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Gravity from Local Lorentz Violation*

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Abstract

In general relativity, gravitational waves propagate at the speed of light, and so gravitons are massless. The masslessness can be traced to symmetry under diffeomorphisms. However, another elegant possibility exists: masslessness can instead arise from spontaneous violation of local Lorentz invariance. We construct the corresponding theory of gravity. It reproduces the Einstein-Hilbert action of general relativity at low energies and temperatures. Detectable signals occur for sensitive experiments, and potentially profound implications emerge for our theoretical understanding of gravity.

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Gravity is curvature of spacetime, and gravitational waves are ripples in its fabric that transport energy and momentum density. The ripples propagate at the speed of light, so their quanta, the gravitons, are massless. Remarkably, these features of the ripples contain enough information to reconstruct the full theory of general relativity. Indeed, one derivation of the Einstein equations is to start with the free theory for a massless field $h_{\mu\nu}$ propagating in Minkowski spacetime, and then to impose a self-consistent coupling to the energy-momentum tensor $T_{\mu\nu}$. The self-consistency requirement leads uniquely to the Einstein equations, and the resulting series of corrections to the Minkowski metric sums to give the Riemann metric and the familiar spacetime geometry [1, 2].

In this derivation, the reason for starting with a symmetric field $h_{\mu\nu}$ is readily understood, since it is needed in the action to couple to the symmetric energy-momentum tensor $T_{\mu\nu}$. But why are the gravitons massless?

Masslessness is often taken to be the consequence of a symmetry. In quantum electrodynamics the masslessness of the photon is normally attributed to gauge invariance, or symmetry under local changes of phase. In quantum chromodynamics, the theory of the strong interaction, masslessness of the gluons is likewise attributed to a gauge invariance, albeit a nonlinear one. In general relativity, the masslessness of gravitons can be traced to symmetry under active diffeomorphisms: no diffeomorphism-invariant mass term exists.

Nature and mathematics allow, however, for an alternative reason why a field might be massless. Surprisingly, this alternative explanation involves the breaking of a symmetry rather than its existence. A general result, the Nambu-Goldstone theorem [3], states under mild assumptions that there must be a massless particle whenever a continuous global symmetry of an action isn't a symmetry of the vacuum. This result is readily understood for the simple case of an action with a kinetic term K and a nonderivative potential V. The vacuum solution has zero K and is a minimum of V. By assumption, a symmetry of the theory transforms any given minimum of V to a different minimum, so V has at least one flat direction. Small vibrations perpendicular to the flat direction are massive modes, with mass related to the curvature of the potential at the minimum, but vibrations parallel to it are massless because the potential is flat. The corresponding massless particles are called Nambu-Goldstone modes.

In this essay, we show that an alternative description of gravity can be constructed from a symmetric two-tensor without the assumption of masslessness. In this picture, masslessness is a consequence of symmetry breaking rather than of exact symmetry: diffeomorphism symmetry and local Lorentz symmetry are spontaneously broken, but the graviton remains massless because it is a Nambu-Goldstone mode. Remarkably, the Einstein-Hilbert action of general relativity is recovered at low energies and temperatures. However, the new theory is the same as general relativity only in this limit, so differences between the two theories can emerge under suitable circumstances. It follows that the origin of the graviton's masslessness is an issue with potentially profound theoretical implications and is experimentally testable.

To derive the new theory, we adopt the construction beginning with a basic action in Minkowski spacetime and then imposing self-consistency of the coupling to the energy-momentum tensor. The cardinal object in the theory is a symmetric two-tensor, denoted by $C_{\mu\nu}$. The Lagrange density for the basic theory is

$$\mathcal{L}_C \;=\; \tfrac{1}{2} C^{\mu\nu} K_{\mu\nu\alpha\beta} C^{\alpha\beta} - V(C^{\mu\nu} C_{\mu\nu} - c^2). \tag{1}$$

Here, $K_{\mu\nu\alpha\beta}$ is the usual quadratic kinetic operator for a massless spin-2 field, V is a potential with the functional form shown, and c^2 is a real number. This theory is invariant under local Lorentz transformations and under diffeomorphisms.

The vacuum is found by minimizing V. The functional dependence of V ensures that the cardinal field has a constant nonzero vacuum value $C_{\mu\nu} = c_{\mu\nu}$, where $c^{\mu\nu} c_{\mu\nu} = c^2$. This spontaneously breaks local Lorentz and diffeomorphism invariances. The massless Nambu-Goldstone fields are the excitations $\delta C_{\mu\nu} \equiv C_{\mu\nu} - c_{\mu\nu}$ about this solution, generated by the broken symmetries and maintaining the potential minimum. In what follows, we show that these excitations play the role of the graviton field $h_{\mu\nu}$.

How many massless fields are generated in this way? Since there are six Lorentz transformations (three rotations and three boosts) and four diffeomorphisms, there could in principle be up to 10 massless modes: six Lorentz modes contained in an antisymmetric field $\mathcal{E}_{\mu\nu}$, and four diffeomorphism modes contained in a field Ξ_μ. However, the kinetic operator in the Lagrange density is purely quadratric in derivatives, a feature known to imply that the diffeomorphism field Ξ_μ decouples from the action at leading order in small fields [5]. This can be confirmed directly: denoting by D_μ the covariant derivative in general coordinates in Minkowski spacetime, at leading order in small fields we find $D_\lambda C_{\mu\nu} \approx \partial_\lambda \mathcal{E}_\mu{}^\alpha c_{\alpha\nu} + \partial_\lambda \mathcal{E}_\nu{}^\alpha c_{\alpha\mu}$, which depends only on the Lorentz field $\mathcal{E}_{\mu\nu}$. We can therefore write

$$\delta C_{\mu\nu} \equiv M_{\mu\nu}{}^{\alpha\beta} \mathcal{E}_{\alpha\beta} = \mathcal{E}_\mu{}^\alpha c_{\alpha\nu} + \mathcal{E}_\nu{}^\alpha c_{\alpha\mu}, \tag{2}$$

where $M_{\mu\nu\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}c_{\nu\beta} + \eta_{\nu\alpha}c_{\mu\beta} - \eta_{\mu\beta}c_{\nu\alpha} - \eta_{\nu\beta}c_{\mu\alpha})$. Since there are only six independent fields in $\mathcal{E}_{\mu\nu}$, the 10 independent components of $\delta C_{\mu\nu}$ must satisfy four identities. For generic $c_{\mu\nu}$, these are

$$\mathrm{tr}(\delta C\ c^m) = 0, \tag{3}$$

with $m = 0, 1, 2, 3$.

The properties of the candidate graviton field $\delta C_{\mu\nu}$ are given by its equations of motion. Varying the Lagrange density with respect to the independent degrees of freedom $\mathcal{E}_{\mu\nu}$ gives $M^{\mu\nu\rho\sigma}K_{\mu\nu\alpha\beta}\delta C^{\alpha\beta} = 0$. These equations can be solved using Fourier decomposition. The solutions obey the usual massless wave equation,

$$\partial^\lambda\partial_\lambda\delta C_{\mu\nu} = 0, \tag{4}$$

subject to the Hilbert condition $\partial^\mu\delta C_{\mu\nu} = 0$. The latter imposes another four constraints on $\delta C_{\mu\nu}$. This means only two combinations of the massless Lorentz modes $\mathcal{E}_{\mu\nu}$ propagate.

These results are promising: they show that for weak fields the new theory contains the expected number of modes obeying the massless wave equation. However, if the new theory indeed contains general relativity, the weak-field limit of the new theory must match exactly the usual description of a massless graviton field $h_{\mu\nu}$ in Minkowski spacetime. How does this match arise?

To answer this, let's start with the Lagrange density for a free massless graviton field $h_{\mu\nu}$,

$$\mathcal{L}_h = \tfrac{1}{2}h^{\mu\nu}K_{\mu\nu\alpha\beta}h^{\alpha\beta}. \tag{5}$$

Initially, $h_{\mu\nu}$ has 10 degrees of freedom. Since the theory is diffeomorphism invariant and there is no potential V to drive spontaneous symmetry breaking, diffeomorphisms can be used to make four coordinate choices. These choices represent a gauge restriction on $h_{\mu\nu}$. Most studies of gravitational waves adopt the so-called transverse-traceless gauge, but here we pick instead a different gauge that yields a direct match to the new theory. For generic $c_{\mu\nu}$, we choose the four conditions

$$\mathrm{tr}(h\ c^m) = 0 \tag{6}$$

with $m = 0, 1, 2, 3$. A short calculation verifies that this is an acceptable gauge choice.

The behavior of the graviton field $h_{\mu\nu}$ is fixed by the equations of motion, $K_{\mu\nu\alpha\beta}h^{\alpha\beta} = 0$. These can be solved by performing a Fourier decomposition and using the specific gauge choice. The solutions obey the Hilbert condition $\partial^\mu h_{\mu\nu} = 0$ and satisfy the wave equation

$$\partial^\lambda \partial_\lambda h_{\mu\nu} = 0. \tag{7}$$

They describe two graviton degrees of freedom propagating as massless waves, as usual.

We can now see how the match between the two theories arises. Both start with symmetric fields, $h_{\mu\nu}$ and $\delta C_{\mu\nu}$. In the usual picture, diffeomorphism symmetry permits some components of $h_{\mu\nu}$ to be fixed, while in the new theory only six components of $\delta C_{\mu\nu}$ are independent from the start. In both cases, the Hilbert condition holds and the two graviton degrees of freedom obey the massless wave equation. At this level, the equations for the two theories are in direct correspondence, even though the symmetry structures of the two theories are radically different.

How can the full nonlinear Einstein equations be obtained? In the usual $h_{\mu\nu}$ theory, this can be done by adding a coupling to the energy-momentum tensor $T_{\mu\nu}$ and imposing self-consistency of its conservation. An elegant single-step procedure has been given by Deser [2]. For our purposes, the key feature of this derivation is its gauge independence, which means our specific gauge choice (6) for $h_{\mu\nu}$ still allows construction of the Einstein-Hilbert action. Since the two theories match when this gauge is used, it follows that the new theory contains general relativity too, via the same construction.

We have thus come to the surprising result that the conventional description of gravity via general relativity follows as a consequence of spontaneous breaking of local Lorentz symmetry. This result may seem paradoxical, since the existence of local Lorentz symmetry is among the underlying foundations of general relativity. However, the paradox is superficial: although the new theory contains general relativity, the two theories are different.

Do the differences lead to physical effects? These are still being explored, but the answer is definitely yes. The new theory includes various corrections to the action of general relativity. Effects arise in both the pure-gravity sector and the matter sector, and also from the structural differences. Let's take a brief tour of some results obtained so far.

In the pure-gravity sector, the new theory has subleading corrections to the Einstein equations in vacuum, which generate potentially observable effects in sensitive experiments. For nearly Minkowski spacetimes and hence in laboratory or solar-system tests, including gravitational-wave experiments, the effects are nonzero but small because they enter at higher order in small fields. However, in more extreme contexts such as black holes or the early Universe, the deviations from general relativity can be significant.

Effects in the matter sector arise because the coupling of the cardinal field to the energy-momentum tensor generates an extra term $c_{\mu\nu}T^{\mu\nu}$ in the action. This has potential physical implications for the behavior of matter in laboratory searches for Lorentz breaking, including ones that aren't traditionally viewed as gravitational tests. For example, terms of this type can modify signals in clock-comparison experiments, in neutrino oscillations, and in tests with light, among others. A framework for the comprehensive treatment of such effects exists [4], and numerous searches are presently under way.

Structural differences between the two theories include the nonzero vacuum value $c_{\mu\nu}$, which may help explain dark energy, and effects at energies and temperatures near the Planck scale. At high energies, the extra degrees of freedom in the cardinal field become relevant because the modes perpendicular to the flat direction of the potential V can be excited. High temperatures restore the local Lorentz symmetry because V acquires corrections [6] that change its shape, making stable a zero value for $C_{\mu\nu}$. The former gravitons then become oscillations about this new minimum and acquire large masses. This may have profound implications for the very early Universe.

The quantum properties of gravity are also significantly changed, which may shed light on the perennial problem of the consistent quantization of gravity. For example, recent results for vector fields [7] can be adapted for the new theory to show that nonpolynomial and superficially unrenormalizable potentials V can become renormalizable and stable when quantum corrections are included. These theories have spontaneous Lorentz violation, so the requirements of renormalizability and stability of the basic theory (1) suffice to ensure a massless graviton without diffeomorphism symmetry, a surprising result.

In our derivation leading to the new description of gravity, the geometrical properties of the theory are obscured, just as the geometry is obscured in the similar construction of general relativity. A geometrical formulation should exist and is actively being sought. It must be both beautiful and unconventional, describing gravity as spacetime curvature and tying spacetime curvature to rippling cardinal fields coupled in a self-consistent way.

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