Secret Life of the Spacetime

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Abstract

Just as the thermal properties of normal matter *demands* the existence of microscopic degrees of freedom, the thermal properties of null surfaces — perceived as local Rindler horizons by accelerated observers — demands the existence of microscopic degrees of freedom to spacetime. The distortion of the null surfaces, just like the deformation of an elastic solid, costs entropy. I show how, just like in the case of an elastic solid, one can describe the dynamics of the *spacetime solid* by introducing an entropy density to the distortion of null surfaces in the spacetime.

1 Warm up: Secret life of matter

Imagine for a moment that you are a physicist working in the late eighteenth century. Solids, fluids and gases appear to be continuous media to you described by density field, $\rho(t, \mathbf{x})$, velocity field, $\mathbf{v}(t, \mathbf{x})$, etc. which are continuous functions on the spacetime. You also understand, by and large, the behaviour of such systems *except for* two puzzling features: (a) The dynamical equations describing the system require certain parameters (like the elastic constants of the solid, viscosity of a fluid, specific heat of a gas etc.) and you have no clue how to determine these. (b) You need to attribute an extra physical variable called temperature which could also be, in general, a function on the spacetime, $T(t, \mathbf{x})$, related to another mysterious quantity which you have been calling the heat content of the body through a set of empirical laws of thermodynamics. For example, an ideal gas seems to obey an equation like $PV = RT$ connecting two mechanical, kinematical variables P, V to a thermodynamical variable T. You also have a notion of the Avogadro's number for a gas but you are not clear what it counts!

Then comes Boltzmann and reveals the secret life of matter. He demolishes the idea that the matter is a continuum. Instead he suggests that it has discrete microscopic structures, the random motion of which leads to what you have

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been calling heat energy. The impact of these microscopic particles on the walls of the container during their random motion can provide a *microscopic* explanation for the law $PV = RT$ if we rewrite it as $PV = Nk_BT$ where N measures the number of microscopic degrees of freedom and k_B is the new constant introduced by Boltzmann. This constant plays a crucial role in relating the microscopic degrees of freedom N to two thermodynamic parameters: The energy E and the temperature T through the law of equipartition:

$$\Delta N = \frac{\Delta E}{(1/2)k_BT}; \qquad E = \frac{1}{2}k_B \int T \frac{dN}{dV} \, dV \tag{1}$$

which tells you how many microscopic degrees of freedom are needed to store an energy ΔE in a region of gas, say, having a temperature T. The right hand side of the first equation is made of thermodynamical variables while the left hand side describes a purely microscopic quantity which has no meaning in the continuum physics.

Boltzmann also provided a firmer foundation for the concept of entropy S (through the relation $S = k_B \ln \Omega$). In fact, we now base the macroscopic description of matter on the thermodynamic function $S(E, V)$ or any other related thermodynamic potentials (like free energy, enthalpy, ...) derivable from S. You can have a metal rod or a glass of water at the same temperature and hence T cannot really distinguish the properties of the system. On the other hand, the function $S(E, V)$ will be quite different for the two systems and the dynamics is encoded in the form of the entropy function.

If matter is made of discrete structures, how come it appears continuous? We now know — a century and more after — that this has to do with the length scales involved in the problem. The discrete structures occur at the scales of, say, 10^{-7} cm which is 7 orders below the everyday scales of physics which the eighteenth century physicists are familiar with. But this secret structure of matter, hidden from direct experimentation, reveals itself through thermodynamics. No micro structure, no storage of energy in microscopic degrees of freedom and hence no heat content or temperature. *So, even without direct evidence, just by knowing the existence of temperature and heat, one can reason out the necessity for micro structure for matter.*

2 Secret life of Spacetime

Move on from matter to spacetime. All evidence suggests that we physicists today are probably at the similar level of ignorance regarding the microscopic structure of spacetimes at Planck scales (characterized by $L_P \equiv (G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm) as the late eighteenth century physicists were about the micro structure of matter! We describe the spacetime by the fields like metric $g_{ab}(t, \mathbf{x})$, curvature $R_{ijkl}(t, \mathbf{x})$ etc. just as the eighteenth century physicists described matter in terms of density, velocity etc. The spacetime might appear to be continuous even at the highest energies ($\sim$ TeV) we have probed because the Planck scale (10^{19} GeV) is still 16 orders of magnitude away! So, is spacetime a continuous structure all the way or does it have discrete microstructure like in the case of matter?

In the absence of any hope for direct observations, we again need to use the thermodynamic arguments just as Boltzmann did. Remarkably enough, these arguments yield rich dividends and gives us a window to peek into the secret life of spacetime. This is based on a whole series of peculiar circumstances related to spacetime dynamics which we will now review rapidly.

Principle of equivalence and a judicious choice of thought experiments [1] tell us that gravity can be described in terms of the metric and hence directly influences the causal structure of spacetime. Around any event $\mathcal{P}$ in the spacetime, one can introduce a locally inertial frame (LIF) of freely falling observer as well as local Rindler frames (LRF) of uniformly accelerated observers. What is more, the null surfaces of the LIF will act as a local Rindler horizon in LRF and will be perceived to have a temperature $T = \kappa/2\pi$ where κ is the acceleration of the observer. (The acceleration sets a timescale $(1/\kappa)$ and the local arguments are valid if $(\dot{\kappa}/\kappa^2) \ll 1$ holds; this can always be achieved for a suitable class of observers.) These observers will also attribute an entropy per unit area of the Rindler horizon, which — in simplest context — will be a constant.

These features tell you that: (a) Spacetimes, like matter, can be hot and possess temperature and entropy. (b) All thermodynamics is observer dependent. The latter conclusion should not come as a surprise since any material can be thought of as a highly excited state of the quantum vacuum; the Davies-Unruh effect [2] tells us that the temperature attributed to the *vacuum* itself depends on the observer and hence the temperature attributed to any highly excited state — say, a cup of water — will also be observer dependent.

Further theoretical evidence about spacetime micro structure comes in a form [3] very similar to the eighteenth century physics of matter [4]. For example, in a wide class of theories of gravity called Lanczos-Lovelock models — which include Einstein's theory as a special case — the gravitational field equations reduce to a thermodynamic identity on any horizon. What distinguishes the different theories — i.e., the different dynamics the "spacetime solid" can exhibit — is encoded in a divergence free entropy tensor P^{ab}_{cd} which has the symmetries of curvature tensor and is related to the Lanczos-Lovelock Lagrangian by $P^{cd}_{ab} \equiv (\partial L/\partial R^{ab}_{cd})$. More remarkably, in any static spacetime, one can prove an equipartition law similar to the one in Eq. (1) and identify the "Avogadro number of the spacetime". It turns out that we now obtain a relation very similar to the second equation in Eq. (1) relating the energy E in a region $\mathcal{V}$ of the spacetime to an integral over the boundary $\partial\mathcal{V}$ of that region:

$$E = \frac{1}{2} k_B \int_{\partial\mathcal{V}} T \, \frac{dN}{dA} \, dA \tag{2}$$

In this sense, gravity is holographic and a patch of area δA in the transverse dimensions contribute a surface density of microscopic degrees of freedom given by

$$\frac{dN}{dA} = 32\pi \, P^{ab}_{cd} \; \epsilon_{ab} \, \epsilon^{cd} \to \frac{1}{L_P^2} \tag{3}$$

where ϵ_{ab} is the bi normal to the surface. The second expression holds in $D = 4$ Einstein's theory telling us that every patch of area L_P^2 contributes one microscopic degree of freedom. This rigorously provable result is probably the direct link with the discrete micro structure of spacetime just as Eq. (1) was in the case of matter.

The above results were obtained starting from the field equations of classical gravity and introducing one single quantum mechanical input, viz. the existence of Davies-Unruh temperature for the local Rindler horizon as perceived by accelerated observers near every event. If this paradigm that classical gravity is an emergent phenomenon like fluid mechanics is correct, one should be able to reverse the logic and obtain the field equations from a thermodynamic extremum principle directly. I will now describe how this can be done thereby making the analogy between thermodynamics of matter and thermodynamics of spacetime complete.

3 Deformations of the spacetime solid

Let us again take a clue from the eighteenth century physics, this time from the elasticity. The central quantity in studying elastic deformations of a solid is a deformation field $\mathbf{q}(t, \mathbf{x})$ which tells you that points on the solid have been displaced by an amount $(\bar{\mathbf{x}}' - \mathbf{x}) = \mathbf{q}(t, \mathbf{x})$. The thermodynamic potentials like entropy, free energy etc. are usually taken to be quadratic functionals of the gradient of the field $\mathbf{q}$, and $\mathbf{q}$ itself when external forces are present. Extremising a suitable thermodynamic functional like entropy density, free energy density etc. will then lead to the dynamics of the solid.

Incredibly enough, the same idea works for spacetime. Any 4-dimensional vector field $q^a(x^i)$ in the spacetime can be thought of as inducing a deformation of the spacetime by the amount $q^a = \bar{x}^a - x^a$. In general, such a deformation does not cost any entropy except when it distorts a null surface thereby changing the causal structure. When the null surfaces are distorted, the local Rindler observers will find that the amount of information accessible to them has changed. This suggests that we should attribute an entropy density with every null vector field $n^a(x)$ (with $n^a n_a = 0$) in the spacetime since one can associate a deformation of a small patch of null surface in the direction of the null vector which is normal to it. It can be shown that this idea indeed works [5]. We can associate a quadratic expression as a thermodynamic potential for the deformation of the null surface along n^a:

$$\Im[n^a] = \Im_{\rm grav}[n^a] + \Im_{\rm matt}[n^a] \equiv -4P_{ab}^{cd}\nabla_c n^a \nabla_d n^b + T_{ab} n^a n^b\,, \qquad (4)$$

where P_{ab}^{cd} is a tensor having the symmetries of curvature tensor and is divergence-free in all its indices and T_{ab} is a divergence-free symmetric tensor. The divergence-free condition generalizes in a natural fashion the idea of elastic *constants* to the spacetime. As can be easily verified [4], demanding the extremum of $\Im[n^a]$ for all null vectors leads to the gravitational field equation for a theory for which T_{ab}

is the source stress-tensor and P^{ab}_{cd} is the entropy tensor, which counts the surface density of microscopic degrees of freedom (see Eq. (3)). Let us explore the physics behind it a little more especially the concept of distorting a spacetime solid.

The most natural way of describing the distortion of a null surface $\mathcal{S}$ (taken to to be described by x^1 = constant in a suitable set of coordinates) associated with a null congruence ℓ^a is in terms of the Weingarten coefficients defined as follows. Because $\boldsymbol{\ell}\cdot\nabla_\mu\boldsymbol{\ell} = (1/2)\partial_\mu\boldsymbol{\ell}^2 = 0$ (where Greek letters like μ run through 0,2,3) the covariant derivative of $\boldsymbol{\ell}$ along vectors tangent to $\mathcal{S}$ is orthogonal to $\boldsymbol{\ell}$ and hence is (also!) tangent to $\mathcal{S}$. Therefore $\nabla_\alpha\boldsymbol{\ell}$ is a vector which can be expanded using the coordinate basis $\boldsymbol{e}_\mu = \partial_\mu$ on $\mathcal{S}$. Writing this expansion with a set of coefficients (called Weingarten coefficients) $\chi^\alpha{}_\beta$ we have

$$\nabla_\alpha\boldsymbol{\ell} \equiv \chi^\beta{}_\alpha\partial_\beta = \chi^\beta{}_\alpha\boldsymbol{e}_\beta; \qquad \nabla_\alpha\ell^\beta = \chi^\beta{}_\alpha \tag{5}$$

The gravitational entropy is then just:

$$\Im[\ell^a] = -4P^{\alpha\beta}_{\mu\nu}\chi^\mu{}_\alpha\chi^\nu{}_\beta \to -[Tr(\chi^2) - (Tr\chi)^2] \tag{6}$$

The second expression is for Einstein's theory, which is similar to the well-known term involving the extrinsic curvature in ADM action.

The gradient $\nabla_a q_b$ of a vector field can always be separated into a symmetric part $S_{ab} = (1/2)\nabla_{(a}q_{b)}$ and antisymmetric part $F_{ab} = (1/2)\nabla_{[a}q_{b]}$ both of which have simple physical meanings: S_{ab} induces a distortion in the metric field given by $\delta g_{ab} = 2S_{ab}$ while F_{ab} leads to a current $J^a = \nabla_b F^{ba}$ which is conserved due to the antisymmetry of F^{ab}. In terms of S_{ab} and F_{ab}, the gravitational entropy density in Eq. (4) becomes:

$$\Im_{\rm grav}[n^a] = 2P^{ibjd}S_{ij}S_{bd} + P^{abcd}F_{ab}F_{cd} \tag{7}$$

When n_j is a pure gradient, F_{ij} will vanish and one can identify the first term with a structure like Tr(K^2)- (Tr K)^2. On the other hand, when n_a is a local Killing vector, the contribution from S_{ij} to the entropy density vanishes and we find that the entropy density is just the square of the antisymmetric potential F_{ab}. For a general null vector, both the terms contribute to the entropy density.

This suggests that we construct a symmetric $\mathcal{S}^{ad} = 4P^{bacd}S_{bc}$ and antisymmetric $\mathcal{F}^{ad} = 2P^{ijad}F_{ij}$ tensors using the entropy tensor P^{abcd} of the theory. Field equations of the theory, arising from extremising the entropy density in Eq. (7) then reduces to the demand that

$$q_a\nabla_b\mathcal{F}^{ba} = T_{ab}\,q^a q^b \tag{8}$$

should hold for all null vectors q^j which satisfies the condition $S_{ab} = 0$ at a given event x^i. This criterion is satisfied by the local Killing vector corresponding to translations along the proper time of the Rindler observer located at the given event. (Obviously, the condition will hold only at a point but that is all we need since we can construct different Rindler observers at different events.) The right hand side, for null vectors q_a, is proportional to the entropy density of matter while the left hand side, proportional to $q_i J^i$, is essentially the gravitational entropy density obtained from the Noether current of the theory. This equation can be interpreted as an entropy balance between that of matter and that of a null surface.

When $S_{ab} \neq 0$, the distortion field induces a metric deformation and the above entropy balance equation holds if we take away this contribution. In this case:

$$q_a \nabla_d (\mathcal{F}^{ad} - \mathcal{S}^{ad}) = T^{ad}\, q_d\, q_a \tag{9}$$

The right hand side is the matter entropy density which is balanced by the gravitational entropy density on the left where we have subtracted the contribution $\mathcal{S}^{ad}$ arising from the metric distortion. (When we consider q_a which satisfies the local Killing condition, this term will vanish.) In the case of Einstein's there the situation is even simpler. In this case $\mathcal{S}^a_b = S^a_b - \delta^a_b S$ and $\mathcal{F}^{ab} = F^{ab}$ so that Eq. (8) becomes something very similar to the one in electrodynamics: $q_a \nabla_b F^{ba} = T_{ab}\, q^a q^b$.

4 Conclusions

The above arguments show that we obtain a fairly simple description of spacetime by studying the entropic response of the spacetime to distortions induced by null vector fields. This makes the analogy between material science and gravitational dynamics quite rigorous and provides a strong case for treating classical gravity as an emergent phenomenon like elasticity or fluid mechanics.

References

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