Black holes as Gravitational Atoms

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Recently, Almheiri et.al. argued, via a delicate thought experiment, that it is not consistent to simultaneosuly require that (a) Hawking radiation is pure, (b) effective field theory is valid outside a stretched horizon and (c) infalling observers encounter nothing unusual as they cross the horizon. These are the three fundamental assumptions underlying Black Hole Complementarity and the authors proposed that the most conservative resolution of the paradox is that (c) is false and the infalling observer burns up at the horizon (the horizon acts as a "firewall"). However, the firewall violates the equivalence principle and breaks the CPT invariance of quantum gravity, leading Hawking to propose that gravitational collapse may not end up producing event horizons, although he did not give a mechanism for how this may happen. In this essay, we support Hawking's conclusion in a quantum gravitational model of dust collapse. We will show that continued collapse to a singularity can only be achieved by combining two independent and entire solutions of the Wheeler-DeWitt equation. We interpret the paradox as simply forbidding such a combination, which leads naturally to matter condensing on the apparent horizon during quantum collapse.

Classical collapse models suggest that a sufficiently massive self-gravitating system will undergo continued collapse until a singularity forms. In 1975, Hawking [1] pointed out that if an event horizon forms and if effective field theory is valid away from a stretched horizon, then radiation from the black hole is produced in a mixed state from the point of view of the observer who remains outside the black hole provided that the freely falling observer detects nothing unusual ("no drama") while crossing the horizon. Under these conditions information is lost if the black hole evaporates completely, which violates unitarity and led Hawking to propose that quantum mechanics should be modified [2] (he has since changed his mind). To preserve unitarity in quantum mechanics one of two possibilities must be true: (a) Hawking radiation is in fact pure or (b) the evaporation leaves behind a long lived remnant, which preserves all the information that collapsed into the black hole. However, if quantum gravity is CPT invariant then remnants are ruled out and only the first of the two options above remains viable. In 1993, building on the work of 't Hooft [3] and Preskill [4], Susskind et. al. [5] proposed that unitarity could be preserved if information is *both* emitted at the horizon *and* passes through the horizon so that an observer outside would see it in the Hawking radiation and an observer who flies into the black hole would see it inside. No single observer would be able to confirm both pictures. One simply cannot say where the information in the Hilbert space is located, so quantum mechanics is saved at the cost of locality. This is the principle of Black Hole Complementarity

Recently, Almheiri et. al. (AMPS) [6] suggested that the three assumptions of Black Hole Complementarity *viz.,* (a) unitarity of Hawking evaporation, (b) validity of effective field theory outside a stretched horizon and (c) "no drama" at the horizon for a freely falling observer are not self-consistent. Briefly, their argument can be stated as follows. Consider a very large black hole so that a freely falling observer crossing the horizon sees an effectively flat spacetime (on scales much smaller than the horizon length). From the point of view of an observer who stays outside the horizon, the purity of the Hawking radiation implies that late time photons are maximally entangled with some subset of the early radiation. However, these late photons when propagated back from infinity to the near horizon region using effective field theory must be maximally entangled with modes inside the horizon from the point of view of the freely falling observer (this is simply a property of the Minkowski vacuum, appropriate to a freely falling observer). This is not permitted by the strong additivity of entanglement entropy. Assuming then that effective field theory is valid and that Hawking radiation is pure, the paradox can only be avoided if the backward propagated photon is not entangled with a mode behind the horizon. But this would lead to a divergent stress tensor near the horizon, so AMPS concluded that the freely falling observer would burn up before she could cross it. This is the "firewall".

Considerable interest has surrounded the proposed firewall [7], all of it assuming that continued collapse will occur, leading to black holes with event horizons. But Hawking has recently raised several objections to the firewall and suggested that the correct resolution of the AMPS paradox is that event horizons do not form, only apparent horizons form [8]. Radiation from the black hole is then deterministic, but chaotic. In this essay, we will justify this proposal in the context of the exact quantum collapse of spherically symmetric dust, showing that in fact continued collapse can only be achieved by artificially combining two independent solutions of the Wheeler-DeWitt equation, each of which covers the entire spacetime.

The classical spherical collapse of inhomogeneous dust in AdS of dimension $d = n + 2$ is described by the LeMaitre-Tolman-Bondi (LTB) family of metrics [9]. The models may be expressed in canonical form after a series of simplifying canonical transformations and after absorbing the surface terms [10–13]. They are then described in the phase space consisting of the dust proper time, $\tau(t, r)$, the area radius, $R(t, r)$, the mass density, $\Gamma(r)$, and their conjugate momenta, $P_\tau(t, r)$, $P_R(t, r)$ and $P_\Gamma(t, r)$ respectively, by two constraints,

$$\begin{aligned} \mathcal{H}_r &= \tau' P_\tau + R' P_R - \Gamma P_\Gamma' \approx 0 \\ \mathcal{H} &= P_\tau^2 + \mathcal{F} P_R^2 - \frac{\Gamma^2}{\mathcal{F}} \approx 0, \end{aligned} \tag{1}$$

where

$$\mathcal{F} \stackrel{\text{def}}{=} 1 - \frac{F}{R^{n-1}} + \frac{2R^2}{n(n+1)l^2}. \tag{2}$$

with $\Lambda = -l^{-2}$ representing the cosmological constant and $F(r)$ the mass function. The condition $\mathcal{F} = 0$ determines the physical radius of the apparent horizon and is an essential singularity of the wave equation. Dirac quantization of the constraints leads to a Wheeler-DeWitt equation which, for a smooth dust distribution, can be regularized on a lattice. Each point (labeled by "i" below) on the lattice will then represent a collapsing dust shell. Assuming that the wave-functional is factorizable and taking σ to be the lattice spacing, it can quite generally be written as

$$\Psi[\tau, R, \Gamma] = \lim_{\sigma\to 0} \prod_i \psi_i(\tau_i, R_i, F_i) = \exp\left[-\frac{i}{\hbar}\int dr \Gamma(r) \mathcal{W}(\tau(r), R(r), F(r))\right], \tag{3}$$

where each ψ_i resides on the lattice point i and can be thought of as a shell wave function. The wave functional automatically obeys the momentum constraint provided that $\mathcal{W}(\tau, R, F)$ has no explicit $r-$dependence. Independence of the wave functional on the lattice spacing implies that the lattice wave functions must satisfy three equations [13, 14], one of which is the Hamilton–Jacobi equation, which was used to describe the Hawking radiation in [13]. The other two equations together uniquely fix the Hilbert space measure and the factor ordering. For the shell wave functions, ψ_i, one finds the exact positive energy solutions

$$\psi_i = e^{\omega_i b_i} \times \exp\left\{-\frac{i\omega_i}{\hbar}\left[a_i \tau_i \pm \int^{R_i} dR_i \frac{\sqrt{1 - a_i^2 \mathcal{F}_i}}{\mathcal{F}_i}\right]\right\}, \tag{4}$$

where $a_i = 1/\sqrt{1 + 2E_i}$ is related to the energy function, $\omega_i = \sigma\Gamma_i/2$ and the factor $e^{\omega_i b_i}$ is a normalization.

These wave-functions are well defined everywhere except at the apparent horizon, where there is an essential singularity. In order to match interior to exterior solutions, we deform the integration path in the complex R_i-plane so as to go around the essential singularity at $\mathcal{F}_i = 0$ [15, 16]. This is similar to the quasi-classical tunneling approach employed in various semi-classical analyses [17] (the deformed path does not correspond to the trajectory of any classical particle). The direction of the deformation is chosen so that positive energy solutions decay. One finds the following solution representing collapse with support everywhere in spacetime [16]:

$$\psi_i^{(1)}(\tau_i, R_i, F_i) = \begin{cases} e^{\omega_i b_i} \times \exp\left\{-\frac{i\omega_i}{\hbar}\left[a_i\tau_i + \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i > 0 \\ e^{-\frac{\pi\omega_i}{\hbar g_{i,h}}} \times e^{\omega_i b_i} \times \exp\left\{-\frac{i\omega_i}{\hbar}\left[a_i\tau_i + \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i < 0 \end{cases} \quad (5)$$

where $g_{i,h}$ is the surface gravity at the apparent horizon. It represents dust shells condensing to the apparent horizon on both sides of it but the interior, outgoing wave appears with a relative probability of $e^{-2\pi\omega_i/\hbar g_{i,h}}$, which is the Boltzmann factor for a shell at the "Hawking" temperature, $T_{i,H} = \hbar g_{i,h}/2\pi k_B$.

Another independent solution exists, with support everywhere and the same deformation of the integration path,

$$\psi_i^{(2)}(\tau_i, R_i, F_i) = \begin{cases} e^{-\frac{\pi\omega_i}{\hbar g_{i,h}}} \times e^{\omega_i b_i} \times \exp\left\{-\frac{i\omega_i}{\hbar}\left[a_i\tau_i - \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i > 0 \\ e^{\omega_i b_i} \times \exp\left\{-\frac{i\omega_i}{\hbar}\left[a_i\tau_i - \int^{R_i} dR_i \frac{\sqrt{1-a_i^2\mathcal{F}_i}}{\mathcal{F}_i}\right]\right\} & \mathcal{F}_i < 0 \end{cases} \quad (6)$$

Here, dust shells move away from the apparent horizon on either side of it but this time the exterior, outgoing wave is suppressed by the Boltzmann factor at the Hawking temperature for the shell.

In principle we may take the general solution representing the collapsing dust ball to be a linear combination of the two solutions (5) and (6),[1]

$$\psi_i = \psi_i^{(1)} + A_i \psi_i^{(2)}. \quad (7)$$

However, there is nothing within the theory that suggests a value for A_i and further input is needed to determine these amplitudes. Note that if $0 < |A_i| \leq 1$, the dust will ultimately pass through the apparent horizon in a continued collapse on its way to a central singularity and an event horizon will form. This process will be accompanied by thermal radiation in the exterior. We therefore see that the AMPS paradox provides the required additional input. To avoid a firewall, $|A_i|$ must vanish and therefore (5) *alone* is a complete description of the quantum collapse. But this solution says that each shell will condense to the apparent horizon and will not undergo further collapse; there is no tunneling into the exterior and no firewall. As each shell converges to the apparent horizon, a "dark star" forms. The density profile of such a dark star will depend on the initial data, but we can expect that it will attain very high densities in the central regions. Even so, provided that the initial data respect cosmic censorship, no central singularity can form.

[1] Strictly speaking, one superposes the wave functionals constructed separately out of $\psi_i^{(1)}$ and $\psi_i^{(2)}$ according to (3). Then one finds a gauge invariant relative probability for the exterior, outgoing wave-functional of $|\mathcal{A}|^2 e^{-S}$, where $\mathcal{A} = \prod_i A_i$ and S is the Bekenstein-Hawking entropy of the AdS black hole [16].

Thus, in view of the AMPS paradox, an entirely new picture of the black hole has emerged. Instead of a spacetime singularity covered by an event horizon we will have an essentially quantum object, an extremely compact dark star, which is held up not by any degeneracy pressure but by quantum gravity just as ordinary atoms are sustained by quantum mechanics. Astronomical observations [18, 19] indicate that astrophysical black holes possess dark surfaces and this is consistent with the picture we have just described.

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