Probing Quantum Violations of Equivalence Principle

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Abstract

The joint realm of quantum mechanics and general-relativistic description of gravitation is becoming increasingly accessible to terrestrial experiments and observations. In this essay we study the emerging indications of the violation of equivalence principle (VEP). While the solar neutrino anomaly may find its natural explanation in a VEP, the statistically significant discrepancy observed in the gravitationally induced phases of neutron interferometry seems to be the first indication of a VEP. However, such a view would seem immediately challenged by the atomic interferometry results. The latter experiments see no indications of VEP, in apparent contradiction to the neutron interferometry results. Here we show that these, and related torsion pendulum experiments, probe different aspects of gravity; and that current experimental techniques, when coupled to the solar-neutrino data, may be able to explore quantum mechanically induced violations of the equivalence principle. We predict quantum violation of the equivalence principle (qVEP) for next generation of atomic interferometry experiments. The prediction entails comparing free fall of two different linear superpositions of Cesium atomic states.

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I. INTRODUCTION

In classical physics, if a system can be described by a Lagrangian, then it possesses a precise energy-momentum tensor, $T^{\mu\nu}(x)$. In this context, the general theory of relativity asserts equality of the inertial and gravitational masses. This equality, as embedded in the principle of equivalence, has been confirmed by all classical tests, so far. This contrasts the realm of quantum mechanics, where the precision with which one measures the values of the various components of $T^{\mu\nu}(x)$, associated with that of a source *and* necessarily that of a test particle (see below, footnote #6), are constrained by the fundamental uncertainty relations. Usually, it is assumed that any violations arising out of such considerations are beyond terrestrial experiments and carry no observable consequences. In this essay we shall argue to the contrary. We shall show that in the quantum realm the principle of equivalence suffers a tiny violation. But these tiny violations, under certain circumstances, can lead to significant physical consequences. To establish this thesis we first review why the canonical wisdom believes in the non-observability of the quantum-gravity effects in the laboratory. Arguments of this kind generally run as follows:

(i) Since quantum electrodynamics (QED) and quantum-gravity effects should arise via the dimensionless constants $\alpha_e = e^2/\hbar c \approx 7.3 \times 10^{-3}$,

$$\alpha_G^j = \frac{(M_j^2 G)/(\hbar/M_j c)}{M_j c^2}, \quad j = \text{electron, proton, etc.} \tag{1}$$

which for proton yields $\alpha_G^p \approx 6 \times 10^{-39}$, the quantum-gravity effects fade into insignificance when compared with effects of QED by some forty orders of magnitude.

(ii) All quantum-gravity spacetime fluctuations, and the associated violations, are governed by the dimensions of the Planck length

$$\lambda_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-33} \text{ cm}, \tag{2}$$

and that this length scale is twenty orders of magnitude smaller than the charge radius of a proton, for instance.

In the context of the existing experimental techniques these arguments are misleading, and in fact, at times, incorrect. A simple argument to establish this requires us to note that the dimensionless gravitational potential on the surface of the Earth is

$$\Phi_{grav}^{Earth} \approx -7 \times 10^{-10} \times \frac{R_\oplus}{R_\oplus + z} \tag{3}$$

where $R_\oplus$ is radius of the Earth, and z stands for the vertical distance from

the surface of the Earth.[2] Now if we consider a neutron (or a proton, or even an electron) in this potential, then it is this number that is relevant, and not ratios like α_j/α_e (or, similarly defined other small numbers). In addition, for a table-top neutron interferometry experiment a thermal neutron carries a de Broglie wavelength of about $\lambda_n \approx 1$ Å. Compared to dimensions of a table-top neutron path, $L \approx 1$ m, this yields

$$\frac{L}{\lambda_n} \approx 10^{10} \tag{4}$$

It is this circumstance, as encoded in the interplay of equations (3) and (4), that in 1975 led to the first detection of a gravitationally induced quantum effect in neutron interferometry by Colella, Overhauser, and Werner (COW) [1]. Similarly, about a year ago Amelino-Camelia was to falsify the impression that quantum-gravity induced spacetime fluctuations are beyond the scope of terrestrial probes. The Amelino-Camelia argument is fast becoming a classic and the reader is referred to Refs. [2,3] for its origins. A series of other recent papers also point toward a change of perspective in an independent manner [4–6]. The reader is also referred to Refs. [7–9] for additional data on equivalence principle tests. A paper that advocated, in part, the view presented here is by Viola and Onofrio [10].

II. RECENT EXPERIMENTS ON THE EQUIVALENCE PRINCIPLE

The classical tests, based on sophisticated torsion pendulums, reveal no violations of the equivalence principle. Quantitatively, in an experiment published in December 1999 [11], the observed differential accelerations of the Copper (Cu) and Lead (Pb) test bodies toward a 3 ton ^{238}U attractor was found to be

$$a_{Cu} - a_{Pb} = (1.0 \pm 2.8) \times 10^{-13} \text{ cm/s}^2 \tag{5}$$

as compared to the corresponding gravitational acceleration of 9.2×10^{-5} cm/s^2. While this test, in its essence, is purely classical, and was designed to probe $\exp(-r/\lambda)$ type deviations in the Newtonian limit of general relativity, another experiment (reported in August 1999 [12]) compared gravitationally induced accelerations of a classical object (a macroscopic glass object) with that of a quantum object (a Cesium atom in a linear superposition of two different energy eigenstates). Denoting by a_G the acceleration of the glass object, and

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[2] In writing equation (3) we have ignored contributions from the local supercluster of galaxies. For neutron interferometry that contribution, even though several orders of magnitude larger, is irrelevant because of its few part in 10^{11} constancy in the solar system.

by a_{Ce}, the acceleration of the indicated Cesium atom; a_G and a_{Ce} were found identical to 7 parts in 10^9.

In the last twenty five years the COW experiments have become more sophisticated. The latest (September 1997 [13]) neutron interferometry experiments report a statistically significant discrepancy between the experiment and theory, and it has been suspected (by the experimenters) to carry any of the following two sources: (a) some systematic error in the measurements, (b) "they (the discrepancy) may also represent a difference between the ways in which gravity acts in classical and quantum mechanics," to directly quote the authors [13]. The gravitationally induced differences in phases that the experiment probed depend on the following combination of the inertial and gravitational masses

$$u = \frac{m_i \, m_g \, g}{2\pi\hbar^2} \times \text{a geometrical factor} \tag{6}$$

where g is the acceleration due gravity. Assuming the equality of the inertial and the gravitational masses for neutron, $m_i = m_g$, the experiment found this phase factor to be about 1% lower than predicted:

$$\frac{(u)_{expt.} - (u)_{m_i=m_g}}{(u)_{m_i=m_g}} = \begin{cases} -(1.5 \pm 0.12) \times 10^{-2}, \\ -(0.8 \pm 0.11) \times 10^{-2} \end{cases} \tag{7}$$

where the top value corresponds to the skew-symmetric interferometer while the bottom value is associated with the symmetric interferometer.

It would thus appear that the latest neutron interferometry experiments are in conflict with the more precise tests of the equivalence principle conducted via atomic interferometry, and with those based on torsion pendulum. In fact, authors of Ref. [12] write "we may conclude that there are aspects of neutron interferometry that are not well understood." While that may be so in part, here we shall argue that the answer is not necessarily so simple. In fact, we show that each of these experiments probes a different aspect of the equivalence principle. Furthermore, we predict that the atomic interferometry experiments, as the precision improves further, should begin to see a violation of the equivalence principle.

We deliberately enter details, which may appear common sense, in order to raise relevant questions that may otherwise escape our attention.

III. QUANTUM VIOLATIONS OF THE EQUIVALENCE PRINCIPLE: ATOMIC INTERFEROMETRY

That one should expect a tiny violation of the equivalence principle in the

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quantum regime is obvious. Since every physical system carries an inherent energy uncertainty determined by $\Delta E \Delta t \sim \hbar$, its inertial and gravitational properties must carry unavoidable fluctuations. In particular, these fluctuations affect the equality of the inertial and gravitational masses, and may even emerge as violation of the equivalence principle. However, as time of observations takes on macroscopic values these fluctuations become vanishingly small and only very clever experiments, perhaps along the lines suggested by Amelino-Camelia (in an entirely different context [2]), could be hoped to probe these fluctuations in the equality of the inertial and gravitational masses.

Here, we take a far more readily accessible experimental situation and study a possible violation, and associated fluctuations, of the equivalence principle. In effect, we choose a system for which certain quantum fluctuations evolve coherently.

To model an experimental set up, such as that used in the atomic interferometry experiments [12], consider two "flavors" of Cesium atoms:

$$\begin{bmatrix} |^{\alpha}Ce\rangle_{\xi} \\ |^{\beta}Ce\rangle_{\xi} \end{bmatrix} = \begin{bmatrix} \cos(\xi) & \sin(\xi) \\ -\sin(\xi) & \cos(\xi) \end{bmatrix} \begin{bmatrix} |^{E_1}Ce\rangle \\ |^{E_2}Ce\rangle \end{bmatrix} \tag{8}$$

Here, $|^{E_1}Ce\rangle$ and $|^{E_2}Ce\rangle$ represent two different *energy* eigenstates of the Cesium atom. The "flavor" states, $|^{\alpha}Ce\rangle_{\xi}$ and $|^{\beta}Ce\rangle_{\xi}$, are linear superposition of the energy eigenstates. These are characterized by the flavor indices $\{\alpha, \beta\}$, and by the mixing angle ξ. In a given gravitational environment these flavors oscillate from one flavor to another as is now well understood [14–19]. The oscillation of the flavors provides a flavor-oscillation clock, and the flavor-oscillation clocks red-shift as required by the theory of general relativity. Here we shall concentrate on an entirely different issue, and exploit the fact that the flavor-oscillations carry the fluctuations $\Delta E \Delta t \sim \hbar$ in a coherent manner — the inherent energy-uncertainty associated with the flavor states is simply related to the inverse of time period of the flavor-oscillation. However, this evolution happens in coherent manner and does not suffer from randomness often associated with the constraint $\Delta E \Delta t \sim \hbar$.

Having emphasized that we are modeling an experimentally accessible situation that can realized at the Stanford laboratory of Steven Chu and, possibly also at the Wineland-Itano's group at NIST, we now introduce a simplified notation:

$$|\psi_{\ell\xi}\rangle = |^{\ell}Ce\rangle_{\xi}, \quad \ell = \alpha, \beta \tag{9}$$

$$|\varphi_{E_j}\rangle = |^{E_j}Ce\rangle, \quad j = 1, 2 \tag{10}$$

We assume that the states $|\varphi_{E_j}\rangle$ are, in comparison to the time of observation,

long lived. The "free" fall experiment that we consider assumes, for simplicity, that flavors do not evolve significantly during their "free" fall from the source to the detector. Relaxing these assumptions to suit a given experimental situation should pose no technical or conceptual problem; and in fact should amplify the considered effect.

The flavor states of the Cesium atoms carry an inherent uncertainty in their energy

$$\Delta E_{\ell\xi} = \sqrt{\langle\psi_{\ell\xi}|H^2|\psi_{\ell\xi}\rangle - \langle\psi_{\ell\xi}|H|\psi_{\ell\xi}\rangle^2} \tag{11}$$

where

$$H|\varphi_{E_j}\rangle = E_j|\varphi_{E_j}\rangle, \quad j = 1, 2 \tag{12}$$

For this reason, the equality of their gravitational and inertial mass carries an inherent violation of the equivalence principle. The associated quantum violation in the equivalence principle (qVEP) can be characterized by the fractional accuracy

$$f_{\ell\xi} = \frac{\Delta E_{\ell\xi}}{\langle E_{\ell\xi}\rangle} \tag{13}$$

where $\langle E_{\ell\xi}\rangle \equiv \langle\psi_{\ell\xi}|H|\psi_{\ell\xi}\rangle$

Thus, if we study two sets of Cesium atoms, with flavors characterized by angles ξ_1 and ξ_2, then qVEP predicts a *difference in the spread in their accelerations* (as observed in identical free fall experiments by a stationary observer on Earth) to be:

$$|\Delta a_{\ell\xi_2}| - |\Delta a_{\ell\xi_1}| = \left\{\left|\frac{\sin(2\xi_2)}{\langle E_{\ell\xi_2}\rangle}\right| - \left|\frac{\sin(2\xi_1)}{\langle E_{\ell\xi_1}\rangle}\right|\right\}\frac{\delta E}{2}, \tag{14}$$

where $\delta E \equiv E_2 - E_1$. For flavor states of Cesium atoms prepared with $\delta E \approx 1$ eV, this difference is of the order of a few parts in 10^{12}, and should be observable in refined versions of experiment reported in Ref. [12]. How difficult this refinement in techniques at Stanford and NIST would be is not fully known to us. However, the extraordinary accuracy in similar experiments and the already achieved absolute uncertainty of $\Delta g/g \approx 3 \times 10^{-9}$, representing a million fold increase compared with previous experiments, makes us cautiously optimistic about observing qVEP in atomic interferometry experiments pioneered by the group of Steven Chu at Stanford.

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IV. Quantum Violations of the Equivalence Principle: Solar Neutrino Anomaly

It has long been conjectured that the solar neutrino anomaly may be related to a flavor-dependent violation of the equivalence principle. The suggestion first came from Gasperini [20,21]. The argument presented above can – with appropriate generalization, and on interpreting the flavor index ℓ to represent the three neutrino flavors (ν_e, ν_μ and ν_τ) – be extended to neutrino oscillations. Here we simply provide an outline of this argument and show how a quantum violation (of the needed magnitude) of the equivalence principle naturally arises.

To estimate the qVEP effects it suffices to restrict to a two-state neutrino oscillation framework. A simple calculation shows that the difference in fractional measure of qVEP turns out to be exceedingly small:

$$\Delta f_{\ell\ell'} \equiv f_\ell - f_{\ell'} = 6.25 \times 10^{-26} \left[\frac{(\Delta m^2)^2}{\text{eV}^4}\right] \left[\frac{\text{MeV}^4}{E^4}\right] \sin(4\xi_V) \tag{15}$$

where ℓ (say, ν_e) and ℓ' (say, ν_μ) refer to two different neutrino flavors, and ξ_V is the vacuum mixing angle between the underlying mass eigenstates (whose superposition leads to different flavors of neutrinos). The difference in the squares of the underlying mass eigenstates, $m_2^2 - m_1^2$, has been represented by Δm^2; and E is the expectation value of the neutrino energy. For the solar neutrinos the existing data spans the approximate range $0.2 \text{ MeV} \leq E \leq 20 \text{ MeV}$ in energy.

Interestingly, following standard arguments that yield an oscillation length from a flavor-dependent violation of the equivalence principle [22], this is precisely this *smallness* of $\Delta f_{\ell\ell'}$ that gives rise to a *large* oscillation length for solar neutrinos that compares well with the Earth-Sun distance. Scenarios of the violation of equivalence principle are currently under active investigation [22–25] to explain the solar neutrino anomaly. They have the advantage that none of the three separate evidences (the atmospheric, the accelerator, and the solar) for neutrino oscillations need to be ignored to make a consistent fit to all existing data. The qVEP induced oscillation length not only matches the Earth-Sun distances, but it also differs from the standard scenario of Gasperini where one assumes a energy-independent violation of the equivalence principle. The qVEP induced oscillation length carries a E^3 energy dependence, and would be clearly distinguishable from Gasperini's conjecture as more data becomes available, and if (as is true for all such analysis) the constant gravitational potential due to local supercluster of galaxies turns out to have the expected value. It is to be noted that such a gravitational potential carries little significance for planetary orbits because it is essentially constant over the solar system. However, it turns out to be important for the

gravitationally induced phases that determine the qVEP induced effects (or even those that arise from the VEP conjectured by Gasperini).

V. Quantum Violations of the Equivalence Principle: The discrepancy in Neutron Interferometry Experiments

Unlike an electron, which is the active player in the atomic interferometry, a neutron is a complicated object containing the *udd* quark configurations coupled to $\bar{q}q$ sea quarks and the gluonic degrees of freedom. These degrees of freedom, in terms of their spatial distribution, spend a fraction of their time in the classically forbidden region. To gain physical insight into the question of neutron-Earth gravitational interaction[3] we note that the baryonic spectra reveals a series of excited neutron states, and that these states are roughly equally spaced. Thus in "back of the envelope spirit" we can treat neutron as a ground state of an harmonic oscillator. This would not alter our general qualitative results in any significant manner.

For the ground state of an harmonic oscillator[4] we evaluate the kinetic and potential energy contributions from the classically allowed region (CAR), and from the classically forbidden region (CFR). We find that for the ground state of an harmonic oscillator, the CAR contributions to the potential and kinetic energy are, respectively:

$$E_{0,P}^{CAR} = \left(\frac{\mathrm{erf}(1)}{2} - \frac{1}{\mathrm{e}\sqrt{\pi}} \right) \frac{\hbar\omega}{2} \tag{16}$$

$$E_{0,K}^{CAR} = \left(\frac{\mathrm{erf}(1)}{2} + \frac{1}{\mathrm{e}\sqrt{\pi}} \right) \frac{\hbar\omega}{2} \tag{17}$$

Similarly, the contributions from the CFR read:

$$E_{0,P}^{CFR} = \left(\frac{1}{2} + \frac{1}{\mathrm{e}\sqrt{\pi}} - \frac{\mathrm{erf}(1)}{2} \right) \frac{\hbar\omega}{2} \tag{18}$$

$$E_{0,K}^{CFR} = \left(\frac{1}{2} - \frac{1}{\mathrm{e}\sqrt{\pi}} - \frac{\mathrm{erf}(1)}{2} \right) \frac{\hbar\omega}{2} \tag{19}$$

In these expressions "e" represents the natural base of logarithms (and roughly equals 2.218), $\mathrm{erf}(x)$ is the standard error function, and other symbols have their usual meaning. The total ground state energy, $E_0 = (1/2)\hbar\omega$, carries the following proportions: $\mathrm{erf}(1) \times E_0$ from the CAR, and $[1 - \mathrm{erf}(1)] \times E_0$ from the CFR. Since $[1 - \mathrm{erf}(1)] \approx 0.16$, roughly sixteen percept of the

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[3] The latter in turn carrying more than half its mass in neutronic matter.

[4] We restrict to a one-dimensional non-relativistic case because this serves as a good representative example to study classically forbidden region.

ground state energy is contributed by the CFR. Moreover, as is not unexpected, the total contribution to the kinetic energy from CFR is negative definite and equals $\approx -0.13E_0$.[5] In fact, the kinetic energy density[6]

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\begin{aligned}
\rho_{0,K}(x) &\equiv \psi_0^*(x) \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right] \psi_0(x) \\
&= \frac{1}{2} \sqrt{\frac{m\omega}{\pi\hbar}} \left(\hbar\omega - m\omega^2 x^2 \right) \exp\left[-\frac{m\omega x^2}{\hbar} \right]
\end{aligned}
\tag{20}
$$

is positive definite only for CAR. It is negative definite for the entire CFR as a simple graphical analysis reveals. This is not surprising at all since momentum is formally imaginary in *all* CFRs.

The point to be made now is as follows. Generally, it is argued that this formally imaginary momentum does not create any paradoxical situation because any attempt to confine the system to a classically forbidden region and to *measure* its momentum always imparts the system enough energy to destroy the very system one wishes to observe. It is only in the last decade that the CFRs have becomes experimentally accessible. Recent experiments on tunneling times indicate that the standard wisdom may not be the entire story[26–31]. Specifically, experiments such as described in refs. [26,29] do not probe the *spatial extent* of the classically forbidden region. Instead, they simply assure that a photon (or, some similar probe) encountered a CFR on it way from the source to the detector. In the context of neutron interferometry, the general-relativistic gravity not only probes the total energy, as encoded in the the time-time component of the energy-momentum tensor, $T^{\mu\nu}(x)$, of the test particle, but it is also sensitive to the energy currents (such as those represented by momentum flux coming from CFR and encoded in the time-space part of the $T^{\mu\nu})(\mathrm{x})$.[7] For this reason, and because these enter

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[5] Note that CAR contribution combined with CFR contribution adds to $(1/4)\hbar\omega$ for kinetic as well as the potential energy.

[6] In the expression below $\psi_0(x)$ represents the ground state wave function for 1-D harmonic oscillator. Also note that $\int_{-\infty}^{\infty} \rho_{0,K}(x)dx = (1/4)\hbar\omega$, consistent with results given in Eqs. (17) and (19).

[7] The initial theoretical lesson learned from the neutron interferometry was that there exist quantum gravitational effects that depend on the test-particle mass. This circumstance arises from the fact that the test-particle mass does not cancel out from the quantum equations of motion despite the equality of the inertial and gravitational masses. What we are now proposing is that since the "gravitational charge" is related to the $T^{\mu\nu}(x)$, one needs to look beyond the time-time component of $T^{\mu\nu}(x)$ in neutron interferometry and consider the neutron $T^{\mu\nu}(x)$ as its gravitational charge. For extended test particles, such as neutrons, this may carry non-trivial physical consequences. The source-$T^{\mu\nu}(x)$ already has a starring role in the theory of general relativity. By fully extending that role to test particles we shall not only introduce a "source–test-particle" symmetry but at the same time open a

phases in quantum mechanical evolution of a quantum systems, CFR may affect gravitational evolutions of a system. Since neutron (and Earth) must carry CFR contributions, the discrepancy observed in the gravitationally induced phases in in the latest neutron interferometry experiments may be probing this anomalous neutron-Earth interaction. The gravitational role played by the energy-momentum tensor is far more intricate in a general-relativistic description of gravity. It becomes even more intricate in a quantum mechanical settings as already suspected by Littrel, Allamn, and Werner because of their experimental findings [13].

Already in 1975 [1], it was *experimentally* established that the equality of gravitational and inertial masses does not imply that the mass of a test particle shall drop out of the quantum-gravity equations of motion. The latest neutron interferometry results suggest that these experiments may have evolved to an extent that some of them are probing the CFRs. The effects of the latter identically vanish if one simply describes the test particle by expectation value of the test-particle $T^{\mu\nu}$.

VI. CONCLUDING REMARKS

In the last decade the propagation through tunneling regions has probed classically forbidden regions with some dramatic, and still controversial, results. Paralleling this development, neutron interferometry, we suggest, may be probing the classically forbidden region where the momentum-density is pure imaginary, and kinetic energy density negative definite. That matter inside a neutron must exist in classically forbidden regions is a general expectation of all quantum mechanical considerations and is independent of specific models in its qualitative aspects.

However, there is more than one aspect of quantum mechanical structure that requires a deeper study in the context of gravity. This has been made abundantly clear in recent years. Fortunately, many of these aspects, such qVEP, may be studied in atomic interferometry and may lie at the heart of the explanation of the solar neutrino anomaly. If this was proven, then in the spirit of the concluding remarks in Ref. [3], we may once again say that while Planck scale physics seems so remote it does not make quantum gravity a science where humans cannot venture to probe her secrets.

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