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A Strategy for a Vanishing Cosmological Constant
in the Presence of Scale Invariance Breaking

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ABSTRACT

Recent work has shown that complex quantum field theory emerges as a statistical mechanical approximation to an underlying noncommutative operator dynamics based on a total trace action. In this dynamics, scale invariance of the trace action becomes the statement $0 = \text{ReTr}T_\mu^\mu$, with $T_{\mu\nu}$ the operator stress energy tensor, and with Tr the trace over the underlying Hilbert space. We show that this condition implies the vanishing of the cosmological constant and vacuum energy in the emergent quantum field theory. However, since the scale invariance condition does not require the operator T_μ^μ to vanish, the spontaneous breakdown of scale invariance is still permitted.

Perhaps the most baffling problem in current theoretical physics [1] is that of understanding the smallness of the observed cosmological constant Λ . The naive expectation is that one should find $\Lambda \sim M_{\text{Planck}}^4$, whereas current observational bounds are 120 orders of magnitude smaller than this, suggesting that there is an exact symmetry principle enforcing vanishing of the cosmological constant. Unfortunately, in standard quantum field theory no such symmetry principle is evident. The two natural candidates are scale invariance and supersymmetry, but the empirical facts that particles have rest masses, and that bosons and fermions have different mass spectra, tell us that both of these symmetries are broken in the observed universe. It is very difficult to understand how either of these symmetries can be broken without the breaking communicating itself to the vacuum sector, thereby leading to an unacceptably large cosmological constant.

Let us explicitly illustrate the problem in the case of scale invariance symmetry, which is the focus of this essay. We consider matter fields quantized on a background metric $g_{\mu\nu}$, with effective stress energy tensor operator $T_{eff\ \mu\nu}$. In the limit of a flat background metric, Lorentz invariance implies that the vacuum expectation of the stress energy tensor has the structure

$$\langle 0|T_{eff\ \mu\nu}|0\rangle = -Cg_{\mu\nu} \quad . \quad (1)$$

This corresponds to a matter vacuum energy contribution of

$$\langle 0|T_{eff\ 00}|0\rangle = C \quad , \quad (2)$$

and to a matter-induced contribution to the cosmological constant of

$$G^{-1}\Lambda_{\text{ind}} = 8\pi C \quad , \quad (3)$$

with G Newton's constant. Contracting Eq. (1) with $g^{\mu\nu}$, we can express C in terms of the vacuum expectation of the Lorentz index trace of the effective matter stress energy tensor, giving

$$C = -\frac{1}{4}\langle 0|T_{eff\ \mu}^{\mu}|0\rangle \quad , \quad (4)$$

$$G^{-1}\Lambda_{ind} = -2\pi\langle 0|T_{eff\ \mu}^{\mu}|0\rangle \quad .$$

Suppose now that we lived in an exactly scale invariant world. Then the bare cosmological constant would have to vanish, and since scale invariance implies [2] that $T_{eff\ \mu}^{\mu} = 0$, by Eq. (4) the matter induced contribution to the cosmological constant would vanish as well, giving a vanishing observed cosmological constant. But, as we have already noted, the assumption of exact scale invariance is not viable: even if we restrict ourselves to theories in which scale invariance anomalies cancel, to be relevant to physics these theories must break scale invariance so that rest masses are present, leading at least to soft mass terms that break the vanishing of $T_{eff\ \mu}^{\mu}$. Part of the problem in trying to use scale invariance as a symmetry to enforce vanishing of the cosmological constant is that it is a much stronger condition than is necessary, since to get the vanishing of a single real number, the cosmological constant, we must impose the vanishing of an operator $T_{eff\ \mu}^{\mu}$, a condition equivalent to the vanishing of an infinite number of real numbers.

To seek a way out of this impasse, we turn to a new kinematic framework [3-6] that we have termed *Generalized Quantum Dynamics (GQD)*. In GQD the fundamental dynamical variables are symplectic pairs of operator-valued variables $\{q_r\}, \{p_r\}$ acting on an underlying Hilbert space; for simplicity we describe here only the case of bosonic operators in a complex Hilbert space, although fermions and real and quaternionic Hilbert spaces are readily incorporated into the formalism. The distinguishing feature of GQD is that no a

priori commutativity properties are assumed for the q 's and p 's. Nonetheless, a theory of flows in the operator phase space can be set up by focusing on *total trace functionals*, defined as follows. Let $A[\{q_r\}, \{p_r\}]$ be any polynomial in the phase space variables (the specification of which depends on giving the ordering of all noncommutative factors). The corresponding real-number valued total trace functional $\mathbf{A}[\{q_r\}, \{p_r\}]$ is defined as

$$\mathbf{A}[\{q_r\}, \{p_r\}] = \text{ReTr}A[\{q_r\}, \{p_r\}] \equiv \mathbf{Tr}A[\{q_r\}, \{p_r\}] \quad , \quad (5)$$

where Tr denotes the ordinary operator trace. We assume sufficient convergence for $\mathbf{Tr} = \text{ReTr}$ to obey the *cyclic property*

$$\mathbf{Tr}\mathcal{O}_1\mathcal{O}_2 = \mathbf{Tr}\mathcal{O}_2\mathcal{O}_1 \quad . \quad (6)$$

Although noncommutativity of the phase space variables prevents us from simply differentiating the operator A with respect to them, we can use the cyclic property of \mathbf{Tr} to define derivatives of the total trace functional \mathbf{A} by forming $\delta\mathbf{A}$ and cyclically reordering all of the operator variations $\delta q_r, \delta p_r$ to the right. This gives the fundamental definition

$$\delta\mathbf{A} = \mathbf{Tr} \sum_r \left(\frac{\delta\mathbf{A}}{\delta q_r} \delta q_r + \frac{\delta\mathbf{A}}{\delta p_r} \delta p_r \right) \quad , \quad (7)$$

in which $\delta\mathbf{A}/\delta q_r$ and $\delta\mathbf{A}/\delta p_r$ are themselves operators. Introducing an operator Hamiltonian $H[\{q_r\}, \{p_r\}]$ and a corresponding total trace Hamiltonian $\mathbf{H} = \mathbf{Tr}H$, the time derivatives of the operator phase space variables (denoted by a dot) are generated by the operator Hamilton equations

$$\frac{\delta\mathbf{H}}{\delta q_r} = -\dot{p}_r \quad , \quad \frac{\delta\mathbf{H}}{\delta p_r} = \dot{q}_r \quad . \quad (8)$$

Applying Eq. (7) to \mathbf{H} and substituting Eq. (8), we learn that \mathbf{H} is a constant of the motion.

Corresponding to the Hamiltonian formalism for GQD there is also a Lagrangian formalism following from a total trace Lagrangian $L[\{\mathbf{q}_r\}, \{\dot{\mathbf{q}}_r\}]$, obtained as the Legendre transform of \mathbf{H} . Using the Lagrangian formalism, repeating the standard Noether analysis [3, 4] shows that for a Lagrangian symmetry parameterized by a c -number parameter κ , there is a conserved *total trace* charge \mathbf{Q}_κ . Thus, in a Poincaré invariant theory there is a conserved total trace stress energy tensor $\mathbf{T}_{\mu\nu}$, and in a Poincaré and scale invariant theory the Lorentz index trace of the total trace stress energy tensor vanishes,

$$\mathbf{T}_\mu^\mu = 0 \quad . \quad (9)$$

There is, however, no operator analog of Eq. (9), and this plays a crucial role in our argument.

We have discussed now two apparently unrelated theories: standard quantum field theory on the one hand, and the classical, noncommutative operator dynamics GQD on the other. In recent work with Millard [7], we have established a surprising relation between the two, by showing that *the statistical mechanics of GQD has a structure isomorphic to complex quantum field theory*. The argument is based on the observation that in addition to \mathbf{H} , there are two other generic conserved quantities in GQD. One is the anti-self-adjoint operator \tilde{C} defined by

$$\tilde{C} = \sum_r [q_r, p_r] \quad , \quad (10)$$

and the other is the natural integration measure $d\mu$ for the underlying operator phase space. Conservation of $d\mu$ gives a GQD analog of Liouville's theorem, and permits the application of statistical mechanical methods [7, 8]. For example, the canonical ensemble [7] is given by

$$\rho = \rho(\tilde{C}, \tilde{\lambda}; \mathbf{H}, \tau) = Z^{-1} \exp(-\text{Tr} \tilde{\lambda} \tilde{C} - \tau \mathbf{H}) \quad , \quad (11)$$

with Z a normalization constant (the partition function) chosen so that $\int d\mu\rho = 1$, and with the ensemble parameters τ and $\tilde{\lambda}$ (the latter an anti-self-adjoint operator) chosen so that the ensemble averages

$$\langle \mathbf{H} \rangle_{AV} = \int d\mu\rho \mathbf{H} , \quad \langle \tilde{\mathbf{C}} \rangle_{AV} = \int d\mu\rho \tilde{\mathbf{C}} , \quad (12)$$

have specified values. In general $\langle \tilde{\mathbf{C}} \rangle_{AV}$ can be brought to the canonical form

$$\langle \tilde{\mathbf{C}} \rangle_{AV} = i_{eff} D , \quad i_{eff} = -i_{eff}^\dagger , \quad i_{eff}^2 = -1 , \quad [i_{eff}, D] = 0 , \quad (13)$$

with D a real diagonal and non-negative operator. The analysis of Ref. 7 studies the simplest case, in which D is a constant multiple of the unit operator in the underlying Hilbert space, and demonstrates an isomorphism between thermodynamic averages in GQD and vacuum expectation values in a complex quantum field theory, with i_{eff} acting as the imaginary unit and with the constant magnitude D playing the role of Planck's constant \hbar . Under this isomorphism, an Hermitian operator \mathcal{O} in GQD corresponds to an effective complex quantum mechanical operator

$$\mathcal{O}_{eff} = \frac{1}{2}[\mathcal{O} - i_{eff}\mathcal{O}i_{eff}] , \quad (14)$$

and the ensemble average $\langle \mathcal{O}_{eff} \rangle_{AV}$ can be interpreted as the vacuum expectation $\langle 0|\mathcal{O}_{eff}|0\rangle$ in the effective quantum field theory.

Let us now assume that the observed universe of quantum fields is in fact the effective field theory arising from the statistical mechanics of an underlying GQD. Then Eq. (4) for the matter-induced cosmological constant can alternatively be written as a GQD ensemble average

$$G^{-1}\Lambda_{ind} = -2\pi \langle T_{eff\ \mu}^\mu \rangle_{AV} = -2\pi \int d\mu\rho T_{eff\ \mu}^\mu . \quad (15)$$

Taking the Tr of Eq. (15), and using the relation $\text{Tr}\mathcal{O}_{eff} = \text{Tr}\mathcal{O}$ which follows from the definition of Eq. (14) and the cyclic property, we get

$$G^{-1}\Lambda_{\text{ind}}\text{Tr}1 = -2\pi\langle\text{Tr}T_{\mu}^{\mu}\rangle_{AV} \quad . \quad (16)$$

Since $\text{Tr}1$ is nonzero (it is the dimension of the underlying Hilbert space), comparing Eq. (16) with Eq. (9) we see that when the underlying GQD is scale invariant, the induced cosmological constant Λ_{ind} vanishes. However, the vanishing of the total trace $\mathbf{T}_{\mu}^{\mu} = \text{ReTr}T_{\mu}^{\mu}$ does not require the vanishing of the operator T_{μ}^{μ} , and so the effective quantum field theory can still break scale invariance and develop nonzero particle masses. By the Lorentz invariance argument of Eqs. (1-4), the vanishing of the right hand side of Eq. (16) also implies the vanishing of $\langle\mathbf{H}\rangle_{AV}$, giving a condition [9] relating the ensemble parameter τ , which has the dimension of an inverse mass, to the dynamically acquired mass scale of the effective theory.

Intuitively, we can describe our picture as follows: In GQD, chaotic motions of the coupled operator degrees of freedom give rise to fluctuations, which in the statistical mechanical limit take the form of the quantum vacuum fluctuations of the emergent complex quantum field theory. Scale invariance in GQD imposes on the underlying fluctuations a single constraint, Eq. (9), that at the complex quantum field theory level translates into a single restriction on observable parameters, the vanishing of the vacuum energy or cosmological constant.

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