Losing information outside the horizon[1]

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Abstract

If a system falls through a black hole horizon, then its information is lost to an observer at infinity. But we argue that the *accessible* information is lost *before* the horizon is crossed. The temperature of the hole limits information carrying signals from a system that has fallen too close to the horizon. If we attempt to bring the system back to infinity, acceleration radiation destroys the information. For systems in string theory where we pack information as densely as possible, this acceleration constraint is found to have a geometric interpretation. Thus in theories of gravity we should measure information not as a quantity contained inside a given system, but in terms of how much of that information can be reliably accessed by another observer.

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1 Introduction

The notions of gravity, entropy and information form a closely linked triangle. It is plausible that unlocking the mysterious relations in this triangle will provide a deep insight into the nature of spacetime and quantum theory.

Consider a system that falls towards a black hole. If the system crosses the horizon, then its information will be lost to an outside observer. But in some approaches to the black hole problem, the region outside the hole is supposed to give a complete and self-consistent description of physics. The idea of black hole complementarity is one such situation, where one assumes that the infalling observer would get destroyed at the horizon (and have its information re-radiated to infinity); it is only in a second complementary description that he falls through the horizon [1, 2]. Recently efforts have been directed to showing that the backreaction of Hawking radiation may be severe enough to prevent a shell from falling through its horizon [3]. In string theory one finds that black hole microstates do not have regular horizons; instead they are 'fuzzballs' [4].

In such situations, one may think that the information in an infalling bit would be preserved until the infalling object reaches the horizon (or the surface of the fuzzball). But in this essay we will show that the information available to the observer at infinity actually starts losing its fidelity as the system falls towards the horizon, and is effectively wiped out before the horizon is crossed.

Consider a system having mass m, falling radially from rest at infinity, towards a black hole with metric

$$ds^2 = (1 - \frac{2M}{r})dt^2 + \frac{1}{1 - \frac{2M}{r}}\, dr^2 + r^2\, d\Omega_2^2 \tag{1}$$

When the system has reached a radius $\bar{r}$, its inward proper velocity is $U^r = \frac{dr}{d\tau} = -\sqrt{\frac{2M}{\bar{r}}}$. For $\bar{r} \approx 2M$, we get $U^r \approx -1$, so the system is falling in very fast indeed. Now suppose the system tries to send the information about its state to the observer at infinity, by encoding this information in the spin of an emitted photon. By local momentum conservation, at most $\frac{1}{2}m$ of the system's energy can be given to this photon. This photon is redshifted as it reaches infinity, arriving with an energy

$$E_\gamma \approx \frac{m}{4}\,\frac{(\bar{r} - 2M)}{2M} \tag{2}$$

But if $E_\gamma \lesssim kT$, where T is the temperature of the hole, then this information carrying photon cannot be distinguished from the photons in the thermal bath of Hawking radiation, and we cannot hope to recover the information of the system. Thus we find that the information in the system is effectively lost when

$$\frac{\bar{r} - 2M}{2M} \sim \frac{4kT}{m} \tag{3}$$

As an example, if the system has energy $m = 20\, kT$, then this condition is $\bar{r} - 2M \sim 0.4M$.

But one may argue that we do not need to have the system send back its information using its own internal energy; the observer at infinity is in possession of infinite resources, and can just send a device to scoop up the system and bring it back to infinity, where he can measure its state at leisure. Here we will encounter the following difficulty: to prevent the system at position $\bar{r}$ (and $U^r \approx -1$) from falling past the horizon, we need some minimum acceleration a to be maintained for a certain proper time $\Delta\tau$. This acceleration leads to Unruh radiation being felt by the system [5], and its information carrying bits can then get knocked out of their state into new states. Thus we again find a degradation of the information in the system.

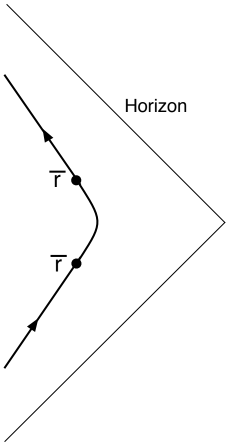

Figure 1: The trajectory of the system in the $t-r$ plane. We consider free infall until the position $\bar{r}$, then a period $\Delta\tau$ of constant acceleration a which returns us to $\bar{r}$ with velocity reversed, then free motion again out to infinity.

To compute this effect, we let the information be encoded in a 2-level system with energy gap ΔE and a coupling $H_{int} = g\hat{O}\hat{\phi}$ to a massless scalar field ϕ. Letting $g\langle\psi_2|\hat{O}|\psi_1\rangle \equiv \alpha$, we find that the probability per unit proper time for the system to get exited by absorbing a $\hat{\phi}$ quantum from the Rindler bath is

$$\Gamma = \frac{|\alpha|^2 \Delta E}{2\pi} \frac{1}{e^{2\pi a^{-1}\Delta E} - 1} \tag{4}$$

We let the system be in free fall till radius $\bar{r}$ as before, then maintain a constant a until the time that we return at position $\bar{r}$ with the sign of U^r reversed; then we let the system coast back to infinity under free motion. For such a path (shown in fig.1) we find that [6]

$$\Delta\tau = \frac{2}{a}\sinh^{-1}\frac{1}{2[(\bar{r}-2M) - a^{-1}]} \tag{5}$$

The probability of excitation is $P = \Gamma\Delta\tau$, and we should choose a to minimize P. If we take a too large (sudden turnaround) then we are almost certain to change the state of the 2-level system. If we take $a \lesssim [4(\bar{r} - 2M)]^{-1}$, then we fall through the horizon. Assuming $|\alpha|^2$ is not parametrically different from unity, one finds that $P \sim 1$ if

$$\bar{r} - 2M \sim \frac{1}{\Delta E} \tag{6}$$

Let us compare this to (3). If we let the energy scale ΔE of the system be $\sim m$, and note that $T = \frac{1}{8\pi M}$, then we find that the two estimates of the critical radius are of the same order. This is interesting, because it is not a priori obvious why two such different processes of recovering information should lead to the same condition.

It remains true however that the critical radius we have obtained depends on the details $\Delta E, \alpha$ characterizing the system. If we are to hope for a universal condition, then we should make our system using the objects present in a full theory of quantum gravity; in this case the couplings and energy levels would be presumably related to the gravitational coupling and density of states in gravity.

Thus consider string theory, where we compactify 6 directions on small circles. For our infalling system we take a string wrapped n_1 times around one of the circles. We add n_p units of momentum along the string; this momentum is carried by transverse vibrations $w(t-y)$ where y is the coordinate along the circle. Since the momentum can be partitioned among different harmonics in many ways, we find $N = Exp[S_{ex}]$ possible states, where

$$S_{ex} = 2\sqrt{2}\pi\sqrt{n_1 n_p} \tag{7}$$

turns out to agree with the expression for the entropy of an extremal black hole with charges n_1, n_p [7]. Thus this vibrating string - called the NS1-P system in string theory – allows us to store information very densely; choosing one of the allowed N states stores $\ln N = S_{ex}$ bits of information. It is plausible that systems like these (whose entropy agrees with the entropy of a black hole with the same mass and charges) would be the most dense information storing devices in any complete theory of quantum gravity.

This time we know the energy levels of our system – they are just the vibration levels of the string, described by left and right moving transverse deformations $w(t \pm y)$. For the scalar field ϕ we can take any one of the gravitons h_{ij} with indices in the 6 compact directions. The coupling of these scalars to the vibrations has the form $S_{int} = C \int dtdy\ \phi\, \partial_{t-y}w\, \partial_{t+y}w$, where the constant C was computed in [8]. When our system accelerates, the coupling S_{int} can excite it to any other energy level, so we have to perform a weighted sum over final energy levels, getting a factor $\int dE_f\, \rho(E_f)\, |\alpha_{i\to f}|^2 \equiv Q$.

We now have all the tools available to compute the excitation rate, but at this point we note that the result of this computation can be already extracted from known results. Suppose we send a beam of ϕ quanta onto our NS1-P system in flat space, and compute the absorption cross section σ. The couplings and level density again appear in the combination Q. Thus for our accelerating system we can package all the needed system characteristics into its cross section σ. But σ for the NS1-P system was computed in [8], and has a surprisingly simple form; we have $\sigma = A_{ex}$, where A_{ex} is the horizon area of a black hole with charges n_1, n_p. Putting these results into the above computation of excitation by acceleration, the critical radius for information degradation ($P \sim 1$) is found to be

$$\bar{r} - 2M \sim r_{ex} \tag{8}$$

where $r_{ex} \sim A_{ex}^{\frac{1}{2}}$ is the radius of the small extremal hole that would have charges n_1, n_p in string theory. The result (8) is found to hold even if we use many other kinds of charges and consider holes in different dimensions D.

Thus we again see a remarkable convergence of two different computations: if we take a system that packs information as densely as possible in a full theory of gravity, then the critical radius determined by acceleration radiation agrees with the radius where a black hole with the quantum numbers of the system would be big enough to touch the horizon.

This agreement between the results of apparently different estimates suggests a change in our perspective on information in theories of gravity. First, we should think in terms of how much information we can store for *given energy*. Second, we should measure information not as a quantity contained in a given system, but in terms of how much of that information can be reliably *accessed* by another observer. If the system and the observer are separated by a gravitational potential, then there is a reduction in the fidelity of the information of the system from the viewpoint of the observer, even though the system has not crossed a horizon. This degradation of information becomes infinitely strong as a horizon is approached, so we should not think of the information as lost 'suddenly' when the horizon is crossed, but as a gradual loss when the horizon is approached. In fact we do not need a horizon to have information degradation, which ties up well with the discovery in string theory that energy eigenstates do not form horizons, but rather form horizon sized quantum 'fuzzballs'.

Acknowledgements

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