A microscopic model for an emergent cosmological constant

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Abstract

The value of the cosmological constant is explained in terms of a noisy diffusion of energy from the low energy particle physics degrees of freedom to the fundamental Planckian granularity which is expected from general arguments in quantum gravity. The quantitative success of our phenomenological model is encouraging and provides possibly useful insights about physics at the scale of quantum gravity.

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Naive vacuum energy estimates of the value of the cosmological constant produces results that are 120 orders of magnitude larger than those extracted from observations [1]. Alternative considerations based on protective symmetries set their value to zero. The usual analysis largely relies on standard notions of spacetime and fields which are smooth to all scales; however, various arguments suggest that the physics of the continuum should only emerge from an underlying discrete reality at the fundamental scale. We will remain agnostic about the exact nature of the underlying fundamental physics, yet we will take seriously the central idea that nature is discrete at the Planck scale. We shall argue that this assumption opens the door for a fresh new look into the cosmological constant problem.

The molecular structure of matter has important macroscopic consequences. The most ubiquitous being friction: the generic tendency of energy to leak from macroscopic scales to the underlying microscopic chaos. Similarly, if the continuum is emergent, energy should 'diffuse' from the large-scale degrees of freedom (represented by matter fields in spacetime) down to the fundamental granular structure. For reasons stated below, the diffusion we postulate is tied to the presence of a non-trivial curvature. This makes it virtually undetectable in local laboratory searches involving essentially flat spacetime regions [1]. The exception however is provided by the physics of the very early universe, where curvature and matter densities become large.

For an individual particle, such diffusion effects are most reasonably encoded in a deviation from geodesic motion. The discreteness that sources friction is associated with the Planck scale $\ell_p = m_p^{-1}$ which, in a relational spirit, must be identified within the rest frame of the particle which only exists if the excitation is massive. The presence of massive degrees of freedom (and the associated breaking of scale invariance) is signaled by a non-vanishing value of the trace of the energy momentum tensor $T = g^{\mu\nu}T_{\mu\nu}$ which, via Einstein's equations [2], can be related to the scalar curvature $R = -8\pi GT$.

Therefore, it is natural to postulate that such a 'friction' force must be proportional to R. In addition, the force should depend on the mass m, the 4-velocity u^μ, the spin s^μ of the

[1] The simplistic view where Planckian discreteness is tied to a globally defined preferred frame seems very tightly constrained [2, 3]. The idea, inspiring in part the present model, that the granularity should rather be associated to a frame locally determined by geometrical features (e.g. curvature) and/or the matter distribution and some of its phenomenological implications susceptible to laboratory testing were considered in [4–8].

[2] In our model Einstein's equations suffer only small corrections.

classical particle (the only intrinsic features defining a particle), and a time-like unit vector ξ^μ specifying the local frame defined by the matter that curves spacetime. Dimensional analysis gives an essentially unique expression which is compatible with the above requirements [3]

$$u^\mu \nabla_\mu u^\nu \;=\; -\alpha\, m\, \mathrm{sign}(s\cdot\xi) \frac{R}{m_p^2}\, s^\nu \tag{1}$$

where $\alpha > 0$ is a dimensionless coupling [4].

In cosmology $\xi = \partial_t$ is the time-arrow of the co-moving cosmic fluid. The factor $\mathrm{sign}(s\cdot\xi)$ makes the force genuinely friction-like. This is apparent when one computes the behaviour of the mechanical energy of the particle $E \equiv -mu^\nu \xi_\nu$ (in the frame defined by ξ^μ) which yields

$$\dot{E} \equiv -mu^\mu \nabla_\mu (u^\nu \xi_\nu) = -\alpha \frac{m^2}{m_p^2} |(s\cdot\xi)| R - mu^\mu u^\nu \nabla_{(\mu}\xi_{\nu)}. \tag{2}$$

The last term in (2) encodes the standard change of E associated to the non-Killing character of ξ^μ. The first term on the right encodes the friction that damps out any motion with respect to ξ^μ. Energy is lost into the fundamental granularity until the particle is at rest with the cosmological fluid, i.e., $u^\mu = \xi^\mu$, and thus $\dot{E} = 0$.

Fermions are fundamentally quantum objects which arguably interact directly with the physics at the Planck scale. The presence of spin on the r.h.s. of (1) is of course consistent with this view. Another important peculiarity of fermions is captured by the spin-statistics theorem (Pauli's exclusion principle). Fermions are sources of torsion which can be viewed as local defects in the Riemannian geometry [5] . This implies that Fermions require the use of tetrads and connections (e, ω) when considering their coupling to gravity (e.g. the Einstein-

[3] Higher curvature corrections could be added, but these are highly suppressed by the Planck scale and are thus negligible for the central point of this essay.

[4] It is important to point out that the violations of the equivalence principle and Lorentz invariance implied by (1) can be checked to avoid conflict with present observational bounds by many orders of magnitude [9]. A simple indication comes from comparison of the values of curvature at the EW transition in cosmology to that associated with, say, the gravitational effect of a piece of lead, which gives $\frac{R_{lead}}{R_{EW}} \sim 10^{-24}$.

[5] In this respect, notice that the characterization of WKB-trajectories of the Dirac theory on a pseudo-Riemannian geometry [10] which, to lowest order in $\hbar$, is given by

$$u^\nu \nabla_\nu (mu_\mu) = -\frac{1}{2} \tilde{R}_{\mu\nu\rho\sigma} u^\nu \langle S^{\rho\sigma} \rangle + \mathscr{O}(\hbar^2). \tag{3}$$

The previous is aquivalent to (1) if we introduce an effective $\tilde{R}_{\mu\nu\rho\sigma} \propto m^2/m_p^2\, \mathrm{sign}(s\cdot\xi) R \epsilon_{\mu\nu\rho\sigma}$ which encode a pure torsion structure as $\tilde{R}_{[\mu\nu\rho]\sigma} \neq 0$ (from the first Bianchi identities). Note also the similarity between (1) and the Mathisson-Papapetrou-Dixon equations [11].

Cartan formalism or any of its generalizations). The torsion part of ω can be integrated-out producing effective Fermi four-fermion-interactions with a coupling constant $\ell_p^2 = m_p^{-2}$ [12, 13]. As for gravity itself, the emergence of such non-renormalizable interactions implies that a consistent gravitational coupling of fermions must necessarily have an effect at the fundamental scale. This has led to appealing pure-fermion proposals for the unification of interactions [14–16]. In non-perturbative approaches to quantum gravity (e.g. loop quantum gravity [17]) Fermions produce Planck scale defects in the quantum geometry. We have become accustomed to treat them through the use of Grassmann variables in path integrals, but, in our view, Fermions remain enigmatic objects whose intrinsic nature is tightly related to the fundamental structure of spacetime. All this naturally suggests that Fermions interact directly with the Planckian granular structure from which classical spacetime is expected to emerge. Although Eqn. (1) is expressed in quasi-classical terms, which, for fermions, is strictly speaking problematic (see however footnote 5), we nevertheless find it significant that it encodes our expectations in such a natural way.

The non-conservative force in (1) leads to a specific form of violation of the energy momentum tensor describing a fluid composed of such particles. Yet this violation is inconsistent with general relativity (GR). Fortunately, there is a slight generalization of GR [18], called *unimodular gravity* (UG), where this limitation is overcome [19]. In UG Einstein's equations are replaced by the trace free equations

$$R_{\mu\nu} - \frac{1}{4} R\, g_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right). \tag{4}$$

Defining $J_\mu \equiv (8\pi G) \nabla^\nu T_{\nu\mu}$, assuming the unimodular integrability $dJ = 0$ [19], and using Bianchi identities, one obtains

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \underbrace{\left[\Lambda_* + \int_\ell J \right]}_{\text{Dark Energy } \Lambda} g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{5}$$

where Λ_* is an integration constant. Note that the current J sources a term in effective Einstein's equations that behave as dark energy [6].

Let us focus on the dynamics of the early universe when its macroscopic geometry is well

[6] As noted in [20, 21], 'vacuum energy' needs not gravitate in the context of UG, aleviating the tension between quantum field theory and cosmology [22].

approximated by the flat FLRW metric

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \tag{6}$$

and where the local frame $\xi = \partial_t$ is identified with comoving observers. The cosmological fluid is defined by an ensemble of quasi-classical particles characterized by a distribution in phase space satisfying a relativistic Boltzman equation with the external force coming from (1) [23]. With this one can compute the exact form of $J_\nu \equiv (8\pi G)\, \nabla^\mu T_{\mu\nu}$ in thermal equilibrium at temperature $\mathscr{T}$. Considering $\mathscr{T} \gg m$ (the relevant regime for what follows) one obtains:

$$J_\nu \equiv (8\pi G)\, \nabla^\mu T_{\mu\nu} = \alpha \frac{4}{3\pi} \frac{\mathscr{T}}{m_p^2} R^2 \xi_\nu. \tag{7}$$

As only massive particles with spin are subjected to the frictional force (1), the diffusion mechanism in cosmology starts when such particles first appeared. According to the standard model—whose validity is assumed from the end of inflation—this corresponds to the electroweak (EW) transition time. We further assume that a protective symmetry enforces $\Lambda_* = 0$ (see for instance [24, 25]). From (5) we get

$$\Lambda = \frac{4}{3} \frac{\alpha}{\pi m_p^2} \left(\int_{t_0}^{t} \mathscr{T}(t) R(t)^2 dt \right) \tag{8}$$

The integral in (8) can be performed using the standard model of cosmology. The results are shown in Figure 1: the dark energy component in (8) rapidly approaches a constant, and its value fits the observed value for $\alpha \approx 1$ and $\mathscr{T}_{ew} \approx 100\,\text{GeV}$. A rough estimate of the calculation can be expressed in terms of the scales involved. The result is dominated by the top quark; the most massive excitation at $\mathscr{T}_{ew}$, thus

$$\Lambda \approx \frac{\overline{m}_t^4 \mathscr{T}_{ew}^3}{m_p^7} m_p^2 \approx \underbrace{\left(\frac{\mathscr{T}_{ew}}{m_p} \right)^7}_{10^{-120}} m_p^2, \tag{9}$$

where $\overline{m}_t$ is the mean top mass m_t that goes from zero to 170 GeV during the EW-transition[7]. We have taken $\overline{m}_t \approx \mathscr{T}_{ew}$ in the second approximation. This indicates that the present accelerated expansion of the universe might simply be the result of a noisy diffusion of energy into the underlying granularity of spacetime.

[7] Massive gauge bosons do not change the order of magnitude estimate, as $m_Z/m_t \approx 1/2$ and $g_{ZW^\pm}/g_{t\bar{t}} = 3/4$. In (8) this leads to a factor $(3/4)^2(1/2)^4$ which is about only 3.5% of the top-quark contribution.

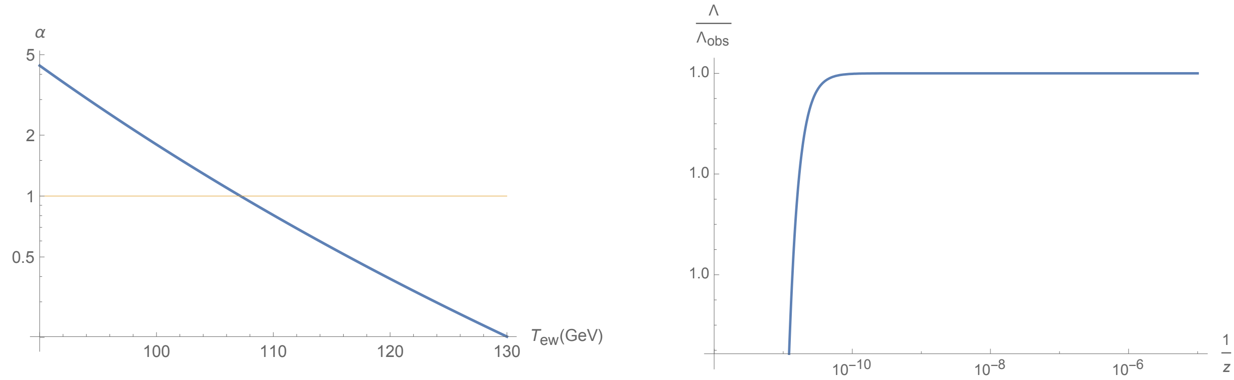

Figure 1: **Left:** The value of the phenomenological parameter α that fits the observed value of Λ_{obs} as a function of the EW transition scale $\mathscr{T}_{\rm ew}$ in GeV. We see that for $\mathscr{T}_{\rm ew} \approx 100GeV$ $\alpha \approx 1$. **Right:** The time dependence of Λ expressed in terms of the inverse redshift factor $1/z$.

Torsion might be the simplest and most direct characterization of the defects in the emergent spacetime geometry that is responsible for our diffusion effect. But it can be at best only a small part of a bigger, as yet unknown, story.

The situation may be likened to that present during the birth of the quantum theory. From the very beginning, 'discretization' of phase space was strongly indicated (e.g. the Rayleigh-Jeans catastrophe). However, going from simple subdivision of phase space into cells *á la* Boltzmann to Dirac's ultimate formulation ('replace Poisson brackets by commutators') required three decades of a great deal of experimental and theoretical work. With regard to spacetime discretization, we should expect no less.

The above quantum-theory example underlines the importance of finding the descriptive language most appropriate for the problem at hand. In addition to the relevance of torsion, as expressed in the first order O(3,1) gauge-gravity formalism, there are other options available. The relevance of the vacuum topological structure is suggested by the O(4,1) extension of gauge gravity, as well as loop quantum gravity. Quantized areas and volumes are also suggested by loop quantum gravity. And a variety of theory extensions (e.g. M-theory) are suggested by string theory.

We are used to the expectation that discoveries of physics at microscopic scales often provide clues about physics at larger scales. But the reverse happens as well. Could it be that in this case the physics of the largest scales (cosmology) is providing us with clues about the physics at extremely small scales? We might be reassured of this possibility by considering the manner in which regularities found in the study of chemical reactions uncovered the first empirical insights for what would become the atomic theory, in contrast with the purely conceptual considerations that inspired old dear Democritus.

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