearly connected eigenstates of $\mathcal{C}$.— There is a slight, and only apparent, asymmetry in the manner in which we introduced $\lambda(\mathbf{p})$ spinors. A linearly related, and very useful, set of eigenstates of the $\mathcal{C}$ operator are:

$$\rho^S(\mathbf{p}) = \begin{pmatrix} \phi_R(\mathbf{p}) \\ -\sigma_2\phi_R^*(\mathbf{p}) \end{pmatrix}, \quad \rho^A(\mathbf{p}) = \begin{pmatrix} \phi_R(\mathbf{p}) \\ \sigma_2\phi_R^*(\mathbf{p}) \end{pmatrix} \tag{.1}$$

with $\mathcal{C}\,\rho(\mathbf{p}) = \pm\,\rho(\mathbf{p})$, where the plus sign is for self conjugate spinors and minus sign is for antiself conjugate spinors. In the above equation $\phi_R(\mathbf{p})$ are massive right-handed Weyl spinors. Now, under Lorentz boosts $\sigma_2\phi_R^*(\mathbf{p})$ transform as left-handed spinors.

It can be shown that the sets $\lambda(\mathbf{p})$ and $\rho(\mathbf{p})$ are not linearly independent.

The *Majorana dual* is defined as:

$$\lambda^S(\mathbf{p}): \quad \overset{\neg S}{\lambda}_{\pm,\mp}(\mathbf{p}) \stackrel{\text{def}}{=} +\left[\rho^A_{\mp,\pm}(\mathbf{p})\right]^\dagger \gamma^0 \tag{.2}$$

$$\lambda^A(\mathbf{p}): \quad \overset{\neg A}{\lambda}_{\pm,\mp}(\mathbf{p}) \stackrel{\text{def}}{=} -\left[\rho^S_{\mp,\pm}(\mathbf{p})\right]^\dagger \gamma^0, \tag{.3}$$

where

$$\gamma^0 \stackrel{\text{def}}{=} \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{.4}$$

With this definition, the self/antiself conjugate $\lambda(\mathbf{p})$ spinors satisfy the following orthonormality

$$\overset{\neg S}{\lambda}_{\eta}(\mathbf{p})\,\lambda^S_{\eta'}(\mathbf{p}) = +\,2m\,\delta_{\eta\eta'}, \tag{.5}$$

$$\overset{\neg A}{\lambda}_{\eta}(\mathbf{p})\,\lambda^A_{\eta'}(\mathbf{p}) = -\,2m\,\delta_{\eta\eta'}, \tag{.6}$$

and completeness relations:

$$\frac{1}{2m}\sum_{\eta}\left[\lambda^S_\eta(\mathbf{p})\,\overset{\neg S}{\lambda}_{\eta}(\mathbf{p}) - \lambda^A_\eta(\mathbf{p})\,\overset{\neg A}{\lambda}_{\eta}(\mathbf{p})\right] = I, \tag{.7}$$

In the above equations, the subscript η ranges over two possibilities: $\{+,-\}, \{-,+\}$. The relations enumerated here help in obtaining results given in the main text. In

Essay for Gravity Research Foundation, 2004

particular, these relations allow for interpretation of the discussed oscillations with $\nu_e \rightleftharpoons \bar{\nu}_e$ oscillations. Detailed analysis and interpretational issues shall be presented in Ref. [6]

Essay for Gravity Research Foundation, 2004