

Dark energy with rigid voids versus relativistic voids alone

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Abstract

The standard model of cosmology is dominated—at the present epoch—by dark energy. Its voids are rigid and Newtonian within a relativistic background. The model prevents them from becoming hyperbolic. Observations of rapid velocity flows out of voids are normally interpreted within the standard model that is rigid in comoving coordinates, instead of allowing the voids' density parameter to drop below critical and their curvature to become negative. Isn't it time to advance beyond nineteenth century physics and relegate dark energy back to the “no significant evidence” box?

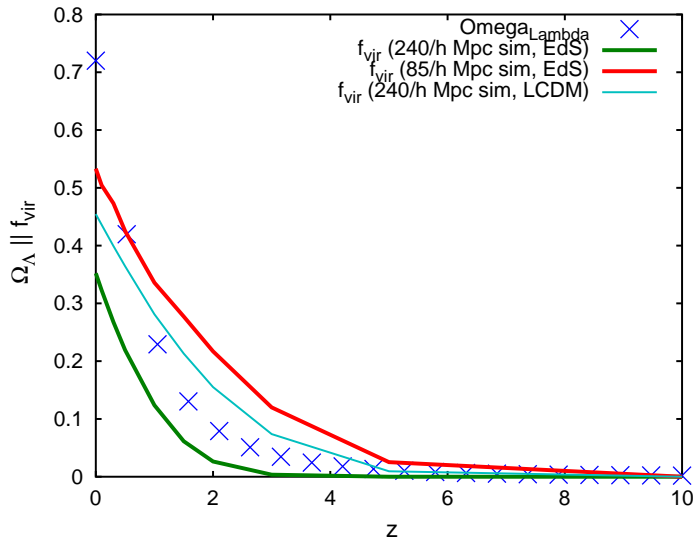


Figure 1: Virialisation fraction $f_{\text{vir}}(z)$ in Virgo Consortium 256³-particle simulations [1, 2] with 240 h^{-1} Mpc and 85 h^{-1} Mpc box sizes, shown as continuous thick curves for the Einstein-de Sitter model, and dark energy parameter $\Omega_{\Lambda}(z)$ evolution for $\Omega_{\Lambda 0} = 0.72$. Thin curve: 240 h^{-1} Mpc Λ CDM simulation.

1 “Dark energy” traces inhomogeneity

The standard model of cosmology is generally accepted to be a spacetime with a Friedmann–Lemaître–Robertson–Walker (FLRW) metric [3, 4, 5, 6, 7], solving the Einstein equation rather simply thanks to the assumption of homogeneous density on any spatial slice, and measured to have the Concordance Model [8] values of the matter density and dark energy parameters, $\Omega_{\text{m}0} \approx 0.32$ (e.g. [9]) and $\Omega_{\Lambda 0} := 1 - \Omega_{\text{m}0}$, respectively (hereafter, Λ CDM). However, we live in the inhomogeneous epoch: galaxies and voids certainly exist today. As found by [10], in an Einstein-de Sitter (EdS) FLRW model (i.e. with $\Omega_{\text{m}0} = 1, \Omega_{\Lambda 0} = 0$), the fraction of matter in a large region that is virialised, f_{vir} , evolves in a very similar way to that of the dark energy parameter in a flat FLRW model with negligible radiation density,

$$\Omega_{\Lambda}(z) = 1 - \frac{\Omega_{\text{m}0}}{a^3 \Omega_{\Lambda 0} + \Omega_{\text{m}0}}. \quad (1)$$

This is shown in Fig. 1, where $\Omega_{\Lambda 0} = 0.68$.

This seems like an extraordinary coincidence. Over the same redshift range during which one *expects* that the Universe is inhomogeneous, the degree of inhomogeneity, as expressed by $f_{\text{vir}}(z)$ in an EdS model, approximately follows the proportion of the critical density represented by dark energy, if the dark energy is inferred from forcing a *homogeneous model* on the observational data. To first order, f_{vir} is not sensitive to the choice of FLRW model (see Fig. 1), so the coincidence also exists for inhomogeneity in a Λ CDM model.

The simplest inference is that a homogeneous-model-inferred non-zero dark energy parameter is really just *a measurement of inhomogeneity*.

What physical link could there be between this inhomogeneity parameter and homogeneous-model-inferred “dark energy”?

2 Void dominance: low matter density, high critical density

The most obvious physical link between inhomogeneity and homogeneous-inferred dark energy is the volume dominance of voids compared to virialised regions at recent epochs [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. This is because gravitational collapse implies an increase in density, i.e., a reduction in volume, by a factor of about $\delta_{\text{vir}} \sim 100\text{--}200$ (e.g., $8\text{--}18\pi^2$ [21]), so that, to first order, the collapsed matter occupies a negligible fraction of the spatial volume. Thus, recent-epoch spatial volume is overwhelmingly dominated by low-density regions.

Moreover, there are velocity flows out of the voids, since otherwise, the voids couldn’t be nearly empty. Thus, the critical density defining spatial flatness in the voids is higher than it would be in a homogeneous calculation. The standard, FLRW approach insists that space expands uniformly with spatially constant curvature, i.e. space is rigid in comoving coordinates—it is forbidden from bending under the influence of gravity. N -body simulations are typically used to study the *Newtonian* formation of overdense structures and voids within this rigid, comoving background model. However, both the low density of the voids and the velocity flow out of them imply that the matter density parameter in the voids is sub-critical. Thus, the voids are hyperbolic. Geometrically, this hyperbolicity should also be taken into account, implying an even lower matter density parameter (conservatively, let us ignore this: the effect is small).

3 Volume-weighted averaged metric

These arguments can be formalised using the volume-weighted averaging approach to modelling inhomogeneous spatial slices [22, 23, 11, 24, 16], in which the Friedmann equation is generalised to (12) of [13]. For simplicity, let us (i) set the dark energy term to zero, (ii) neglect the kinematic backreaction as much smaller than the curvature backreaction (see [16] for numerical justification), and (iii) combine the curvature parameters into a single full curvature parameter as suggested in [13]. Writing “k” instead of “ \mathcal{R} ”, this gives the domain-averaged, effective Friedmann equation

$$\Omega_{\text{k}}^{\text{eff}}(z) = 1 - \Omega_{\text{m}}^{\text{eff}}(z). \quad (2)$$

Along a typical, large-scale, random, spacelike or null geodesic over recent epochs, what proportions of the geodesic lie in the emptied and virialised

regions? The proportions at a given z are, on average, $(1 - f_{\text{vir}}/\delta_{\text{vir}}) : f_{\text{vir}}/\delta_{\text{vir}}$, respectively. Given that $\delta_{\text{vir}} \sim 100\text{--}200$ and

$$0 \leq f_{\text{vir}} \leq 1, \quad (3)$$

less than about 1% of the geodesic falls within the virialised regions. Thus, by starting with a large-scale, high-redshift, “background” FLRW model—in this case, an EdS model—an effective metric can be written by assuming that the virialised matter contributes negligibly. As in [10], the effective expansion rate is

$$H^{\text{eff}}(z) = H(z) + H_{\text{pec}}(z), \quad (4)$$

combining the FLRW expansion $H(z)$ with the peculiar velocity gradient (physical, not comoving) across voids, $H_{\text{pec}}(z)$, estimated numerically from an N -body simulation ([1, 2], a $240 h^{-1}$ Mpc box-size EdS simulation; see [10]) The background Hubble constant H_{bg} is set to make the present value consistent with low redshift estimates [25, 26], i.e.

$$H_{\text{bg}} := 74 \text{ km/s/Mpc} - H_{\text{pec}}(0) \sim 50 \text{ km/s/Mpc}. \quad (5)$$

Thus, the loss of matter from the voids and the higher critical density in voids both decrease the EdS background value from $\Omega_{\text{m,bg}} = 1$ to an effective value of

$$\begin{aligned} \Omega_{\text{m}}^{\text{eff}}(z) &\approx (1 - f_{\text{vir}}) \left(\frac{H}{H^{\text{eff}}} \right)^2 \Omega_{\text{m}} \\ &= (1 - f_{\text{vir}}) \left(\frac{H_{\text{bg}}}{H^{\text{eff}}} \right)^2 \Omega_{\text{m,bg}} a^{-3}. \end{aligned} \quad (6)$$

The effective radius of curvature is

$$R_{\text{C}}^{\text{eff}}(z) = \frac{c}{a H^{\text{eff}}(z) \sqrt{\Omega_{\text{k}}^{\text{eff}}(z)}}. \quad (7)$$

The effective metric (cf [17]) is

$$ds^2 = -dt^2 + a^2(t) \left[d\chi^{\text{eff}2} + R_{\text{C}}^{\text{eff}2} \left(\sinh^2 \frac{\chi^{\text{eff}}}{R_{\text{C}}^{\text{eff}}} \right) (d\theta^2 + \cos^2 \theta d\phi^2) \right], \quad (8)$$

where the radial comoving component is

$$d\chi^{\text{eff}}(z) := \frac{c}{a^2 H^{\text{eff}}(z)} da. \quad (9)$$

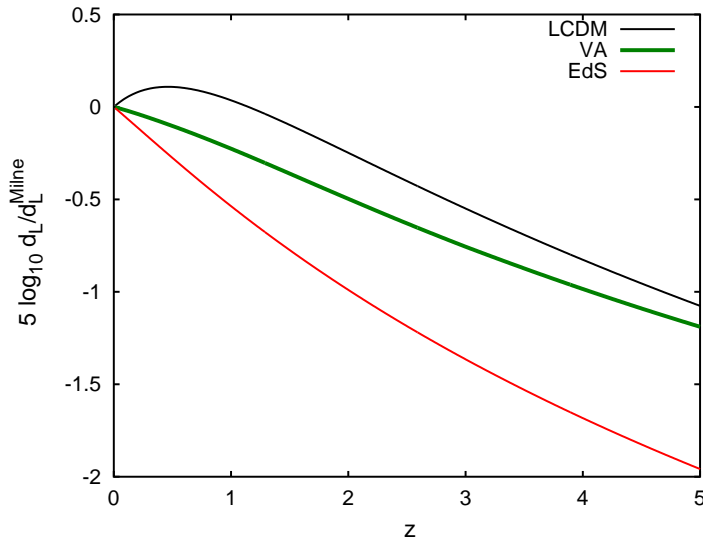


Figure 2: Distance modulus normalised to the Milne model ($\Omega_m = 0, \Omega_\Lambda = 0, \forall z$) for the homogeneous Λ CDM model (top, black), the uncorrected, homogeneous EdS model (bottom, red), and the void-corrected EdS model (middle, thick, green; “VA” = virialisation approximation).

4 Matter density parameter and luminosity distances

Without any attempt to fit this approximation to observational data, apart from (5) above, the correction of the EdS model as presented above gives an effective matter density parameter (6) that drops slowly from its background value of unity at high redshift down to $\Omega_m^{\text{eff}} = 0.27$ at the present epoch $z = 0$, remarkably close to the last two decades’ local estimates of the matter density parameter.

The effective luminosity distance follows directly from the radial comoving distance and hyperbolicity,

$$d_L^{\text{eff}} = (1 + z)R_C^{\text{eff}} \sinh \frac{\chi^{\text{eff}}}{R_C^{\text{eff}}}. \quad (10)$$

Figure 2 shows that despite the rough nature of the virialisation approximation, it shifts the homogeneous EdS magnitude–redshift relation by a substantial fraction towards the homogeneous Λ CDM relation, and thus, towards the observational supernovae type Ia relation.

5 Conclusion

A handful of simple formulae, lying at the heart of homogeneous, spatially rigid cosmology, remain approximately valid when generalised to inhomogeneous, spatially flexible cosmology [13] and applied to what observationally

and theoretically dominate the present-day spatial volume—the voids. The result is a correction to a large-scale, high-redshift, background Einstein-de Sitter cosmological model. The correction approximately gives the observed low-redshift matter density parameter and nearly matches the type Ia supernovae luminosity distance relation. The amplitude of the correction is unlikely to be much smaller than estimated here. At the present epoch, direct observations [27], N -body simulations [1, 2], and the existence of the cosmic web itself establish inhomogeneous peculiar velocity gradients of $\sim 20\text{--}30$ km/s/Mpc, forcing, at least, a factor of $\sim (75/50)^2$ reduction of the Einstein-de Sitter matter density to $\Omega_m^{\text{eff}} < 0.5$, via Eqs (4), (5), and (6). A virialisation fraction of the order of $\sim 50\%$ reduces this to ~ 0.25 . Even when forcing the homogeneous FLRW models onto the data, the initial analysis of the Planck Surveyor cosmic microwave background data finds $H_{\text{bg}} = 67.3 \pm 1.2$ km/s/Mpc at $z \approx 1100$ [9]—not as low as the velocity gradients imply, but still significantly lower than the low redshift estimates of $H^{\text{eff}}(0) = 74.0 \pm 1.6$ km/s/Mpc ([26, 25], standard error in the mean).

How could such a simple, back-of-the-envelope calculation have been missed for so long? While the volume-averaging approach to cosmology has been developed over many years (see e.g. [13] for a review), possibly the answer lies, ironically, in confusion between the *spacelike, unobserved, comoving*, present time slice and the past light cone. A gigaparsec-scale void in the former should have a very weak ($\delta \sim 10^{-5}$) underdensity, and our would-be location at its centre would be uncomfortably anti-Copernican. But these are both moot points! On the past light cone, an *average*, gigaparsec-scale, sub-critical ($0 < \Omega_m^{\text{eff}} < 0.8$) void is perfectly natural, since it is defined by the onset of the virialisation epoch at $z \lesssim 3$. Moreover, we are naturally located at this pseudo-void’s centre, by the nature of the past light cone.

What is simpler: relativistic, hyperbolic voids, observed by an observer at the tip of the past light cone, with no dark energy parameter? Or rigid, Newtonian voids together with a dark energy parameter that traces the virialisation fraction?

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